The Condor UAV System
A Concept Study
DENNIS ANDRÉ RAMIREZ ALVAREZ
Sammanfattning


Resultatet blev en relativt liten drönare som döptes till The Condor och fick en vikt på 25,6 kg, med ett vingspann på 2,5 m och som opererar på 3500 m flyghöjd med en marschfart på 81 knop. Drönarens räckvidd är 70 nautiska mil och den kan därtöver operera i upp till sex timmar. Den bör klara av en landningsbana på 300 m. Som vingprofil valdes NACA 1412 med en maximal tjocklek och camber på 12 % respektive 1 % av kordalängden. För stabilisatoren valdes den symmetriska profilen NACA 0012.

En så kallad ”constraint analysis” genomfördes för fastställande av motorval och vingbelastning. Motorn som valdes blev en 3.1 hästkrafters pistongmotor från Ricardo. Flygplanskroppens dimensioner utformades endast för att få plats med nyttolasten och ingen noggrannare analys genomfördes. Den blev 2,3 m lång med en maximal diameter på 0,3 m.

Abstract

In this degree project in aerospace engineering, a preliminary design of a UAV (Unmanned Aerial Vehicle) was performed. The UAV was intended to be used as a complement to the Swedish maritime administration’s helicopters, which cannot operate under limited visibility conditions. Its main mission would consist of surveillance. The UAV was therefore designed for some series criteria that were based on the customers’ requirements. The primary literature that was used was John D. Andersons *Aircraft performance and design*. Otherwise, historical statistical data from other aircraft was used and numerous assumptions were made.

The result was a relatively small UAV named The Condor, weighing 25.6 kg with a wingspan of 2.5 m and operational in an altitude of 3500 m with a cruise speed of 81 knots. The UAV’s range is 70 nautical miles and is also able to operate in up to six hours. It should be able to manage a 300 m long runway. The chosen wing profile was the NACA 1412 with a maximal thickness and camber of 12 % and 1 % of the chord length, respectively. As for the stabilizer, the symmetric wing profile NACA 0012 was chosen.

A so called constraint analysis was performed in order to determine the engine choice and the wing loading. The chosen engine was a 3.1 horsepower piston engine provided by Ricardo. The dimensions of the fuselage were designed only to fit the payload and no detailed analysis was done. It became 2.3 m long and with a maximal diameter of 0.3 m.
Preface
This thesis was fulfilled at KTH Royal Institute of Technology in Sweden, from January to May 2016. Firstly, I would like to thank Fredrik Edelbrink for his tutoring, inspiration and support during this project. I would also thank Fredrik Lundell and professor Christer Fuglesang for providing this thesis opportunity and for their participation in this project. Furthermore, I would like to thank Hanna, Maria, Jens, John and Ahmed for their participation in the seminars, sharing their ideas and thoughts.
Lastly, may this thesis be useful for readers and future research about similar topic or any other related field.

Stockholm, 30 April 2016

Dennis André Ramirez Alvarez
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1 Introduction

In this degree project in mechanical engineering, a concept study of a UAV (Unmanned Aerial Vehicle) will be performed in order to develop a preliminary design of a UAV that will live up to a particular technical specification. The UAV will be used for search missions, replacing helicopters in fog or in difficult weather, as the risk of collisions or accidents with helicopters increases under these circumstances. The main tasks of the UAV are surveillance and loiter to identify humans in the waters, with the use of an infrared camera.

2 Mission specification

The study is initiated by estimating and specifying numerous requirements that are crucial to accomplish the mission. These were specified by making several assumptions. Some detailed explanations to these assumptions and requirements are motivated.

2.1 Mission profile

The UAV will have several flight phases, from start up to shut down, and are important to define in order to design a UAV that will be optimal for the specific use. To accomplish this, a mission profile is defined and illustrated in Figure 2.1. The mission profile is subdivided in several mission segments, such as taxi, loiter and landing. This information will be used later when designing the UAV.

![Figure 2.1. The UAV's mission profile.](image)

2.2 Requirements

The requirements are presented in Table 2.1 and are explained in more detail further on. These parameters are taken into account through the project in order to design an aircraft able to satisfy the customer’s demands.
Table 2.1. Requirement specification.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Specification</th>
</tr>
</thead>
<tbody>
<tr>
<td>Engine type/propulsion</td>
<td>2-stroke engine/propeller combination</td>
</tr>
<tr>
<td>Range</td>
<td>70 nm</td>
</tr>
<tr>
<td>Maximum altitude</td>
<td>5000 m</td>
</tr>
<tr>
<td>Cruising altitude</td>
<td>3500 m</td>
</tr>
<tr>
<td>Cruise speed</td>
<td>81 knots</td>
</tr>
<tr>
<td>Stall speed</td>
<td>48 knots</td>
</tr>
<tr>
<td>Endurance</td>
<td>6 h</td>
</tr>
<tr>
<td>Rate of climb</td>
<td>300 m/min</td>
</tr>
<tr>
<td>Take-off and landing</td>
<td>300 m</td>
</tr>
<tr>
<td>Payload</td>
<td>8 kg</td>
</tr>
<tr>
<td>Equipment</td>
<td>Infrared camera for surveillance</td>
</tr>
</tbody>
</table>

2.2.1 Power plant and propulsion

The specified engine is a two-stroke spark ignition engine combined with a propeller for the propulsion. The main reason behind the choice of a piston engine and propeller combination is the propulsion efficiency. The UAV will operate at relatively low velocity and low altitude, and it is difficult for jet engines to acquire high efficiency at low velocities. Also, jet engines have a high maintenance cost.

2.2.2 Range

The UAV will operate up to 20 nautical miles off the outer islands along the Swedish coast line. In order for the UAV to fulfill its mission, it will be designed to have a range of 70 nautical miles. A good margin is included, since some search missions might require more range.

2.2.3 Altitude

Since the air density decreases as altitude increases, the drag will be lower at a higher altitude. The Falco UAV system [1], is used as a reference to determine the cruising and maximum altitude. Thus, the UAV is expected to have a cruising altitude of 3500 m and a maximum altitude of 5000 m.

