Master's Thesis in Mechanical Engineering

Frequency Analysis of Rottne Comfort Line

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Abstract

The European Parliament stipulated regulations concerning the forestry vehicles operators’ health and working conditions. The allowed whole body vibrations were limited, which influenced the design of the vehicles' cabin. Surveys show a strong correlation between operator's comport and their productivity.

The object of the research was Rottne AB Comfort Line Cabin, which was designed to increase the comfort for the forwarder operators. The main objective was to determine the cab's inertia properties, position of the centre of gravity and the resonance frequencies of the cab as well as a system consisting of the cab and its suspension.

The methods used were an impact test with Mass-Line Analysis for the cab's properties and Operational Modal Analysis for the system. For both tests a Leuven Measurement System was used, but a part of the calculations were made in parallel by use of a MATLAB code written for this thesis. In addition a suspension test was made to estimate the centre of gravity and it was here treated as the reference value.

The authors used reference values and the quality of the obtained results to compare the methods used. Further proposals for future research were made together with hints how to use vibration tests more effectively.

Key words: Centre of gravity, Hammer test, Inertia Properties, LMS, Mass line, OMA, Vibration testing,
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List of variables

\( \Delta P \) - distance
\( \eta \) - mode
\( \dot{\theta}_{x0}, \ldots \) - angular acceleration in reference point
\( \dot{\theta} \) - angular acceleration matrix
\( \varphi \) - eigenmode
\( \Phi \) - eigenmatrix
\( \Psi \) - generalised eigenmode
\( \omega \) - natural circular frequency
\( \Omega \) - driving frequency
\( \ddot{a}_0 \) - accelerations vector in the reference point
\( \ddot{a}_i \) - accelerations vector in a i-th point
\( c \) - constants
\( d \) - distance
\( E(f, g) \) - correlation function
\( E \) - reference excitation vector
\( f(t) \) - function with time domain
\( F_{ix}, \ldots \) - excitation force in the i-th point
\( F_{x0}, \ldots \) - reference excitation force
\( F_i \) - excitation force vector in the i-th point
\( H_{ij} \) - Frequency Response Function
\( l_{xx}, \ldots \) - inertia properties of the cab
\( I \) - vector of inertia properties of the cab
\( k \) - generalised stiffness
\( m \) - mass or generalised mass
\( M_{x0}, \ldots \) - reference excitation momentum
\( M \) - reference excitation momentum vector
\( L \) - line in space
\( L \) - line leading vector
\( P \) - point
\( P \) - point’s coordinate vector
\( P(t) \) - excitation
\( R \) - transformation matrix
\( u \) - displacement
\( \ddot{u} \) - acceleration
\( U \) - amplitude
\( t \) - parameter
\( x_{ij}, \ldots \) - coordinates of a point
\( \ddot{x}_p, \ldots \) - acceleration readout/value
\( \ddot{x}_0, \ldots \) - linear acceleration in reference point

List of abbreviations

COG – Centre of Gravity
DOF – Degree-of-freedom
FFT/DFT – Fast/Discrete Fourier Transform
FRF – Frequency Response Function
LMS – Leuven Measurement System
OMA – Operational Modal Analysis
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1. Introduction

Forestry is a developing branch of industry. It has significant meaning in countries having their vast areas being forests. Such countries are: Australia, Canada, Russia, Sweden and USA. One of the harvesting systems is cut-to-length, which consists of two types of machines: harvester and forwarder. To the duties of a forwarder belongs load and transportation of cut and finned timber. Both operations, i.e. driving, and loading of the wood piles are conducted from the cabin. The time spent by the operator during a typical working day in the cabin amounts up to eight hours. For all this time the operator is exposed to vibrations, characterised mostly by high amplitudes but low frequencies (Nyström, M., 2014). The excitations are mainly caused by driving over massive rocks, stumps, and other natural bumps that abounds in the forests. That is the reason why the cabin is such an important component of the forwarder, see Figure 1; it is a working environment for the operator.

![Figure 1 A Forwarder F15 during work](image)

Companies involved in heavy vehicles industry put many efforts into improving both safety and comfort for the driver. This is dictated by the will of satisfaction of forestry machines market’s requirements and expectations.

1.1 Background

In recent years comfort of the operator gained significantly. First of all this is due to the standards regarding the permissible level of the operator’s exposure to vibrations in his workplace. According to European Parliament and the Council Directive 2002/44/EC of 25th June 2002 on the minimum health and safety requirements regarding the exposure of workers to the
risks arising from physical agents (vibration), for the whole body vibration (WBV) standardised to an eight-hour reference period. There were introduced limits to both maximum momentary and time average accelerations. Exceeding them obliges an operator to end the workday. Secondly, according to research conducted on a group of forestry machine drivers (Landekic, et al., 2013) there exists a strong need of operator’s comfort improvement since there is an evident correlation between driver’s tiredness and the drop of productivity.

Forestry industry recognized the need of employees as a need of employers of improvement of safety and comfort during the work. In order to meet the customer expectations, Rottne AB has introduced their solution named Comfort Line. Since the system is constantly developed, there is a need to determine its vibration properties in order to identify and eliminate the weak points.

Knowledge about a cabin’s inertia properties is of paramount importance when it comes to design, and calculation of the suspension system. There are many methods enabling determination of those parameters; they differ in simplicity, costs, etc. However, all available methods can be divided into two groups:

- static methods,
- dynamic methods.

The difference between them is that, static methods reveal only mass and centre of gravity (COG) coordinates, whereas dynamic methods additionally determine the inertia tensor. A comprehensive review of both groups of methods was done by Schedlinski and Link (2000); authors not only present, but also evaluate practical usefulness, amount of time needed to conduct the measurements, etc. One of the static methods is a suspension method. The main advantages of this method are: safety, only basic skills of the staff needed, and no need for use of software. When it comes to dynamic methods, hammer tests are commonly used. It is itself a set of methods that depends on conditions and applied mathematical tools. One more method worth mentioning is measurement robot dedicated for this type of survey. Schedlinski and Link (2000) describes this method as very sophisticated, and requiring specialised software. However, the method itself has major advantages e.g. automated procedure and low time requirements. More in-depth study of this method was done by Brancati, Russo and Savino (2009). Their research showed that this method can gain on meaning in the industry, since it enables to carry out measurements of big objects such as: vehicles, small planes, boats or railway truck components. During measurements they obtained good repeatability and small deviation of results compared to conventionally obtained values. Another method is the multi-cable pendulum (Gobbi, Mastinu and Previati, 2010), which bases on recording pendulum motion, and the forces acting on the system, while the body is in
free motion. Later on, a mathematical procedure is applied to obtain inertia properties. The main advantages of this method are: possibility of being scaled to measure considerably large objects i.e., cars, planes or even ships and short measuring time (about 10 minutes). The authors state that this solution can be applied for commercial use if developed.

The dynamic behaviour of the cabin’s suspension can be described by Frequency Response Functions (FRFs). These functions are here obtained from Operational Modal Analysis (OMA) of collected measurement data. OMA, like other analysis methods, uses Fourier Transform (FT) (“time to frequency” transform), but this particular method is used, when excitation remains unknown. It is common for large structures that input is impossible to determine and the measured system response can be used by system identification algorithms (Cara et al., 2013). OMA has many advantages (Hermans and Van der Auweraer, 1999), that distinguishes it from laboratory test, e.g. it enables to obtain a model of an object being exposed to real loads and it takes into account environmental effects, such as pre-stress of suspension, load-induced stiffening, aero-elastic interaction, etc. It has various applications in industry, e.g. damage detection, diagnosis (Martinod, Betancur and Heredia, 2011).

Measuring procedures and mathematical tools, i.e. Fourier Transform and its modifications applied to obtain FRF, are commonly used according to the latest research (Shi, et al., 2014) in optics, image, signal processing, etc.

1.2 Purpose and Aim

There is a need for exact data regarding a Rottne AB cabin’s inertia parameters. Even though, there exists a CAD model of the cabin, all hardware and instrumentation installed later on, during the manufacturing process, increases both, the cabin’s mass, and the components of the inertia tensor. It changes the location of its centre of gravity as well. More in-depth knowledge about cabin’s and cabin’s suspension dynamic properties will enable and simplify further improvement of the product.

The first aim of this thesis is to determine inertia parameters, i.e. the mass, centre of gravity and inertia tensor for a cabin. At this occasion, it is intended to verify a measurement method along with a mathematical procedure that is applied in this type of survey. The next aim is to find the cabin’s and the cabin’s suspension resonance frequencies; that is to be done by use of OMA and a Leuven Measurement System (LMS®).
1.3 Hypothesis and Limitations

1. Hypothesis: The centre of gravity of a complete cabin is moved to the right side, due to placement of the equipment, such as air conditioner on the right side.