2.2.4 Cruise and max speed

The goal of the search missions is to find a person within one hour after receiving the emergency alarm. Considering the range of 20 nautical miles in which the UAV will operate, a cruise speed of 81 knots will get the UAV to the target point within 15 minutes. Furthermore, if the emergency is of higher urgency, a max speed of 108 knots will get the UAV to the target point within roughly 11 minutes. It should be noted that the time duration for the UAV to start, taxi and climb is not accounted for here.

2.2.5 Stall speed

The aircraft needs a minimum speed in order to generate sufficient lift force to overcome the gravitational force which is applied due to the weight of the aircraft. This minimum speed is called the stall speed and is determined in order to approach the wings area. Due to safety and practical reasons a relatively low stall speed is desired; it should be low enough to prevent the aircraft from stalling while still being able to fly at a low velocity. These scenarios are typically for take-off and landing. The UAV will be used for
surveillance and has a maximum and cruise speed of 108 knots and 81 knots, respectively. Using the Falco UAV [1] as a reference, the stall speed is set to 48 knots.

### 2.2.6 Endurance

The mission of the UAV is to identify humans in the waters, at maximum 20 nautical miles off the outer islands. Considering the goal of rescuing a person within an hour, the surveillance mission will be a maximum of a couple of hours. Including a favorable margin for exceptions, an endurance of 6 hours will be sufficient.

### 2.2.7 Rate of climb

The rate of climb will be set to 300 m/min, once more using the Falco UAV as a reference, due to its similar properties.

### 2.2.8 Take-off and landing

The runway needed must be 300 m long for take-off and landing. The take-off will alternatively be done using a catapult. This will be more fuel-efficient and will not require a runway. Furthermore, the UAV will be equipped with a parachute as an alternative landing method.

### 2.2.9 Payload

The equipment that must be taken into consideration when deciding the payload of the UAV is the infrared camera and the parachute. The weight of the camera is estimated to be 3 kg, including the mount. The parachute with its packaging is estimated to weigh 5 kg. Consequently, the payload needed is 8 kg.

### 3 Weight

The weight of the aircraft is one of the fundamental processes to take into account in the conceptual design analysis. This section provides the preliminary estimation of the gross weight of the airplane, using several weight components of the aircraft and the famous Breguet Range Equation.

#### 3.1 Weight estimation

The weights that will be taken into account are the take-off gross weight, empty weight, payload weight (cargo and passengers), crew weight and fuel weight. This can be presented in one equation:

\[
W_0 = W_e + W_{pl} + W_c + W_f.
\]  

So far the only known parameters are \(W_{pl}\) and \(W_e\). The UAV is by definition unmanned, hence, \(W_0 = 0\). The payload weight is specified as 8 kg. The empty weight is unknown, and will be calculated later on.

The takeoff gross weight of the UAV can be rewritten [2] and become

\[
W_0 = \frac{W_e + W_{pl}}{1 - \frac{W_f}{W_0}}.
\]  

#### 3.1.1 Weight fractions

The mission profile, shown in Figure 2.1, is divided into different segments and is helpful in order to estimate the fuel weight. The weight of the airplane at the end of a segment in the mission profile divided by the weight of the airplane at the beginning of the segment, is called mission segment weight fraction [2]. In Table 3.1, the mission segment weights for the UAV are defined.
Table 3.1. The mission segment weights of the UAV

<table>
<thead>
<tr>
<th>Segment</th>
<th>Weight Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_0$</td>
<td>Take-off gross weight</td>
</tr>
<tr>
<td>$W_1$</td>
<td>Weight of the UAV at the end of take-off</td>
</tr>
<tr>
<td>$W_2$</td>
<td>Weight of the UAV at the end of climb</td>
</tr>
<tr>
<td>$W_3$</td>
<td>Weight of the UAV at the end of cruise</td>
</tr>
<tr>
<td>$W_4$</td>
<td>Weight of the UAV at the end of loiter</td>
</tr>
<tr>
<td>$W_5$</td>
<td>Weight of the UAV at the end of cruise</td>
</tr>
<tr>
<td>$W_6$</td>
<td>Weight of the UAV at the end of loiter</td>
</tr>
<tr>
<td>$W_7$</td>
<td>Weight of the UAV at the end of landing</td>
</tr>
</tbody>
</table>

The weight fractions can then be represented by the following equation,

$$\frac{W_7}{W_0} = \frac{W_1}{W_0} \frac{W_2}{W_1} \frac{W_3}{W_2} \frac{W_4}{W_3} \frac{W_5}{W_4} \frac{W_6}{W_5}$$

(3.3)

The weight change of the aircraft during the flight is only due to the fuel consumption. Assuming that the aircraft consumes all of its fuel, the weight of the fuel [2] may be calculated as

$$\frac{W_f}{W_0} = 1 - \frac{W_7}{W_0}$$

(3.4)

Designing the aircraft with this assumption is not safe, because the mission might be prolonged depending on several factors, such as weather conditions or traffic problems. Thus, aircrafts are designed using reserve fuel, being normally 6% of the total fuel for reserve [1]. Adjusting equation (3.4), the gives

$$\frac{W_f}{W_0} = 1.06 \left(1 - \frac{W_7}{W_0}\right)$$

(3.5)

Historical data shows that the first two fuel fractions in equation (3.3) can be estimated to 0.97 and 0.985, respectively, and the last fraction to 0.995 [3].

The instantaneous weight, $W$, of the airplane is

$$W = W_i + W_f,$$

(3.6)

where, $W_i$ denotes the weight of the airplane when the fuel tank is empty, and $W_f$ the weight of the fuel. This equation, together with some several other expressions (see Appendix 2), can be modified and become

$$R = \eta_\text{dr} \frac{L}{c} \ln \left(\frac{W_0}{W_f}\right),$$

(3.7)

which is the Breguet Range Equation for propeller driven airplanes with reciprocating engine power plants, and is derived in Appendix 2. $W_0$ is the gross weight of the UAV when the tanks are full. For the UAV, the range is specified in the requirements as $R=70$ nm $\approx 130$ km.

When deriving this equation several assumptions have been made. It is assumed that the aircraft is in steady, level flight. Also, a stationary atmosphere is assumed. Furthermore,
the aircraft velocity, the specific fuel consumption and the lift-to-drag ratio are all assumed to be constant. The third and fifth fraction, for which the UAV is cruising, can be calculated using the Breguet range equation [2]. However, this requires an estimate of the lift to drag ratio $L / D$. For a propeller driven aircraft, the most efficient cruise occurs with a velocity yielding maximum lift to drag ratio [3]. The greater the lift-to-drag ratio, the larger payload will the aircraft be able to carry over a longer time, and over a longer distance [4]. The Falco UAV, produced by Galileo Avionica, has very similar performance specifications as the one required in this project [1]. Using the Falco UAV as a reference, as well as historical data from several different aircraft types [5], the lift-to-drag ratio is estimated to

$$\left( \frac{L}{D} \right)_{\text{max}} = 13. \quad (3.8)$$

In order to use the Breguet equation, $c$ and $\eta_{pr}$ are needed. Since the UAV uses a reciprocating engine with a propeller combination, the specific fuel consumption, $c$, can be estimated to $2.02 \times 10^{-7} \text{lb} \cdot \text{s}/(\text{ft} \cdot \text{lb/s})$ [2]. The propeller efficiency, $\eta$, depends on the type of propeller and its geometry, and is provided by the manufacturer. Typical values range from 0.7 to 0.9 [6]. It will therefore be assumed that the propeller efficiency is 0.8 for all flight conditions.

Using the weight fraction from the Breguet equation (3.7), together with the estimated parameters and modifications (see Appendix), calculates the cruise segments to 0.99.

The fraction for which the aircraft is descending is based on historical data, and estimated to 0.99 [5].

As for the loiter segment, it is calculated using the endurance formula [2],

$$E = \frac{\eta_{pr}}{cV_\infty} \frac{L}{D} \ln \left( \frac{W_L}{W_a} \right)$$

which is calculated to 0.95.

The weight fraction $W_L / W_0$ can now be calculated by inserting the weight fractions into equation (3.3), obtaining

$$\frac{W_L}{W_0} = (0.97)(0.985)(0.99)(0.95)(0.99)(0.99)(0.995) = 0.880 \quad (3.10)$$

Finally, equation (3.10) into equation (3.5) gives

$$\frac{W_L}{W_0} = 0.128 \quad (3.11)$$

### 3.1.2 Takeoff gross weight and empty weight

The weights left unknown are $W_0$ and $W_e$. The empty-gross weight ratio can be expressed as

$$\frac{W_e}{W_0} = aW_0 + b \quad (4.1)$$

The constants $a$ and $b$ are chosen from historical data for a GA-single engine aircraft [5], which estimates the values to $1.543 \times 10^{-5}$ and 0.57, respectively. Using Newton-Raphson’s method, as shown in Appendix 1, together with equation (3.2) and (4.1), the takeoff gross weight is approximated numerically. Thus,

$$W_0 = 26.5 \text{ kg.} \quad (4.2)$$

Inserting the value in equation (4.1) and solving the empty weight gives

$$W_e = 15.1 \text{ kg.} \quad (4.3)$$
The fuel weight can now be calculated by using equation (3.1) and all known values, hence,

\[ W_f = 3.4 \text{ kg}. \quad (4.4) \]

### 4 Wing configuration

The wing design is a crucial part of the aircraft. The wings generate the majority of the lift force of the aircraft, and must therefore be designed with respect to the altitude, velocity and angle of attack the aircraft will have at cruise condition.

#### 4.1 Airfoil profile and flaps

The airfoil chosen for the UAV is a NACA airfoil of the four-digit series, and is generated by choosing the parameters maximal thickness and camber of the chord, which were set to 12% and 1%, respectively. Thus, the airfoil profile NACA 1412 was chosen, and is generated using the airfoil generator provided by Airfoil Tools [7]. This specific airfoil has, according to the given specification, approximately a maximal lift coefficient of 1.3 at 16° angle.

![Figure 4.1. The NACA 1412 airfoil [7].](image)

#### 4.2 High lift device

With the purpose of augmenting the lift of the UAV at low speeds, a so-called high lift device will be used in the design. This high lift device can be mounted on the trailing edge or the leading edge of the wing and are called flaps. Here, a single slotted flap will be mounted on the UAV, which increases the maximal lift coefficient by 65% [8]. Thus, the theoretical maximal lift coefficient will become 1.3 \( \times \) 1.65 = 2.145. Taking into account the three-dimensional effect of the finite aspect ratio [2], the maximal lift coefficient becomes 1.93.

#### 4.3 Wing loading and reference wing area

The wing loading \( W / S \) is the ratio between the aircraft weight and the wing area. The total wing area should be designed according to the stall speed in order to make sure that the wing will generate sufficient lift force in all flight conditions. This scenario would be obtained when the density is at its maximal value, which would be at ground level, and where the speed is at stall speed. Thus,

\[ \left( \frac{W}{S} \right)_{\text{stall}} = \frac{1}{2} C_{L,\text{max}} \rho V_{\text{stall}}^2. \quad (4.5) \]

where \( \rho \) is the air density, \( V \) the aircraft velocity and \( C_{L,\text{max}} \) is the maximal lift coefficient and is related to a specific airfoil profile. Calculating this for a ground level air density of 1.2 kg/m\(^3\), the wing loading becomes 706 N/m\(^2\). The reference wing area can now be calculated, using the equation

\[ S_{\text{ref}} = \frac{W_0}{W / S}. \quad (4.6) \]

Hence, the reference wing area is 0.368 m\(^2\).
4.4 Wing-tip vortices and aspect ratio
A characteristic feature of the wings is that they induce a flowfield which trail downstream, a vortex sheet, and are especially strong at near the wing tips [9]. A downward component is induced by the vortex sheet, and is called downwash. The vortex wake behind the wing is illustrated schematically in Figure 4.2.