Limitations:

The cabin’s COG is obtained in three ways; by an inclination test and analytical geometry, LMS® toolbox called “Modal Rigid Body” calculator and finally by code made by the authors, using FRFs as an input.

2. Hypothesis: OMA allows using measurement data to obtain FRF.

Limitations:

FRFs are obtained only for the cabin, not for the whole vehicle. OMA would be carried out by MATLAB® code written for this thesis.

3. Hypothesis: FRFs can be used to find resonance frequencies, evaluate the dynamic behaviour and compare it with Finite Element (FE) models of the system.

Limitations:

Resonance frequencies are determined, dynamic behaviour is examined. Creating FE model of the object is out of the scope of this research.

4. Hypothesis: Hammer test can be applied to determine inertia properties of various objects.

Limitations:

Only cabin’s measurements are carried out. Results are compared with those obtained from one of the commonly used method.

1.4 Reliability, validity and objectivity

Vibration measurements are carried out with use of accelerometers, which are attached to chosen places on the cabin and its suspension. The number of accelerometers depends on the aim of the measurements; however, for every considered point it is desired to mount three accelerometers, one for each degree-of-freedom (DOF) in x-, y- and z-direction. Analysis in three dimensions increases reliability due to comprehensive description of motion. Each and every accelerometer has its own serial number with date of calibration. This must be taken into account during the measurements since accelerometers are sensitive for shocks or impacts; every incident should be reported to sustain the reliability of the measurements. Aforementioned
information guarantees accuracy of collected data. The mass of the accelerometers is small in comparison to the mass of the elements that they are attached to, so their influence on the results can here be neglected. The placement of the accelerometers has a great impact on the reliability of the results. Accelerometers must be placed on stiff elements to eliminate the possibility of recording local vibrations. What is also worth mentioning is that the points of the sensors attachments must be chosen very carefully, since the space in cabin’s suspension is very limited and consists of many movable components. When it comes to operationally collected data they can be affected by the driver’s skills and their ability to keep constant velocity during the test. The final results are affected by many factors such as: temperature, road and tires condition, etc., but they all have replicable character. It means that qualitative data (shapes of FRFs) remain the same, while quantitative (response values) can differ insignificantly.

Validity of the measurements relies on LMS® data acquisition system and measured quantities. Since most of the researches are based on collecting data, not on observations, they are not affected by human errors or personal biases. Data processing is carried out by use of computer programme. Only one of the tests demands human perception and this is the inclination test. A laser level is then used to increase accuracy.
2. Theory

The measurements are carried out with use of accelerometers. Records are collected by a Leuven Measurement System (LMS®). Next, using the MATLAB® code mentioned in the previous paragraph, obtained characteristics of accelerations as functions of time, are transformed into Frequency Response Functions (FRFs). These results are used in the determination of the frequencies.

2.1 Fourier Transform (FT)

For a periodic function \( f(t) \), the Fourier Transform, used in engineering applications, is represented by:

\[
\hat{f}(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-i\omega t} dt \tag{2.1.}
\]

and it is called time-to-frequency transformation.

2.1.1 Discrete Fourier Transform (DFT)

Since FT deals with continuous input functions that occurs in simple or theoretical cases, it cannot be applied to practical cases. In real-world problems, output is in the form of discrete data including both, leading function (if such exists) and concomitant. Discrete data need Discrete Fourier Transform (DFT); following Craig and Kurdila (2006, p.185).

\[
S(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos(\omega nt) + b_n \sin(\omega nt)) \tag{2.2.}
\]

Where:

\[
a_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \cos(\omega nt) dt \tag{2.3.}
\]

\[
b_n = \frac{2}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} f(t) \sin(\omega nt) dt \tag{2.4.}
\]

This formula presents decomposition into an infinite number of harmonic functions and presents an exact solution for the considered number going to infinity. However, this formula still needs continuous functions.

To split these analogue elements, the function is presented discretely, and a summation is introduced instead of integration. Discrete input function
(measurements) is presented in a form of a real vector: \([a_0, a_1, a_2, \ldots, a_N]\) and complex (harmonic) output: \([A_0, A_1, A_2, \ldots, A_N]\).

\[ A_k = \sum_{n=0}^{N-1} (a_n \omega_N^{-kn}) \quad (2.5.0) \]

Where \(0 < k < N - 1\) and \(\omega_N = e^{\frac{2\pi i}{N}}\)

Since the summation is just a multiplication of the vector \(a_n\) with coefficients based on \(-kn\) power, the formula can be presented as:

\[ A_k = M a_n \quad (2.6.0) \]

where \(M_{kn} = \omega_N^{-kn}\) (assuming first row and column is “0”, not “1”).

2.1.2 Fast Fourier Transform (FFT)

FFT is the DFT algorithm calculated in a much more efficient way (Cooley and Tukey, 1965). It is based on recurrent calculations of the transforms \(N_1, N_2\) that fulfill \(N_1 N_2 = N\). Divide and conquer method allows to reduce computational complexity, but it introduces additional conditions on the number of samples. Basically, the computational complexity of FFT is \(O(N \log_2 N)\) instead of \(O(N^2)\), but then the number of samples taken must be \(2^m\), where \(m \in \mathbb{N}\).

2.2 Single Degree of Freedom (SDOF)

Single-degree-of-freedom (SDOF) systems (Figure 2) are systems whose position can be precisely defined using only one variable (degree-of-freedom). As a consequence, independently on the number of elements in the system, an SDOF can be presented as:

Figure 2 An SDOF system

where elements \(m\) and \(k\) represent respectively inertia element (i.e. both mass and mass moment of inertia) and stiffness element (of linear and rotational springs). Since it is a general model, the displacement \(u(t)\) can be either linear or angular. To get particular values of \(k, m,\) and \(P(t)\), the system must be solved using Newtonian (forces) method as most basic example, or any other, e.g. virtual displacement (work) method.
For an undamped SDOF like the one in Figure 2, the differential equation of motion is:

\[ m\ddot{u} + ku = P(t) \]  
(2.7.)

When \( P(t) = 0 \), the case is called free vibration and then the solution is:

\[ u(t) = A \cos(\omega_n t) + B \cos(\omega_n t) \]  
(2.8.)

where \( \omega_n \) is the natural frequency of the system

\[ \omega_n = \sqrt{\frac{k}{m}} \]  
(2.9.)

When the system has a nonzero excitation, then

\[ u(t) = u_{\text{homogenous}} + u_{\text{particular}} \]  
(2.10.)

\[ u(t) = A \cos(\omega_n) + B \cos(\omega_n) + u_{\text{particular}} \]  
(2.11.)

The homogenous part is connected to the system’s natural harmonic motion and the particular part is connected to the excitation.

The FRF is the response of the system to a harmonic excitation, having a unit amplitude. Assuming excitation:

\[ P(t) = P_0 \cos(\Omega t) \]  
(2.12.)

then

\[ m\ddot{u} + ku = P_0 \cos(\Omega t) \]  
(2.13.)

Assuming that the system’s steady-state response follows the driving frequency:

\[ u(t) = U \cos(\Omega t) \]  
(2.14.)

\[ \ddot{u}(t) = -U \Omega^2 \cos(\Omega t) \]  
(2.15.)

Implementing this into equation (2.13):

\[ m(-U \Omega^2 \cos(\Omega t)) + kU \cos(\Omega t) = P_0 \cos(\Omega t) \]  
(2.16.)

The harmonic part can be reduced, and keeping \( U \) on the left side:

\[ U = \frac{P_0}{k - m\Omega^2} \]  
(2.17.)

Implementing the natural frequency:

\[ U = \frac{P_0}{m \left( \frac{k}{m} - \Omega^2 \right)} = \frac{P_0}{m \left( \omega_n^2 - \Omega^2 \right)} \]  
(2.18.)
Normalising the FRF to a unit load (1N):

\[ H = \frac{U}{P_0} = \frac{1}{m(\omega_n^2 - \Omega^2)} \]  

(2.19.)

The function \( U(\Omega) \) is shown in Figure 3.

This formula informs about two main features:

- displacement’s amplitude is dependent on the driving frequency, and this function is called FRF,

- the resonance occurs when the driving frequency is the same as the natural frequency, since the denominator approaches zero, the amplitude is going to infinity.

An FRF can consist of either, displacement, velocity or acceleration as a function of frequency. They are called: receptance, mobility and accelerance respectively.