![Figure 4.2. The vortex wake [9].](image)

Examining Prandtl’s lifting line theory [2], it is clear that the aspect ratio plays an important role for the induced drag; if the aspect ratio is increased by a factor of two, the induced drag is reduced by a factor of two. Additionally, the aspect ratio is defined as

\[ AR \equiv \frac{b^2}{S}, \quad (4.7) \]

where \( b \) is the wing span and \( S \) the wing planform area. Thus, the higher the aspect ratio the farther from the center of the wing the wing tips move. This is desirable because the induced wing-tip vortices are produced farther out on the wings, and therefore reduce the induced drag. Although, increasing the aspect ratio too much leads to a narrow wing, which is not desirable. Thus, a compromise must be made between the width and the induced drag of the wing. As for the UAV, the aspect ratio is set using current and historical data from 15 different aircraft models [10] as a reference, hence, the aspect ratio is set to 9.3.

5 Design criteria
This section covers the main design criteria used to acquire the initial aircraft parameters. It is vital to do this since the main goal of the UAV is to meet the requirements. Thus, several decisions will be taken in order to evolve the design of the aircraft.

5.1 Constraint analysis
The constraint analysis is used to estimate the significance of performance constraints on the design. A number of constraint, which are specified more detailed below, are plotted into a two-dimensional graph called the constraint diagram. The requirements presented in Table 2.1 are parts of the constraints. The constraints are then expressed by specific equations, which are all developed from a general equation that expresses the thrust-to-weight ratio, referred to as the “Master Equation” [11], shown in Appendix 3. Since the UAV will be using a propeller as propulsion, the expression thrust-to-weight ratio will be converted to power-to-weight ratio, using the equation

\[ \frac{P}{W} = \left( \frac{T}{W} \right) \frac{V}{\eta_p} \quad (5.1) \]

where \( V_\infty \) is the aircraft velocity and \( \eta_p \) the propeller efficiency.
5.1.1 Constant velocity turn constraint

There are several assumptions that must be made in order to calculate the CVT (constant velocity turn). It will therefore be assumed that the UAV must be able to maintain a $45^\circ$ bank angle at cruise condition. The bank angle is converted into a load factor $n$, which is equal to $1/\cos(45^\circ)$. Furthermore, the minimum drag coefficient is based on data from a GA single class with fixed gear [12], and is set to 0.03.

The Oswald span efficiency factor, $e$, is a constant used for calculations of finite wings that has not an elliptical lift distribution, which is considered to be the optimal lift distribution [11]. It can be calculated using the empirical formula for relatively straight wings [11], which in turn allows the factor $k$ in the lift induced drag term [13] to be calculated. The calculations are presented in Appendix 1.

The Nasa Atmosphere Model [14] is used to calculate the density at 3500 m, which is needed to calculate the dynamic pressure at cruise condition. The CVT constraint can then be plotted using these values.

5.1.2 Rate-of-climb constraint

The UAV is required to climb 300 m/min. Assuming this must be done at sea level, and that the aircraft’s velocity during the climb is 51 knots, which is approximately 63% of the cruising speed, the dynamic pressure is also calculated at sea level. Hence, the rate-of-climb constraint, RoC, can be plotted.

5.1.3 Take-off criteria

When defining the take-off criteria, it is assumed that the liftoff speed will be done about 50 knots, which is approximately 62% of the cruising speed. Also, the UAV is required to operate in a runway no greater than 300 m. The drag and lift coefficients for the take-off condition are set to 0.0415 and 0.7, respectively, based on historical data [12]. Lastly, the ground friction for a Type 2 runway is between 0.025 and 0.04 and is recommended to be set to 0.03 for coursework [15], which has been done in this study.

5.1.4 Maximal speed and cruising conditions

Finally, the maximal speed and cruising constraint are calculated. The assumptions made for these conditions is the altitude of 3500 m, which is the cruising condition. Though, the constraints are calculated for maximal and cruising speed, respectively.

5.2 Constraint diagram and engine selection

The calculations of the constraints were made in MATLAB and are shown in Appendix 1. The plot is presented in Figure 5.1. The y-axis is shows the power-to-weight ratio and the x-axis the wing loading.
Using the constraint diagram a design point [2] marked as a red dot in the diagram is chosen. The purpose of the design point is to obtain as low $P/W$ as possible, in order to minimize the engine size and power requirement. Thus, the point where the power-to-weight ratio is 85 W/kg is chosen. This gives a wing loading of 380 N/m$^2$. This information can now be used in order to choose an engine, since the power-to-weight ratio is known, the horsepower needed can be calculated, which is 3.0 horsepower. It will be assumed that the UAV will be using a supercharger in order to maintain a constant density of the air that flows inside the engine.

The engine chosen for the UAV is the Wolverine 3 [16], which is a 2-cylinder 2-stroke combustion engine with 3.1 horsepower, which is appropriate for the requirement of 3.0 horsepower, giving a margin of 0.1 horsepower. The engine is designed for heavy fuel such as JP-8. The length, width and height of the engine are 193, 267 and 175 mm, respectively.

6 Configuration layout

6.1 Fuel tank and payload positions

The fuel tank will be placed in the wing. As mentioned before, the engine will use JP-8 fuel which has a density of 0.840 kg/L [18]. Since the fuel capacity needed is 3.38 kg, the fuel tank must have a volume of $(3.38/0.840) \text{ L} = 4.0 \text{ L}$. The fuel tank will therefore be divided into two tanks, one in each wing. The dimension of each tank will be 2 dm x 4 dm x 0.25 dm, which will contain the required volume of fuel.

The parachute will be placed in the middle of the fuselage. As for the infrared camera, it will be placed behind the engine. The weight of the installed engine is 1.4 times the dry weight [2] and is therefore 4.48 kg.
6.2 Wing design

The taper ratio of the wing is defined as

\[ \lambda = \frac{c_t}{c_r} \] (6.1)

where \( c_t \) and \( c_r \) is the tip chord and root chord, respectively, which are illustrated in Figure 6.1.

![Figure 6.1. Illustration of the wing geometry seen from above.](image)

The main tradeoff when picking a suitable wing taper is between structural merit and satisfactory resistance to tip stall [19]. To compromise the tradeoff, the taper ratio is set to 0.5 for the UAV.

Since the aspect ratio is set to 9.3, the wing span can be calculated using equation (4.7), giving the value of 2.52 m. The wing design shape is a trapezoid, and using the formula of a trapezoidal area [2], the tip and root chord can be calculated to 0.18 m and 0.36 m, respectively. Based on the wing loading discussed in Section 5.2, the wing area is determined to be 0.684 m².

6.3 Horizontal and vertical tail geometry

The UAV will use a conventional location for the tail location, due to its relatively low structural weight and fair stability and control [2]. The shape of the tail will somewhat be the same as the wing shape, and the wing profile for both the horizontal and vertical tail will have no camber and be symmetric. Thus, the NACA 0012 airfoil will be chosen. Wings with lower aspect ratio stall at higher angles of attack than wings with higher aspect ratio. Thus, for safety reasons, the horizontal tail’s aspect ratio will be lower than 9.3. Typical aspect ratios are about 4 to 5 [20] and the UAV’s will be set to 5. As for the vertical tail, the aspect ratio usually ranges from 1.3-2.0 [2], and will be set to 1.7. The volumes for the vertical and horizontal tails are set to 0.04 and 0.7, respectively. Estimating the moment arm from the tails’ aerodynamic center to the center of gravity of the UAV, the proportions of the tails can be calculated.

<table>
<thead>
<tr>
<th>Table 6.1. Dimensions of the stabilizer.</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Area [dm²]</strong></td>
</tr>
<tr>
<td>----------------</td>
</tr>
<tr>
<td>Vertical tail</td>
</tr>
<tr>
<td>Horizontal tail</td>
</tr>
</tbody>
</table>

6.4 Center of gravity and fuselage configuration

In order to determine not only the wing placement but also the fuselage’s dimensions, the center of gravity of the UAV must be calculated. This is done by estimating the placement of each of the payload and estimating the distance from each individual payload’s center of gravity to the nose of the fuselage. Also, the weight of the payload must be estimated. The formula for center of gravity is
Given the engine size in Section 5.2, the fuselage will be sized to fit the engine since the payload’s size is small compared to the engine. The engine will be placed in the nose, and therefore, the nose’s cross sectional diameter must be larger than 267 mm. Thus, it will be set to 300 mm. The length of the engine is 193 mm, and the diameter of the camera is estimated to be 200 mm and weigh 6.3 kg, using a Controp TR-STAMP as a reference [21]. Furthermore, the length of the container in which the parachute will be stored is estimated to be 200 mm and is estimated to weigh 1.1 kg, using the SkyCat parachute as a reference [22]. Hence, the length of the fuselage is required to be larger than 743 mm. Furthermore, some space between the payloads are necessary and thus, the fuselage length is set to 2.3 m. Then, using equation (6.2), the center of gravity is determined to be 0.57 m.

With the help of the 3D CAD program Autodesk Inventor 2014, the fuselage was divided into three sections and the wetted area of the fuselage could be calculated. Adding the wetted area of the wings and stabilizer, the total wetted area was estimated to 2.96 m$^2$. Then, using the wetted area, the dynamic pressure and an estimation of the skin friction coefficient, the total skin friction drag is calculated to 8.9 N.

### 6.5 Landing gear and wing placement

The UAV will use a low-wing location due to the convenience of storing the landing gear directly into the wing box, which is one of the strongest elements in the aircraft structure. The placement of the wing is determined assuming a static margin of 7.5 %. Thus, the distance between the wing’s aerodynamic center and the center of gravity of the aircraft is 0.075 times the mean aerodynamic chord of the wing. With the wing installed, the center of gravity of the UAV changes. Thus, a new calculation including the wing is done. The weight of the wing is approximated to be 2.5 times its wing area [2], giving the result of 1.71 kg. The new center of gravity point is then calculated to be 0.66 m.

The main landing gear is placed at the same distance from the nose as the center point of the wing, which is 0.75 m. As for the nose wheel, it is arbitrarily placed 0.3 m from the nose. The main wheels and nose wheel diameters are 9.0 cm and 7.2 cm, respectively. And the widths of the wheels are 3.9 cm and 3.2 cm, respectively.

### 6.6 Refined weight estimation

The main configuration layout is now complete and an even more precise weight estimation can now be made. Approximating a weight buildup for general aviation airplanes [2], the final total weight was calculated by iterations (see Appendix 1) and resulted in 25.6 kg. Furthermore, the final fuel and empty weight resulted in 3.3 kg and 14.3 kg, respectively.

### 6.7 The design

Finally, the UAV was designed in Autodesk Inventor 2014. The figures below present the final model of the UAV.
7 Conclusions

The main objective of the project, which was to develop a preliminary design of an Unmanned Aerial Vehicle, was finally completed. The UAV, called The Condor, resulted in a light weighted, relatively small aircraft, but with favorable conditions to fulfill its mission. It should be stressed that many of the calculations were based on assumptions, since more analytical and precise calculations would require far more time and effort. An aircraft is surely not a one-man job. Furthermore, numerous parameters were based on historical data, as an attempt to be more accurate due to the lack of other information. The UAV had a final weight of 25.6 kg and an engine of 3.1 horsepower, provided by Ricardo. The wingspan and wing area were 2.52 m and 0.68 m², respectively. The fuselage was somewhat arbitrarily designed, since a more accurate design would require a far greater amount of analysis, which could not be done within the time frame.
8 References


[19] Hall, Steven Ray; Coleman, Charles P; Drela, Mark; Lundqvist, Ingrid Kristina; Spearing, S Mark; Waitz, Ian A; Young, Peter W, MIT OpenCourseWare, *Unified Engineering I, II, III & IV*. Massachusetts Institute of Technology.