![Exemplary FRF for SDOF](image)

**Figure 3** Exemplary diagram of SDOF’s FRF

### 2.3 Multiple Degree of Freedom (MDOF)

MDOF system’s motion is described by differential equations with multiple variables, which make MDOF systems more complicated to solve than an SDOF. The general equation of motion for an undamped MDOF is:

\[ \ddot{\mathbf{u}} + \mathbf{K}\mathbf{u} = \mathbf{P}(t) \]  

(2.20.)

The motion of each degree-of-freedom is described by a function \( u_i(t) \); \( \mathbf{M} \) and \( \mathbf{K} \) stand for the mass, and stiffness matrix of the system respectively.
Solving this multivariable differential equation, harmonic motion for each node is assumed:

\[ u_j = U_j \cos(\omega t - \alpha) \]  \hspace{1cm} (2.21.)

This assumption leads to eigenproblem:

\[ (K - \omega^2 M)u = 0 \]  \hspace{1cm} (2.22.)

Non-trivial solution of this problem requires

\[ \det(K - \omega^2 M) = 0 \]  \hspace{1cm} (2.23.)

Solving this problem gives the eigenfrequencies of the system. Then, for each \( \omega_j \), parameterisation of one degree allows to describe other degrees, e.g., for parameterisation of \( u_1 \) (assumption \( \varphi_0 = 1 \))

\[
\begin{bmatrix}
\varphi_0 u_1 \\
\varphi_1 u_1 \\
\vdots \\
\varphi_{n-1} u_1 
\end{bmatrix} = \begin{bmatrix} 1 \\ \varphi_1 \\ \vdots \\ \varphi_{n-1} \end{bmatrix} u_1
\]  \hspace{1cm} (2.24.)

The vectors of coefficients \( \varphi \) are called eigenmodes and are stored in a matrix:

\[ \Phi = [\varphi_1 \varphi_2 \ldots \varphi_n] \]  \hspace{1cm} (2.25.)

If the assumption is made that:

\[ u(t) = \Phi \eta \]  \hspace{1cm} (2.26.)

then:

\[ \Phi^T M \Phi \ddot{\eta} + \Phi^T K \Phi \eta = \Phi^T p(t) \]  \hspace{1cm} (2.27.)

The coefficients matrices are diagonal, so the equations are uncoupled and each \( \eta \) can be solved as an SDOF, and then finally:

\[ u_i = \sum_{r=1}^{n} \varphi_{ir}^T \eta_r \]  \hspace{1cm} (2.28.)

Understanding the idea of modes, eigenfrequencies, and eigenmodes is crucial for understanding a system’s motion and FRFs.

Eigenfrequencies are the resonance frequencies of a system. The motion of an MDOF system is superposition of modes. Under an excitation, modes having natural frequencies closest to the driving frequency, would dominate the motion of structure, but still the rest of the modes would have influence, according to their own FRFs. As already mentioned, eigenmodes are normalised vectors of values. Information they contain can be described in two ways:
- as semi-independent information about the behaviour of the system in a particular frequency, e.g., eigenmode \( \varphi \) in low frequencies can be a rigid body mode connected with rotation or in higher frequencies it could show deflection distribution along a beam,

- as influence of a mode on a degree-of-freedom – due to (2.28) eigenmodes must be known to calculate DOF’s motion; to be more precise they contain distribution of influences on all degrees, as vector is normalised, and single value taken from vector has no physical sense.

One of normalisation method (to one DOF) was shown before. In that general case for mode:

\[
m_r \ddot{\eta} + k_r \dot{\eta} = \varphi_r^T \textbf{P}(t) \tag{2.29.}
\]

\[
\ddot{\eta} + \omega_r^2 \eta = \frac{\varphi_r^T \textbf{P}(t)}{m_r} \tag{2.30.}
\]

Normalising to a DOF makes calculations easier, and allows treating one degree-of-freedom as reference to others; there are different ways to normalise eigenmodes. One of them is the mass normalisation. Eigenmodes are scaled such for all modes.

\[
\ddot{\eta}_r + \omega_r^2 \eta_r = \varphi_r^T \textbf{P}(t) \tag{2.31.}
\]

2.4 Frequency Response Functions (FRFs)

Knowing the general formula for mode displacement, it is easy to derive FRFs. Each FRF is a reaction at one DOF for excitation in another DOF. The excitation \( \textbf{P}(t) \) contains therefore only one non-zero value in the excitation point.

Searching for the \( i^{th} \) DOF response for excitation in the \( j^{th} \) DOF, the following statement is true:

\[
\ddot{\eta}_r + \omega_r^2 \eta_r = \varphi_r^j \textbf{P}(t) \tag{2.32.}
\]

Since \( \varphi_r^j \) is a scalar, there is no need to still denote it as transposed. Now the formula for the amplitude of the \( r^{th} \) mode:

\[
\tilde{\eta}_r = \frac{\varphi_r^j \textbf{P}_0}{\omega_r^2 - \Omega^2} \tag{2.33.}
\]
MDOF FRFs are also force normalised; they show the response to an excitation equal 1N. Using equation (2.28):

$$H_{ij} = \sum_{r=1}^{n} \frac{\varphi_r^i \varphi_r^j}{\alpha_r^2 - \Omega^2}$$

(2.34.)

An example of FRF consisting of three modes is presented in Figure 4:

![FRF diagrams](image)

Figure 4 Exemplary diagrams of MDOF’s FRF

2.5 Operational Modal Analysis (OMA)

There are multiple ways of testing. Hammer test is a Single Input Single Output (SISO) or Single Input Multiple Output (SIMO) test. Measuring large structures with two or more shakers is a Multiple Input Multiple Output (MIMO) test. But an entirely different way of measuring is an operational (in situ) measurement, for which the input may be random. It is still possible to get FRFs if relevant tools are applied.

The input for Operational Modal Analysis (OMA) is a record of accelerations in points of interest. Due to the highly random values connected with non-defined excitation, the Dirichlet condition to apply Fourier Transformation is not fulfilled. To overcome this problem two functions are used:

- Correlation in time domain
- Spectral Density in frequency domain

A correlation of functions shows how coherent they are to each other. In this case autocorrelation is used with a time shift \( \tau \) to see how much a function is ‘repeatable’ for different time shifts.

$$R_{ff}(\tau) = E\left( f(t), f(t + \tau) \right)$$

(2.35.)
It appears that the correlation fulfils the Dirichlet condition and according to Ewins (2000, p.138) a FFT can be calculated; then the result is called a Spectral Density. The FFT of the autocorrelation is called an Auto Spectral Density or Power Spectral Density (PSD).

\[ S_{ff}(\omega) = FFT\left(R_{ff}(\tau)\right) \] (2.36.)

For cross-analysis, the same steps are followed. Firstly, the correlation of \( f_1(t) \) and \( f_2(t) \) is found:

\[ R_{f1f2}(\tau) = E\left(f_1(t), f_2(t + \tau)\right) \] (2.37.)

Then the Cross Spectral Density (CSD):

\[ S_{f1f2}(\omega) = FFT\left(R_{f1f2}(\tau)\right) \] (2.38.)

The FRF of DOF one with excitation in DOF two is:

\[ H_{ij} = \frac{S_{f1f1}(\omega)}{S_{f2f1}(\omega)} \] (2.39.)

The problem with this OMA is that the modes are not normalised

\[ H_{ij} = \sum_{r=1}^{n} \frac{\Psi_{r}^{i} \cdot \Psi_{r}^{j}}{m_{r} (\omega_{r}^{2} - \Omega^{2})} \] (2.40.)

as neither excitation force nor modal mass is known.
3. Method

For better understanding of the cabin’s dynamic behaviour it is required to determine its inertial properties, i.e. the mass, centre of gravity (COG) and inertia tensor.

Scales and inclination test are commonly used in industry. For needs of this paper they act as a reference method, to enable comparison with the results obtained using the MATLAB® code made by the authors and LMS® features dealing with rigid body properties. Measurement of inertial properties is an opportunity to compare different approaches in this type of survey. At first, to obtain the value of the cabin’s mass and to indicate the coordinates of centre of gravity, methods widely applied in the industry so far, were used. This means that mass was measured by use of scales placed between the suspended cabin and the crane.

3.1 Scales

In order to obtain the mass of the cabin it was suspended and lifted by use of a crane. The following way of mounting was applied; two hooks were attached on both sides of the cabin, they were joined with a horizontal beam by stripes. The beam was connected with scales by a stripe. The whole aforementioned set was attached to the crane’s hook with a stripe. Mass readout was displayed on a measure unit that was connected with suspended scales by wire. Since the scales was not attached directly to the cabin, it was tared first and the readout already excluded the mass of a beam.

3.2 Inclination test

3.2.1 Preparations and measurements

When the mass measurement with the use of scales was completed, the cabin’s suspension was changed in a way that is shown in Figure 5, in order to mark vertical lines when cabin reaches the state of equilibrium. The point where two lines crosses each other on one of the four sides of the cabin, see Figure 6, is a point through which one of the axis indicating the position of COG is located.
To find out where the centre of gravity is situated, two axes were determined. In ideal conditions the point where they cross each other indicates the centre of gravity. Axes are set down by two points which were obtained from measurements. Firstly, the cabin had to be suspended in two points, more precisely in two neighbouring corners, in a way that it was tilted. When the cabin reached equilibrium, i.e. no pendulum movement, a laser level was adjusted according to the points of suspension. Vertical line displayed on the cabin indicated the direction of gravitational force. The next step was to firmly mark the aforementioned line with a permanent marker on the cabin. In order not to damage the cabin’s surface, marks were made on a paper tape stuck on it. To complete the process for the first point, the cabin was suspended on the opposite side and lifted. The whole procedure was repeated, the place where two lines intersected each other was the first point. All steps were made for all four sides of the cabin in the same way.
3.2.3 Data preparation

During the measurements, the coordinates of the marked points were noted down with respect to characteristic points on the cabin. In order to obtain global coordinates of the COG, local values had to be transformed. For this purpose, a CAD model of the cabin was used. Distances between the origin of governing frame of reference and particular points, i.e. x, y, and z components, were added to respective local coordinates.

3.2.4 Calculations

Measured data are gathered from four (two pairs of) degrees-of-freedom. Firstly, their position must be read using the reference points in geometry. Then analytical geometry is used to obtain coordinates of COG in the following way. The formula of a line crossing two points A and B with coordinates \( A = [x_A, y_A, z_A] \) and \( B = [x_B, y_B, z_B] \) is:

\[
\begin{align*}
    \frac{x - x_A}{x_B - x_A} &= \frac{y - y_A}{y_B - y_A} = \frac{z - z_A}{z_B - z_A} = t
\end{align*}
\]  

(3.1.)

Solving this equation allows writing parametric formula of line:

\[
L = Lt + P_A
\]  

(3.2.)

for \( t \in \mathbb{R} \) and where:

\[
L = \begin{bmatrix} x_B - x_A \ y_B - y_A \ z_B - z_A \end{bmatrix}
\]  

(3.3.)

\[
P_A = \begin{bmatrix} x_A \\ y_A \\ z_A \end{bmatrix}
\]  

(3.4.)

This means that the line goes from an initial point \( P_A \) (in bold as vector of coordinates) and follows vector \( L \) using the parameter \( t \).

The formula for the distance between two skew lines uses vector multiplications \( d(L_1, L_2) = \frac{|\overrightarrow{P_1} \times \overrightarrow{P_2} \cdot \overrightarrow{v_1} \cdot \overrightarrow{v_2}|}{|\overrightarrow{v_1} \times \overrightarrow{v_2}|} \); this is basically the formula which defines the distance between skew lines through proportion of triple to double multiplication of characteristic vectors, what can be compared to dividing space by area to get distance.

However, this formula does not provide points in which the least distance appears, so this formula is derived from the beginning:

\[
d(t_1, t_2) = |L_1 - L_2| = |L_1t_1 + P_1 - L_2t_2 - P_2|
= |L_1t_1 - L_2t_2 + \Delta P|
\]  

(3.5.)
\[ \sqrt{\sum (L_1 t_1 - L_2 t_1 + \Delta P)^2} = \sqrt{c_1 t_1^2 + c_2 t_2^2 + c_3 t_1 t_2 + c_4 t_1 + c_5 t_2 + c_6} \] (3.6.)

Where:

\[ c_1 = \sum (L_1^2) \] (3.7.)
\[ c_2 = \sum (L_2^2) \] (3.8.)
\[ c_3 = -2 \sum (L_1 L_2) \] (3.9.)
\[ c_4 = 2 \sum (L_1 \Delta P) \] (3.10.)
\[ c_5 = -2 \sum (L_2 \Delta P) \] (3.11.)
\[ c_6 = \sum (\Delta P^2) \] (3.12.)

Multiplications are carried out element-by-element and summation adds values from each dimension. These formulas describe how expression for distance between the lines is transformed from vector form into square root of polynomial of \( t_1 \) and \( t_2 \). Next, minimum of the function is derived:

\[ \frac{\partial d(t_1, t_2)}{\partial t_1} = \frac{2c_2 t_2 + c_3 t_1 + c_5}{2 \sqrt{c_1 t_1^2 + c_2 t_2^2 + c_3 t_1 t_2 + c_4 t_1 + c_5 t_2 + c_6}} = 0 \] (3.13.)
\[ \frac{\partial d(t_1, t_2)}{\partial t_2} = \frac{2c_1 t_1 + c_3 t_2 + c_4}{2 \sqrt{c_1 t_1^2 + c_2 t_2^2 + c_3 t_1 t_2 + c_4 t_1 + c_5 t_2 + c_6}} = 0 \] (3.14.)

Since it is known exactly what this function describes, it has only one extreme and it is global minimum that shows the distance. Using \( d \neq 0 \) since it is known that lines are skew, so the distance between them must be larger than zero. Roots of the function \( T_1 \) and \( T_2 \) are found:

\[ \begin{cases} 2c_1 T_1 + c_3 T_2 + c_4 = 0 \\ 2c_2 T_2 + c_3 T_1 + c_5 = 0 \end{cases} \] (3.15.)

Matrix form of (3.15):

\[ \begin{bmatrix} 2c_1 & c_3 \\ c_3 & 2c_2 \end{bmatrix} \begin{bmatrix} T_1 \\ T_2 \end{bmatrix} = \begin{bmatrix} -c_4 \\ -c_5 \end{bmatrix} \] (3.16.)
And the solution is:
\[
\begin{bmatrix}
T_1 \\
T_2
\end{bmatrix} = \begin{bmatrix}
2c_1 & c_3 \\
c_3 & 2c_2
\end{bmatrix}^{-1} \begin{bmatrix}
-c_4 \\
-c_5
\end{bmatrix}
\]

(3.17.)

Now \(T_1, T_2\) allows getting coordinates of the points:

\[
cog_{L1} = L_1T_1 + P_{A1}
\]

(3.18.)

\[
cog_{L2} = L_2T_2 + P_{A2}
\]

(3.19.)

And the results:

\[
cog = \frac{cog_{L1} + cog_{L2}}{2}
\]

(3.20.)

\[
d_{\text{min}} = |L_1T_1 - L_2T_2 + \Delta P|
\]

(3.21.)

COG coordinates (Table 3) act as reference values for those obtained from FRF analysis in hammer test.

3.3 Hammer test

3.3.1 Preparations

Use of the LMS® is a cutting-edge approach for this type of survey. It allows finding the inertial properties without manoeuvring the body, time-consuming adjustments of the laser indicator and inaccuracies caused by human factor; however the method itself is very sophisticated what entails the need of spending more time on preparations and deep knowledge within solid mechanics. The experimental set used during the measurements included accelerometers, a computer with license key, a data acquisition unit, an excitation hammer and a set of wires.

Preparation of the test object is one of the most important things to do before starting the measurements. The first thing to be decided and having great impact on results validity is the way that the body is supported. There are two possibilities (Ewins, 2000), either free- or grounded support. Freely supported object reveals rigid body modes which are determined by mass and inertial properties. This type of support is the best option in case when mass and inertia properties are surveyed. However, sometimes there is no alternative for grounded support, for example when heavy structures, i.e. buildings or parts of power generating stations are tested.

The cabin’s inertial measurements were carried out as follows. First and foremost, governing reference frame was established and was firmly marked on the floor under the cabin. Secondly all devices necessary to carry out the measurements were connected and plugged in, i.e. the computer with license key and LMS® data acquisition unit. Next step was to determine points on
the cabin where the accelerometers should be attached, to provide valid data. Apart from the validity requirement also availability for hammer impacts had to be taken into account while choosing the points. There were 8 points; 4 on the bottom of the cabin and 4 at the top of it. In each point 3 accelerometers were attached to provide response in each (x, y, z) DOF which gives a comprehensive view on the system’s behaviour. All accelerometers were connected with the LMS® data acquisition unit with wires, which were carefully numbered and labelled in accordance with their relating points. The unit used during measurements has 20 channels, 1 input was assigned to the excitation hammer, 19 were assigned to the accelerometers - 18 for aforementioned points and one additional accelerometer attached to the seat base. Since responses in only 6 of the points could be measured at once, two tests were carried out – in both 12 accelerometers were placed on the bottom points while the top left points were measured in test 1 and the top right in the second one. Figure 7 shows that tests give repeatable results and that is why results from test 2, from top right points can be directly added to the responses from test 1.