Appendix

Appendix 1

close all; clc; format long

syms x;

%% Estimation of takeoff weight and empty weight

V_stall_ft=291653.54330709;%Desired stall speed

kts=88.9km/h=291653ft/h

c=2.02*10^{-7}; %specific fuel consumption [ft^{-1}]
npr= 0.8; %propeller efficiency

LD=13;%Lift-to-drag ratio

R=425328; %Range [ft]

E=6;%Endurance [h]

V_Emax=1.3*V_stall_ft;%Estimated endurance speed

W_1_0=0.97;

W_2_1=0.985;

W_3_2= exp(-c*R/(npr*LD));

W_4_3=exp(-E*c*V_Emax/(0.866*npr*LD));

W_5_4=W_3_2;

W_6_5=0.99;

W_7_6=0.995;

W_7_0=W_1_0*W_2_1*W_3_2*W_4_3*W_5_4*W_6_5*W_7_6;

W_f_0=1.06*(1-W_7_0);

%% Using Newton-Raphson method to solve equation by iterations

x_n+1 = x_n - f(x_n)/fprime(x_n)

%The equation to be solved is

W_0=(W_pl+W_c)/(1-(W_f/W_0)-(W_e/W_0))

%Where W_e/W_0=(a*W_0+b)and W_f/W_0=0.072

%Constants:

a_const=1.543*10^{-5};

b_const=0.57;

%Parameters:

W_pl=8; % Payload [kg]

W_c=0; % 0kg crew weight (UAV)

%% Newton Raphson

%Using x=W_0.
f=@(x) (x-W_pl/(1-W_f_0-a_const*x-b_const)) ;
fprime=@(x) (1-((W_pl*a_const)/(1-W_f_0-b_const-a_const*x)^2));
error=0.001;

U=1;
i=1;
U(i+1)=U(i)-f(U(i))/fprime(U(i));

while abs (U(i+1)-U(i))>error
    i=i+1;
    U(i+1)=U(i)-f(U(i))/fprime(U(i));
end

W_0=U(i+1);
W_e=W_0*(a_const*W_0+b_const);
W_f=W_0-W_e-W_pl;

%% Wing Configuration

CL_max_theo=1.3*1.65; %This constant apply for NACA profile 1412 plus 65%

rho=1.2; %Air density at ground/sea level
V_stall=88.896/3.6; %(48 kts=88.9km/h) Desired stall speed [m/s]

W_S=0.5*CL_max*rho*V_stall^2; % [N/m^2]

%% Wing area reference

S_ref=W_0*g/W_S; %[m^2]

%% Wing Geometry

AR_data=[7.32 7.73 7.19 12.3 9.78 11 11 7.84 12.78 9.2 7.91 8 10.06 9.82 7.4];
AR=round(mean(AR_data),1); %Aspect Ratio
lambda=0.5; %Taper ratio
b=sqrt(AR*S_ref); %Wing span
cr=2*S_ref/(1.5*b); %Root chord
cr=lambda*cr; %Tip chord
MAC=(2/3)*cr*(1+lambda+lambda^2)/(1+lambda); %Mean aerodynamic chord

%% Base line
%% Wing loading values
x = (0:0.2:1000); %Wing loading values

%% Constants
e=1.78*(1-0.045*AR^0.68)-0.64; %Oswald efficiency
k=1/(pi*AR*e); %Induced drag coefficient

%% Constant velocity turn
T=15.04-0.00649*3500; %Temperature at 3500m [degrees celcius]
p=101.29*((T+273.15)/288.08)^5.256; %Static pressure at 3500m [kPa]
rho_3500=p/(.2869*(T+273.1)); %Density at 3500m [kg/m^3]
V =41.67; % Cruising speed [m/s], equal to 81 kts.
q=rho_3500*V^2/2; % Dynamic pressure [Pa]
n=1/cos(pi/4); %Load factor, want to make a 45degree (=pi/4) turn.
cdmin=0.03; %Data from GA single, fixed gear class.

CVT_TW=g*(q*((cdmin./x)+(k*(n/q)^2).*x)); %Sustained turn level
CVT=(V/npr)*CVT_TW;

%% Rate-of-Climb at Sea Level
Vv=5; %RoC [m/s] (equal to 300m/min)
V2=26.2366667; %Velocity [m/s] equals 60knots, when climbing 300m/min (=5m/s)
q2=rho*V2^2/2; % Dynamic pressure at S-L for given values [N/m^2]

RoC_TW=g*(Vv/V2+(cdmin*q2)./x+(k/q2).*x);
RoC=(V2/npr)*RoC_TW;

%% Take-off
VLOF=25.2077778; %Liftoff speed [m/s]
V3=VLOF/sqrt(2);
q3=rho*V3^2/2; % Dynamic pressure for given values [N/m^2]
Sg=300; %ground run distance, [m]
cdto=0.0415; %GA Typical single fixed gear reference, mean value of [0.038 ; 0.045]
clto=.7; %GA Typical single fixed gear reference
mu=0.03; %Ground friction assumption

TO_TW=g*(VLOF^2/(2*g*Sg)+q3*cdto./x+mu*(1-q3*clto./x));
TO=(VLOF/npr)*TO_TW;
%% P/W for max speed at 3500 m altitude
V_{\max}=55.56; \text{ Max speed [m/s] equals 108 kts}
q_4=\rho_{3500}V_{\max}^2/2;

MS_{\text{TW}}=g*(q_4*cd_{\min}/x+(k/q_4)*x);
MS=(V_{\max}/npr)*MS_{\text{TW}};

%% P/W for cruising speed at 3500 m altitude
V =41.67 ; \text{ Cruising speed [m/s], equal to 81 kts.}
q_5=\rho_{3500}V^2/2;

CS_{\text{TW}}=g*(q_5*cd_{\min}/x+(k/q_5)*x);
CS=(V/npr)*MS_{\text{TW}};

%% Stall
STALL=W_S; \text{ Stall speed condition}

%% Constraint diagram
plot(x,CVT,'r',x, RoC, 'g', x, TO, 'b', x, MS, 'k', x, CS, 'm', 'LineWidth',1.5)
title('Constraint diagram');
legend('Constant velocity turn','Rate-of-Climb', 'T-O ground run', 'Max Speed', 'Cruising speed')
axis([0 1000 0 250])
ylabel('Power-to-Weight Ratio, P/W [W/kg]')
xlabel('Wing Loading, W/S [N/m^2]')
y1=get(gca,'ylim');
hold on
plot([STALL STALL],y1)