When the hardware was set up, the system geometry had been introduced to the software along with accelerometers directions and specification - their sensitivity and serial number. This information is stored in accelerometers' memory called Transducer Electronic Data Sheet (TEDS).

In this survey LMS® was applied in two ways. First of all the data acquisition system was used to collect FRFs, and then LMS® rigid body calculator was used. The results from the toolbox were reference values to those obtained from the MATLAB® code, which used the same data.
3.3.2 Measurements

When both the hardware and the software were prepared, measurements could be started. Measurements for each DOF were carried out according to the following procedure. First of all, the direction of impact was determined. Secondly all accelerometers had to be ranged, trial impact indicated if the hit was either too strong, which resulted in overload or not strong enough to be recorded. When scaling was done a series of five hits was recorded. The whole procedure was repeated 18 times, once for each accelerometer in chosen points. Tests were carefully named in connection with point and direction of hit.

Analysing the data as a set of 24 responses on 12 excitations is more valuable than analysis of 18 responses on 18 excitations. The first way covers the whole cabin and fulfils the criterion given by Leuven Measurement Systems (n.d.) whereas the second provides an incomplete view on cabin’s dynamic behaviour despite larger amount of data.

3.3.3 Data preparation

The outcome of cabin’s hammer test was a set of files exported from LMS® to MATLAB® and it is the initial of the analysis. Each file stem from a single hammer test; therefore it contained data about responses of all measured point to an excitation in one DOF. The file itself was a folder with a short description as well as values of the measurement. The most important subfolders were:

- units – information that values are in $g/N$
- x values – defined by the number of elements and increment
- y values – a matrix 19x2049 with FRFs

Before the calculations were carried out, several things had to be straightened out and interpreted. First of all, due to default settings, LMS® puts functions in alphabetical order of points e.g. “Rottne cabin:chair” was before “Rottne cabin:point 1+x”, despite they were respectively the last and first channel, so the matrix had to be reorganized to set FRFs again in channel-like order. While analysing data it also appeared that particular number of mass lines were negative and the reason was found in LMS® algorithms that did not included reversed directions of measured values despite this information is provided when preparing test. The last thing was interpretation of imaginary parts that were substantial a part of some values. A solution to this problem was given by Leurs, et al. (n.d.) and LMS (n.d.), where authors suggested taking absolute value, but following sign of real part.
3.3.4 Mass line

A mass line is a relatively flat part of an FRF plot, lying between a rigid body mode and the first elastic mode, see Figure 4. To provide valid data, a set of requirements must be fulfilled. First of all, according to LMS® guideline by Leurs, et al. (n.d.) it is recommended to use at least 6 excitations and 12 responses. Secondly, since low frequencies are the matter of interest, a soft rubber tip should be mounted in the excitation hammer. This ensures exact data in the frequency range where the mass line occurs, i.e. at low frequencies. The first requirement was fulfilled, i.e. 19 responses were recorded for 18 excitations. The second condition was not satisfied; instead of rubber tip, a harder one was used.

3.3.5 Calculations

The calculations were based on a method described by Leurs, et al. (n.d.), LMS (n.d), along with Okuzumi, (1991). Processing FRFs is more complicated than inclination test calculations. The solution can be divided into several steps. They all use so called “inertia restrains method”, that allows to obtain a centre of gravity through recalculations of the accelerations.

Since the accelerations are measured in different points, a reference point \( P_0 \) must be set and all accelerations transformed.

For each points following formula holds:

\[
\begin{bmatrix}
\ddot{x}_p \\
\ddot{y}_p \\
\ddot{z}_p
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 & c_3 & -c_2 \\
0 & 1 & 0 & -c_3 & 0 \\
0 & 0 & 1 & c_2 & -c_1
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_0 \\
\ddot{y}_0 \\
\ddot{z}_0 \\
\ddot{x}_{x0} \\
\ddot{y}_{y0} \\
\ddot{z}_{z0}
\end{bmatrix}
\]

(3.22.)

What can be written in matrix form:

\[
\ddot{a}_t = R \ddot{a}_0
\]

(3.23.)

I.e. we assume that the linear accelerations in every point are transformed accelerations (linear and angular) from the reference point. In articles sometimes names as “local” and “global” are used, but “reference” sounds more natural, while “local” should refer to, e.g. the accelerometers that were not placed in a defined direction. For the sake of simplicity, the calculations reference point can be just origin of the coordinate system and \([c_1 \ c_2 \ c_3]\) are respectively coordinates of point \( P: [x_p \ y_p \ z_p] \).
For one point it is a system of equations with 6 unknowns $\ddot{a}_0$ and only 3 equations, but as more points are measured, this problem gets solvable and even over-determined for more points. The solution is carried out using the least square method with reference matrix multiplications and for the complete set containing all 8 points:

$$\ddot{a}_{c\{24x1\}} = R_{c\{24x6\}}\ddot{a}_{0\{6x1\}}$$  \hspace{1cm} (3.24.)

$$R_c^T \ddot{a}_c = R_c^TR_c\ddot{a}_0$$  \hspace{1cm} (3.25.)

$$\ddot{a}_0 = (R_c^TR_c)^{-1}R_c^T \ddot{a}$$  \hspace{1cm} (3.26.)

The second step is a similar recalculation. The excitation is translated from the point of hit into the reference point - the force stays the same, but a momentum is added.

$$\begin{bmatrix}
F_{x_0} \\
F_{y_0} \\
F_{z_0} \\
M_{x_0} \\
M_{y_0} \\
M_{z_0}
\end{bmatrix} =
\begin{bmatrix}
1 & 0 & 0 \\
0 & 1 & 0 \\
0 & 0 & 1 \\
0 & -z_i & y_i \\
z_i & 0 & -x_i \\
-y_i & x_i & 0
\end{bmatrix}\begin{bmatrix}
F_{ix} \\
F_{iy} \\
F_{iz}
\end{bmatrix}$$  \hspace{1cm} (3.27.)

This transformation matrix is similar to the acceleration one:

$$E = R^TF_t$$  \hspace{1cm} (3.28.)

While measuring the cabin, accelerometers were placed only in the x-, y-, and z- directions, which simplifies the calculations due to the lack of need for transformation matrices. What is more, excitations were introduced in accordance with the respective axis, so for each hit, force has one component only.

In the following equations, the mass value from the scales is used to increase the accuracy. The centre of gravity is calculated from force equilibrium:

$$\begin{bmatrix}
0 & -m\ddot{y}_{z_0} & m\ddot{y}_{y_0} \\
m\ddot{y}_{z_0} & 0 & -m\ddot{x}_{y_0} \\
-m\ddot{y}_{y_0} & m\ddot{x}_{y_0} & 0
\end{bmatrix}\begin{bmatrix}
x_{cog} \\
y_{cog} \\
z_{cog}
\end{bmatrix} =
\begin{bmatrix}
F_{x_0} - m\ddot{x}_{y_0} \\
F_{y_0} - m\ddot{y}_{y_0} \\
F_{z_0} - m\ddot{z}_{y_0}
\end{bmatrix}$$  \hspace{1cm} (3.29.)

Again solutions are carried out using the least squares method for the complete frequency band.
The last step is solving the momentum equilibrium:

\[
\begin{bmatrix}
\ddot{\theta}_x & 0 & 0 & -\ddot{\theta}_y & 0 & -\ddot{\theta}_z
\end{bmatrix}
\begin{bmatrix}
I_{xx} & & & & & \\
& I_{yy} & & & & \\
& & I_{zz} & & & \\
& & & I_{xy} & & \\
& & & & I_{yz} & \\
& & & & & I_{xz}
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_x \\
\ddot{\theta}_y \\
\ddot{\theta}_z \\
\ddot{\theta}_y \\
\ddot{\theta}_y \\
\ddot{\theta}_z
\end{bmatrix}
\]

(3.30.)

\[
= \begin{bmatrix}
M_x - y_{cog}F_{x0} + z_{cog}F_{y0} \\
M_y + x_{cog}F_{x0} - z_{cog}F_{x0} \\
M_z - x_{cog}F_{y0} + y_{cog}F_{x0}
\end{bmatrix}
\]

The following proposal was made by Okuzumi (1991): before calculating the inertia properties, some of the previous steps were repeated, but this time the reference point was the centre of gravity itself. The author states that it reduces the inaccuracies of calculations. Since most of the equations were solved in least square method, it is advantageous to reduce the arms of forces or at least make them more comparable, especially when the cabin is approximately cuboid in shape or can be considered as if it was, so then each point has similar distance to the reference point. Then (3.30) takes form:

\[
\begin{bmatrix}
\ddot{\theta}_{x,COG} & 0 & 0 & -\ddot{\theta}_{y,COG} & 0 & -\ddot{\theta}_{z,COG}
\end{bmatrix}
\begin{bmatrix}
I_{xx} & & & & & \\
& I_{yy} & & & & \\
& & I_{zz} & & & \\
& & & I_{xy} & & \\
& & & & I_{yz} & \\
& & & & & I_{xz}
\end{bmatrix}
\begin{bmatrix}
\ddot{\theta}_{x,COG} \\
\ddot{\theta}_{y,COG} \\
\ddot{\theta}_{z,COG} \\
\ddot{\theta}_{y,COG} \\
\ddot{\theta}_{y,COG} \\
\ddot{\theta}_{z,COG}
\end{bmatrix}
\]

(3.31.)

\[
= \begin{bmatrix}
M_{x,COG} \\
M_{y,COG} \\
M_{z,COG}
\end{bmatrix}
\]

(3.32.)

Again calculating using the least squares method:

\[
\mathbf{I} = (\ddot{\theta}^T \ddot{\theta})^{-1} \ddot{\theta}^T \mathbf{M}
\]

(3.33.)

gives the inertia components. Also, the mass can be calculated then since the reference frame is in the COG. Then the reference excitation is calculated also for the COG, so forces are separated from momentums as is mass from inertia mass momentums:

\[
\begin{bmatrix}
F_{COG} \\
F_{COG} \\
F_{COG}
\end{bmatrix}
= \begin{bmatrix}
m & 0 & 0 \\
0 & m & 0 \\
0 & 0 & m
\end{bmatrix}
\begin{bmatrix}
\ddot{x}_{COG} \\
\ddot{y}_{COG} \\
\ddot{z}_{COG}
\end{bmatrix}
\]

(3.34.)
Equation (3.35) is transformed into:

\[
\begin{bmatrix}
F_{x,\text{COG}} \\
F_{y,\text{COG}} \\
F_{z,\text{COG}}
\end{bmatrix} =
\begin{bmatrix}
\ddot{x}_{\text{COG}} & 0 & 0 \\
0 & \ddot{y}_{\text{COG}} & 0 \\
0 & 0 & \ddot{z}_{\text{COG}}
\end{bmatrix}
\begin{bmatrix}
m \\
m \\
m
\end{bmatrix}
\] (3.35.)

and solved by the least square method. The masses are not indexed, because in theory they have the same physical quantity (have the same value), but are calculated for each direction and then the resultant mass is taken as the average.

In the calculations 6 excitation points were used with 24 responses which exceeds the minimum and the suggested number.

3.4 Operational Modal Analysis (OMA)

The second part of the survey was based on usage of OMA, in order to analyse the cabin’s suspension system, as well as the cabin itself, during operational conditions. The vehicle used for this survey was Rottne F18, presented in Figure 8, which is the biggest forwarder available in Rottne AB offer. The one used for the measurements was a prototype, but it is still used for tests, validation of new solutions, systems, etc.

![Tested Forwarder F18](image)

Figure 8 Tested forwarder F18

3.4.1 Preparations

To carry out the measurements, 32 accelerometers were placed in preliminary chosen points. Since the cabin and its suspension was the matter of interest, a decision was made to define 12 points. Four of them were at the bottom corners of the cabin, another 4 were in the corners of the
suspension system and the last set of four points included the upper corners of the cabin. To be more precise, 8 bottom points were placed at the ends of the 4 cylinders, one in each corner, which are the main part of the suspension system. Those 8 points were the most important regarding the behaviour of Comfort Line system during operational conditions, so in each point 3 accelerometers were placed (Figure 9 a)), one for each direction in Cartesian coordinate system. This was done to obtain the comprehensive view of the motion of these points. In each point at the top of the cabin two accelerometers were placed, to monitor motion in y- and z- direction as visible in Figure 9 b).

A very important thing was to prepare the surface where the accelerometers were intended to be placed. First of all rust covering the cylinders as can be seen in Figure 10 a), had to be removed, this was done by sandpaper. The next step was to get rid of grease; this was done by paper towel pre-wetted with alcohol. When the surface was prepared and clean, accelerometers were fastened by cyanoacrylate adhesive as presented in Figure 10 b).

Figure 9 Accelerometers placement in bottom points a) and upper points b)

Figure 10 Rusted cylinder a) and same cylinder prepared for measurements b)
Since the operational tests were conducted in outdoor conditions, all devices remaining outside the cabin had to be protected from water and moisture. The most vulnerable were joints between the accelerometers and the wires. To prevent them from soaking, the connections between them were wrapped up with self-amalgamating tape as it can be seen in Figure 11.

![Figure 11 Protection of joints from moisture](image)

The very last thing, before the measurements were run, was to check if all of the accelerometers were properly mounted, and if they were not confused while connected to the channels. To check this, each and every accelerometer was excited and the response in respective channel was verified. This ensured that all accelerometers are in the right place, and were plugged to the corresponding channels. However, in a few cases two neighbouring accelerometers gave very similar responses.

### 3.4.2 Measurements

Operational tests were made during driving on a forest road. Recorded parameters were the values of acceleration as function of time. The duration of each test was 4 minutes. Five tests were carried out; three of them, tests 1, 3, and 4 with 15% of maximum speed; one, test 2, with 25% and one, test 5, with 70% The value of the maximum speed is about 20 \( km/h \). The road chosen to drive on was straight, see Figure 12 a); this was to avoid rapid changes in direction of drive. The road’s surface in detail is presented in Figure 12 b). It is visible that there were small rocks and natural irregularities. During measurements the forwarder remained unloaded.
The following procedure was applied; after the forwarder had moved, the ranging was begun. After 4 minutes software informed that the data collection was completed, the measurements were stopped, and the next test was prepared. The procedure was repeated for every test.

Though the accelerometers were firmly fastened, much attention was paid during the measurements, if there were responses from all of them. While disassembling the accelerometers, none of them neither was come off nor was missing.

3.4.3 Data preparation

In practice, LMS® operates only on cross-powers. The software creates a set of cross-powers for each DOF marked as the reference point. In this case the reference points are points located on the chassis as we treat them as source of the excitation to the cabin. LMS® has a function analysing data in order to choose (propose) resonance frequencies, what is called peak-picking (Hermans and Van der Auweraer, 1999), as most resonance frequencies (except for those heavily damped) are visible as peaks in all cross-powers (in same way as FRFs). The eigenmode of a resonance frequency is the distribution of the peaks’ heights over a set of cross-powers. The mode’s values can be used to present the system’s vibration and interpret it, e.g. as twisting or a kind of rotation. However, Hermans and Van der Auweraer (1999, p.194) state that this method “requires a lot of engineering skills to select the peaks which correspond to system resonances”. It is possible to compare the modes using 3D diagrams (Figure 15) that should look like a modal matrix (2.26) – diagonal, modes separate. If they are not, it means that either there is very similar shape of modes, while differences were not visible or there were not enough points to describe the shapes.

To indicate the resonance frequencies of the cabin occurring in operational conditions, records of accelerations as functions of time, must be processed. This is done using LMS® software. Aforementioned data in LMS® are
called “Throughput data”. They must be added to the input basket first, then
the reference (excitation) points are indicated on the list, and then the cross-
powers are calculated. Since the suspension was measured and the cab’s
motion is the matter of interest, 12 DOFs on the chassis were chosen to be
the reference points, to look on their influence on the cab’s behaviour. It is
important to choose the correct resolution, in this case 1Hz was chosen.
When this is done, the calculated cross-powers must be moved to the basket
for further analysis. After transferring them, a set of cross-powers can be
created in the data section, in this case, all cross-powers are chosen. In
“Operational Time MDOF” toolbox, three windows are used, in ”Band” all
cross-powers are displayed separately if ”Select function” is ticked, the
second option is to display the sum of all the cross-powers; this is done by
marking ”Sum” option. In the second window, named ”Stabilisation”,
modes are marked (Figure 13). This is done with support of LMS® features
that indicate the possible mode’s location.

3.4.4 Calculations

The last window is ”Reference Factors”, which is used to display particular
modes (Figure 14) using pre-defined object’s geometry. It helps to evaluate
the measurements, since motion of the structure can be observed for
respective frequency.
In "Operational Validation" toolbox, the modes correlation matrix can be displayed. This matrix (Figure 15), gives an idea about the correlation between particular modes. This matrix allows verifying if the correct modes were chosen or if some of them were repeated. The matrix is presented in a graphical way, so it is easy to notice if it is diagonal; with weak off-diagonal terms or it contains firm correlation between particular components.