%% Design point
PW=85; \text{ Power-to-weight ratio [W/kg]}
WL=380; \text{ Wing loading [N/m^2]}
P=PW*W_0; \text{ Power [W]}
HP=P/745; \text{ Horsepower}
S=W_0*g/WL; \text{ Wing area [m^2]}

b_2=sqrt(AR*S); \text{ Wing span}
cr_2=2*S/(1.5*b_2); \text{ Root chord}
ct_2=lambda*cr_2; \text{ Tip chord}
max_thick_root=0.12*cr_2; \text{ Max thickness chord at 29.9\% of the root chord [m]}
max_thick_tip=0.12*ct_2; \text{ Max thickness chord at 29.9\% of the tip chord [m]}
thick_root_pos=0.299*cr_2; \text{ Max thickness at root chord position [m]}
thick_tip=0.299*ct_2; \text{ Max thickness at tip chord position [m]}

IV
MAC2=(2/3)*cr2*(1+lambda+lambda^2)/(1+lambda); %Mean aerodynamic chord

%% Engine fuel etc.

fuel_dens=0.84; %JP-8 fuel density [kg/L]
fuel_vol=W_f/fuel_dens; %fuel volume [L]

%% Center of gravity
%Mass and distance to c.g. of each payload object
m1=4.48; %[kg], engine
m2=6.3; %[kg], camera
m3=1.1; %[kg], parachute
x1=0.0965; %[m]
x2=0.7945; %[m]
x3=1.1945; %[m]

mwing=2.5*S; %mass of the wing [kg]
cg=(x1*m1+x2*m2+x3*m3)/(m1+m2+m3); %Center of gravity (c.g.) of the UAV
cg_wing=0.4*MAC2; %The wing's c.g.position
mac=0.25*MAC2; %The wing's mean aerodynamic center
mac_dist=cg_wing-mac; %Mean aerodynamic center distance from c.g. wing
cg_true=(x1*m1+x2*m2+x3*m3+mwing*(cg+mac_dist))/(m1+m2+m3); %c.g. from fuselage nose

VHT=0.7; %Horizontal wing volume ratio

SM=0.075; %Static margin

% % xn=SM*MAC2+cg_true; %Neutral point [m] (aerodynamic center of the plane)
% % xacwb=xn-VHT %Position of the aerodynamic center of the wing body
dist_wingac_cg=SM*MAC2;
x_wing=cg_true+dist_wingac_cg; %Position of the wing's mac from fuselage tip
x_leading_edge=x_wing-mac-(cr2-MAC2)/2; %Position of wing's leading edge from fuselage tip

%% Fuselage
%wetted area
SectionA=0.2279; %Wetted area of first section of fuselage[m^2]
SectionB=1.1134; %Wetted area of middle section of fuselage[m^2]
SectionC=0.3772; %Wetted area of third section of fuselage[m^2]
Wing_wet=2*0.5200; %Wetted area of Wing[m^2]
Stab_wet=2*0.0670+0.0627;%Wetted area of stabilizer [m^2]
S_wet=SectionA+SectionB+SectionC+Wing_wet+Stab_wet;%[m^2]

%% Drag
cf=0.004;%skin friction coefficient (estimated)
Df=q5*cf*S_wet;%Skin friction [N]

%% Vertical and Horizontal Stabilizer
fus_length=2.3;% Fuselage length
IHT=2.22-cg_true;%moment arm from horizontal tail's aerodynamic center (chosen arbitrarily) to c.g.
SHT=VHT*MAC2^2/S/IHT;%Area of horizontal tail (HT)

VVT=0.04;%Vertical tail (VT) volume
IVT=1.55691;%Moment arm from VT's aerodynamic center to c.g. (chosen arbitrarily)
SVT=VVT*b2*S/IVT;%Area of VT

AR_HT=5;%Aspect ratio of HT
b_HT=sqrt(SHT*AR_HT);%Span of HT
cr_HT=2*SHT/((lambda+1)*b_HT);%Chord root of HT
cr_HT=lambda*cr_HT;%Tip of HT
MAC_HT=(2/3)*cr_HT*(1+lambda+lambda^2)/(1+lambda);%Mean aerodynamic chord of HT
mac_HT=0.25*MAC_HT;%Mean aerodynamic center of HT
x_leading_edge_HT=IHT-mac_HT-(cr_HT-MAC_HT)/2;%Position of HT's leading edge from c.g.

AR_VT=1.7;%VT aspect ratio
h_VT=sqrt(AR_VT*SVT);%Root-to-tip height [m]
cr_VT=2*SVT/((1+lambda)*h_VT);%Chord root of VT
cr_VT=lambda*cr_VT;%Chord tip of VT
MAC_VT=(2/3)*cr_VT*(1+lambda+lambda^2)/(1+lambda);%Mean aerodynamic chord of VT
mac_VT=0.25*MAC_VT;%Mean aerodynamic center of HT
x_leading_edge_VT=IVT-mac_VT-(cr_VT-MAC_VT)/2;%Position of VT's leading edge from c.g.