The validity of chosen modes can be verified in another way; the toolbox "Operational Synthesis" creates cross-powers from previously picked modes. Next, the correlation between synthetic cross-powers and those obtained from measurements is presented in a form of graph (Figure 16).
Figure 16 Comparison of synthetic and measured FRF
4. Results

4.1 Mass

4.1.1 Scales

The value of the weight was displayed on the scales’ screen. It indicated that the mass of the cabin amounts to:

\[ m_{\text{sc}a\text{l}e\text{s}} = 1319\text{kg} \]

The measurement was done in accordance with the description given in chapter 3.2.2.

4.1.2 Hammer test

Table 1 Mass values from hammer test

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Mass, kg</th>
</tr>
</thead>
<tbody>
<tr>
<td>X component</td>
<td>1127</td>
</tr>
<tr>
<td>Y component</td>
<td>1076</td>
</tr>
<tr>
<td>Z component</td>
<td>916</td>
</tr>
<tr>
<td><strong>Average value</strong></td>
<td><strong>1040</strong></td>
</tr>
</tbody>
</table>

4.2 Centre of gravity

4.2.1 Inclination test

Table 2 Coordinates of COG axes

<table>
<thead>
<tr>
<th>Pair (line)</th>
<th>Point</th>
<th>X, mm</th>
<th>Y, mm</th>
<th>Z, mm</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left-Right axis</td>
<td>A</td>
<td>929</td>
<td>759</td>
<td>801</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>961</td>
<td>-759</td>
<td>833,5</td>
</tr>
<tr>
<td>Front-Rear axis</td>
<td>A</td>
<td>0</td>
<td>-17,5</td>
<td>773</td>
</tr>
<tr>
<td></td>
<td>B</td>
<td>1900</td>
<td>-14,5</td>
<td>831,5</td>
</tr>
</tbody>
</table>
4.2.2 Hammer test:

The values of the COG obtained from LMS Modal Rigid Body (MRB) Toolbox (Table 4) are presented in Figure 18.

Table 4 COG coordinates from the MATLAB code and LMS MRB

<table>
<thead>
<tr>
<th>MATLAB code</th>
<th>$X_{COG,,mm}$</th>
<th>$Y_{COG,,mm}$</th>
<th>$Z_{COG,,mm}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>LMS MRB</td>
<td>870</td>
<td>-91</td>
<td>698</td>
</tr>
</tbody>
</table>

4.3 Inertia tensor

Inertia tensor computed by use of the MATLAB® code:

$$
\begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{bmatrix} = \begin{bmatrix}
889 & 9 & 55 \\
9 & 1194 & 0.5 \\
55 & 0.5 & 733
\end{bmatrix}, \text{kg} \cdot \text{m}^2
$$

Inertia tensor fed back by LMS MRB Toolbox (Figure 18):

$$
\begin{bmatrix}
I_{xx} & I_{xy} & I_{xz} \\
I_{yx} & I_{yy} & I_{yz} \\
I_{zx} & I_{zy} & I_{zz}
\end{bmatrix} = \begin{bmatrix}
940 & 41 & -8 \\
41 & 1219 & -17 \\
-8 & -17 & 770
\end{bmatrix}, \text{kg} \cdot \text{m}^2
$$
4.4 Operational Modal Analysis (OMA)

4.3.1 Resonance frequencies

Table 5 Modes from test 1

<table>
<thead>
<tr>
<th>Mode nr</th>
<th>( f_i [\text{Hz}] )</th>
<th>( \xi_i [%] )</th>
<th>Nr</th>
<th>( f_i [\text{Hz}] )</th>
<th>( \xi_i [%] )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.98</td>
<td>16.0</td>
<td>12</td>
<td>196</td>
<td>0.09</td>
</tr>
<tr>
<td>2</td>
<td>2.96</td>
<td>55.9</td>
<td>13</td>
<td>213</td>
<td>0.30</td>
</tr>
<tr>
<td>3</td>
<td>6.15</td>
<td>2.85</td>
<td>14</td>
<td>218</td>
<td>0.52</td>
</tr>
<tr>
<td>4</td>
<td>45.0</td>
<td>8.37</td>
<td>15</td>
<td>226</td>
<td>0.36</td>
</tr>
<tr>
<td>5</td>
<td>74.4</td>
<td>0.30</td>
<td>16</td>
<td>229</td>
<td>0.04</td>
</tr>
<tr>
<td>6</td>
<td>88.1</td>
<td>0.31</td>
<td>17</td>
<td>295</td>
<td>0.05</td>
</tr>
<tr>
<td>7</td>
<td>98.3</td>
<td>0.09</td>
<td>18</td>
<td>344</td>
<td>0.08</td>
</tr>
<tr>
<td>8</td>
<td>113</td>
<td>0.42</td>
<td>19</td>
<td>354</td>
<td>0.21</td>
</tr>
<tr>
<td>9</td>
<td>147</td>
<td>0.16</td>
<td>20</td>
<td>373</td>
<td>0.22</td>
</tr>
<tr>
<td>10</td>
<td>164</td>
<td>0.10</td>
<td>21</td>
<td>377</td>
<td>0.14</td>
</tr>
<tr>
<td>11</td>
<td>180</td>
<td>0.21</td>
<td>22</td>
<td>393</td>
<td>0.13</td>
</tr>
</tbody>
</table>
Table 6 Modes from tests 2 and 5

<table>
<thead>
<tr>
<th>Mode nr</th>
<th>Test2</th>
<th>Test5</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$f, [Hz]$</td>
<td>$\xi, [%]$</td>
</tr>
<tr>
<td>1</td>
<td>5.2</td>
<td>2.69</td>
</tr>
<tr>
<td>2</td>
<td>80.4</td>
<td>1.47</td>
</tr>
<tr>
<td>3</td>
<td>93.2</td>
<td>1.53</td>
</tr>
<tr>
<td>4</td>
<td>98.0</td>
<td>0.22</td>
</tr>
<tr>
<td>5</td>
<td>110</td>
<td>0.62</td>
</tr>
<tr>
<td>6</td>
<td>164</td>
<td>0.53</td>
</tr>
<tr>
<td>7</td>
<td>182</td>
<td>1.01</td>
</tr>
<tr>
<td>8</td>
<td>197</td>
<td>0.07</td>
</tr>
<tr>
<td>9</td>
<td>214</td>
<td>1.22</td>
</tr>
<tr>
<td>10</td>
<td>295</td>
<td>0.80</td>
</tr>
<tr>
<td>11</td>
<td>393</td>
<td>0.13</td>
</tr>
<tr>
<td>12</td>
<td>436</td>
<td>2.32</td>
</tr>
<tr>
<td>13</td>
<td>466</td>
<td>0.48</td>
</tr>
</tbody>
</table>

4.3.2 Validation – Modal Correlation matrices

Figure 19 Modal correlation matrix for test 1
Figure 20 Modal correlation matrix for test 2

Figure 21 Modal correlation matrix for test 5
4.3.3 Validation – Modal Cross-correlation matrices

Figure 22 Modal cross-correlation matrix for test 1 and 2

Figure 23 Modal cross-correlation matrix for test 1 and 5
Figure 24 Modal cross-correlation matrix for test 2 and 5

Table 7 Strong correlation of modes

<table>
<thead>
<tr>
<th>Common modes</th>
<th>Test1</th>
<th>Test2</th>
<th>Test5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hz</td>
<td>Hz</td>
<td>Hz</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>-</td>
<td>80.4</td>
<td>78.8</td>
</tr>
<tr>
<td>2</td>
<td>88.1</td>
<td>-</td>
<td>87.0</td>
</tr>
<tr>
<td>3</td>
<td>98.3</td>
<td>98.0</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>113</td>
<td>110</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>164</td>
<td>164</td>
<td>-</td>
</tr>
<tr>
<td>6</td>
<td>180</td>
<td>182</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>197</td>
<td>197</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>213</td>
<td>214</td>
<td>-</td>
</tr>
<tr>
<td>9</td>
<td>392</td>
<td>392</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 8 Secondary correlation of modes