%% Landing gear
x_main=x_leading_edge+cr2/2;%Location of main landing gear relative to fuselage nose
x_nose=0.3;%Location of nose wheel relative to fuselage nose

% forces on wheels
d3=x_main-x_nose;
d1=cg_true-x_nose;
\[ d_2 = d_3 - d_1; \]

\[
FM = ((W_0 \cdot d_1/d_3)/2) \cdot 2.20462262; \text{% Force on each main wheel [lb]}
\]

\[
FN = ((W_0 \cdot d_2/d_3)/2) \cdot 2.20462262; \text{% Force on nosewheel [lb]}
\]

\[ A_d = 1.51; \text{% Wheel diameter constant} \]

\[ A_w = 0.715; \text{% Wheel width constant} \]

\[ B_d = 0.349; \text{% Wheel diameter constant} \]

\[ B_w = 0.3112; \text{% Wheel width constant} \]

Main wheels proportions

\[
\text{Wheel diameter}_1 = 2.54 \cdot A_d \cdot (FM/2)^B_d;
\]

\[
\text{Wheel width}_1 = 2.54 \cdot A_w \cdot (FM/2)^B_w;
\]

Nosewheel proportions

\[
\text{Wheel diameter}_2 = 2.54 \cdot A_d \cdot FN^B_d;
\]

\[
\text{Wheel width}_2 = 2.54 \cdot A_w \cdot FN^B_w;
\]

%% Refined weight estimate

\[
\text{Wing} = 2.5 \cdot \text{Wing wet}; \text{% Wing weight}
\]

\[
\text{Stab} = 2 \cdot \text{Stab wet}; \text{% Stabilizer weight}
\]

\[
\text{Fus} = 1.4 \cdot \text{S wet}; \text{% Fuselage weight}
\]

\[
\text{Eng} = m1; \text{% Engine weight}
\]

\[ Q = 1; \]

\[ z = 1; \]

\[ Q(z+1) = W_0; \]

\begin{verbatim}
while abs(Q(z+1) - Q(z)) > 0.01
    z = z + 1;
    Q(z+1) = Wing + Stab + Fus + 0.0057 \cdot Q(z) + Eng + 0.1 \cdot Q(z) + W_f_0 \cdot Q(z) + W_p;
end
\end{verbatim}

\[ W_0\_final = Q(z+1); \text{% Final weight of the UAV} \]

\[ W_f\_final = W_f_0 \cdot W_0\_final; \text{% Final weight of the fuel} \]

\[ W_e\_final = W_0\_final - W_f\_final; \text{% Final empty weight} \]
Appendix 2

Derivation of the Breguet Range Equation

Proceeding from the expression of the instantaneous weight of the airplane,

\[ W = W_i + W_f \]  \hspace{1cm} (7.1)

where \( W_i \) denotes the weight of the airplane when the fuel tank is empty, and \( W_f \) the weight of the fuel, the Breguet Range Equation can be derived. The weight of the fuel will decrease during the flight, due to the consumption of the fuel. Thus, the total weight of the airplane will decrease during the flight. Differentiating equation (7.1) gives

\[ dW_f = dW \]  \hspace{1cm} (7.2)

The specific fuel consumption for a jet-propelled airplane can be expressed by the following equation:

\[ c_i = \frac{-dW_f}{dt} \]

or

\[ dt = \frac{-dW_f}{c_i T} \]  \hspace{1cm} (7.3)

Making the assumption that the airplane is in steady flight, and that \( s \) denotes the horizontal distance covered over the ground, and also that there is no wind, the airplane’s velocity may be express as:

\[ V_\infty = \frac{ds}{dt} \]

or

\[ ds = V_\infty dt \]  \hspace{1cm} (7.4)

Substituting equation (7.3) into equation (7.4) gives

\[ ds = -\frac{V_\infty}{c_i T} dW_f \]  \hspace{1cm} (7.5)

Due to the assumption of steady flight, level flight, the weight and thrust the equation can be rewritten as:

\[ T = D \]  \hspace{1cm} (7.6)

\[ W = L \]  \hspace{1cm} (7.7)

Substituting equation (7.2), (7.6) and (7.7) in equation (7.5):

\[ ds = \frac{-V_\infty}{c_i} \frac{L}{D} \frac{dW}{W} \]  \hspace{1cm} (7.8)

The equation can be integrated in order to obtain the range. The limits on the left side is set between \( s = 0 \) and \( s = R \); from the starting point to the desired range R. The right limits for the right side of the equation is set between \( W = W_0 \) and \( W = W_f \); at the starting point the weight is equal to the gross weight, and at \( R \) the fuel tank is empty. Equation (7.8) the becomes

\[ R = \frac{V_\infty}{c_i} \frac{L}{D} \int \frac{dW}{W} \]

or

\[ R = \frac{V_\infty}{c_i} \frac{L}{D} \ln \left( \frac{W_f}{W_i} \right) \]
\[ R = \frac{V_x}{c} \frac{L}{D} \ln \left( \frac{W_0}{W_i} \right) \]  

(7.9)

Equation (1.9) is the Breguet range equation, specifically for jet-engine airplanes. The specific fuel consumption can be expressed, unlike equation (7.3), as

\[
ct = \frac{c V_x}{\eta_{pr}}
\]  

(7.10)

where \(\eta_{pr}\) is the propeller efficiency. Substituting this expression in the Breguet range equation gives

\[ R = \frac{\eta_{pr} L}{c} \frac{D}{D} \ln \left( \frac{W_0}{W_i} \right) \]  

(7.11)
Appendix 3
The Master Equation

\[
\frac{T}{W} = \frac{\beta}{\alpha} \left[ \frac{q}{\beta} \left( \frac{C_{D}}{W_{T0}} \right) + k \left( \frac{n\beta}{q} \right)^2 \left( \frac{W_{T0}}{S} \right) \right] + \frac{1}{V} \frac{dh}{dt} + \frac{1}{g} \frac{dV}{dt}
\]

The take-off constraint equation

\[
P = \frac{V}{\eta_p} \left[ \frac{V}{V} + \frac{q}{W / S} \left( C_{d,\text{min}} + \frac{k W}{q} \right) \right]
\]

The sustained level turn equation

\[
P = \frac{V}{\eta_p} \left[ \frac{C_{d,\text{min}}}{W / S} + \frac{k}{q} \left( \frac{W}{S} \right) \right]
\]

Rate-of-climb equation

\[
P = \frac{V}{\eta_p} \left[ \frac{V}{V} + \frac{q}{(W / S)} C_{D,\text{min}} + \frac{k}{q} \left( \frac{W}{S} \right) \right]
\]

Cruise/max speed constraint equation

\[
P = \frac{V}{\eta_p} \left[ q C_{D,\text{min}} \left( \frac{1}{W / S} \right) + k \left( \frac{1}{q} \right) \left( \frac{W}{S} \right) \right]
\]