<table>
<thead>
<tr>
<th>Common modes</th>
<th>Test1</th>
<th>Test2</th>
<th>Test5</th>
</tr>
</thead>
<tbody>
<tr>
<td>Hz</td>
<td>Hz</td>
<td>Hz</td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>74.4</td>
<td>-</td>
<td>78.8</td>
</tr>
<tr>
<td>2</td>
<td>74.4</td>
<td>110</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>88.1</td>
<td>93.2</td>
<td>-</td>
</tr>
<tr>
<td>4</td>
<td>197</td>
<td>-</td>
<td>194</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>197</td>
<td>194</td>
</tr>
<tr>
<td>6</td>
<td>213</td>
<td>214</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>295</td>
<td>-</td>
<td>291</td>
</tr>
<tr>
<td>8</td>
<td>-</td>
<td>295</td>
<td>291</td>
</tr>
<tr>
<td>9</td>
<td>393</td>
<td>393</td>
<td>-</td>
</tr>
</tbody>
</table>
5. Analysis

5.1 Hammer test

5.1.1 Mass line analysis

Peaks representing rigid body modes coincide with the y axis. Due to very light suspension, rigid body modes are close to 0Hz frequency, and their acceleration amplitude values exceed the scale (Figure 25). Then, for hundreds of hertz curves seem to be stable and then they rise again. However, these FRFs show accelerance, not receptance since equation (2.14) along with (2.15) connect these parameters through $\Omega^2$, so it is not true that high frequencies dominates the motion, just accelerations are higher in this region. The mass-line is placed between the last rigid body modes and the first elastic mode, so the frequency band should be limited to $<5Hz, 100Hz >$ to analyse values.

![Figure 25 Complete FRF P1x](image)

Figure 26 shows that despite that a few peaks can be considered as erroneous, the first elastic mode can be found in approximately $50Hz$. There is an indeterminate mode at $10Hz$, that is thought to be the last rigid body mode. A conclusion can be made that the mass line is between $10Hz$ and $50Hz$. Figure 27 presents the FRFs in the hit-direction that should show clear mass lines and help to choose exact band that should be used for calculation.
In Figure 27, in the first two graphs, there are functions which start to break directions after 30Hz, so this is maximal frequency that should be considered. Unfortunately the third one is a measurement failure – parts of the FRFs cross zero randomly. These FRFs would not be taken into consideration, but anyway, for calculations only FRFs with excitations in the bottom four points could be taken, because for them all 24 responses are available. The chosen analysed band is $<20Hz, 30Hz>$.
better view for a wide frequency range. During processing the obtained data it turned out, that indeed, elastic modes were clearly visible, but the application of hard tip deteriorated the mass line. This resulted in obstacles in finding a proper frequency range, in which the mass line occurs.

Processing and displaying FRFs, revealed a serious drawback of the LMS® software. While exporting the data to MATLAB® files, the order of FRFs is switched from channel-like to alphabetical one. This caused difficulties during processing FRFs for the need of inertia parameter calculations, and entailed need to perform additional ordering function. However, the problem occurs only when exporting files, LMS® itself displays them absolutely correct.

Before starting inclination, and hammer tests, the cabin was inspected for possible location of COG. Location of heavy components was checked, in order to determine toward what direction the COG can be moved. This allowed to preliminary evaluate if the inclination test was carried out in a correct way or it was not. Later on, results obtained from the inclination test, served as a reference for those from the hammer test.

5.1.2 Centre of gravity

Many factors influence the calculations of the COG. Especially one thing could affect them and this is the reference system and the positioning of accelerometers. The COG from inclination was calculated using a CAD model, while the position of accelerometers for LMS® and MATLAB® results were measured by hand. Further, the – accelerometers should be placed close to the points they are prescribed to and some accelerometers were placed even 7 centimetres away from the points. It applies to bottom accelerometers in z-direction, because it was not possible to find closer surface perpendicular to the z-axis. Comparison of the suspension results to the others shows that their differences in x directions are significant, but real value lies between them, while in z directions both values are reasonably smaller.

The measurements showed that the COG is approximately in the middle of the cab. A few centimetres deviation does not make a significant difference, but it should be included in the calculations. Figure 5 shows that hooks are placed in the right positions and the cabin is stable when hung in one point.

Consider calculating the COG as an iteration problem. The same thing goes for the inertia properties calculated with reference frame in the COG, a new COG can be gained. Iteration calculation written as:

\[ COG_{n+1} = f(COG_n) \]  

(5.1.)
what should lead to:
\[
\lim_{n \to \infty} (COG_{n+1} - COG_n) = [0; 0; 0]
\]
(5.2.)

This means that the COG should give the same value. For the first iteration:
\[
COG = 10^{-5}[-0.0078; 0.355; -0.265], m
\]
(5.3.)

It means that already in the first iteration the differences are in the order of tenths of millimetres, which allows to say that the original position of the COG is calculated with a very good precision (considering calculations capability, not data input).

The MATLAB® code feeds back results similar in order of magnitude and values to the one from LMS®. Unfortunately this is the test for which there were no reference values available. Results are logic in the sense of cabin geometry and order of magnitude.

5.2 Operational Modal Analysis

One very important property of the investigated object was noticed during data processing. In great part of observed modes, there was a significant difference between the amplitude of the suspension and amplitude of the cabin.

Out of 5 conducted tests, only 1, 2, and 5 are presented in the result paragraph, since tests 3 and 4 were carried out at the same speed as the 1st one. Tests conducted with a lower speed gave more clear modes, whereas in other cross-powers only traces of those could be found.

Chosen modes give good cross-powers match in synthesis (appendix I Figure 34, Figure 35, Figure 36) and modes correlation in validation matrix (Figure 19, Figure 20 and Figure 21). Even in test 1, where there are many off-diagonal values, they rarely regard to close modes, which would be suspicious. Two modes in far frequencies can look exactly the same and it still could be a correct model, just the number of measured points is not sufficient to provide differences in those modes. Even though an exemplary matrix of correlations is diagonal, it occurs only when the number of points is going to infinity. In test 2, the matrix looks well too, while test 5 matrix is close to being purely diagonal. On the other hand it is not a surprise that tests with smaller number of modes give better results; as far as autocorrelation itself is concerned.

The cross-correlations confirm if the chosen modes are credible and indicate the real modes. This is due to that, they connect data from different tests. Such modes are visible for following frequencies: 74.5Hz, 88.1Hz, 98.3Hz, 164Hz, 180Hz, 197Hz, 213Hz and 293Hz. Additionally 393Hz and
456Hz, in spite of being neither clear, nor correlated, can be found in nearly every test.

There are also peaks in some tests that do not appear in other tests, e.g. 45.0Hz from test 1 and 16.9Hz from test 5 and these are not correlated with any other modes.

Interesting property of modes from test 5 are their hidden correlations. There is a number of modes that are certain, some appears locally, but some of them are just shifted, e.g. 194Hz and 291Hz just under 197Hz and 295Hz. Consider also mode 87.0Hz, which shows correlation with peak 88.1Hz in the Figure 23. What is more about this test is that it omits peaks in a range of one to two hundred Hz. In some cross-powers there are small peaks, but this band is either flat or shaky.

Unfortunately rigid body modes are unclear in all tests. Many different values were chosen up to 8Hz. They are barely read from cross-powers and never well correlated. Modes even with close frequency, e.g. 2.96Hz from test 1 and 3.09Hz from test 2 had a very small correlation (Figure 23) and the same high level of error has mode 1 from test 1 what cannot be correlated with first three modes from test 3 (appendix I Figure 30).

There is a number of reasons for this state. Firstly, they could appear together – sum of two rigid body modes can appear to be interpreted as one and they were not separated. Other reason is that with finite resolution, six rigid modes of cab cannot be found. The last thing is that the time of tests had influence on the results; the longer time, the smaller frequency can be analysed. Even the fact that some modes were hardly excited (here mainly vibrations in y and x axis due to straight road and constant speed), what might diminish visibility of these modes, did not clarify the other modes.

It is interesting how, in test 1 (Figure 19), some modes periodically shows correlation. It is possible that these modes are elastic modes with similar motion, but of higher order. Unfortunately autocorrelations from tests 4&5 (Figure 21 and appendix I Figure 33) do not confirm this statement.
6. Discussion

It is advantageous to prepare well for the vibration tests and do as much work in advance as possible. Considering the selected excitation and response points before the measurements was helpful, however, there were things that could have been done in a more efficient way. For example, using cabin’s CAD model, precise coordinates of measured points could have been introduced into the “Geometry Toolbox” in advance, before the measurements were made. Next thing was to segregate wires with respect to their lengths, it would save time during measurements, since the distance between particular points and the data acquisition unit differs from each other. In other words, each wire could have been prescribed to a particular point.

Lack of explicit mass line is a serious problem for the analysis. It is strongly recommended to carry out the measurements in accordance with the theory; if rubber tip had been used, then mass line distortions probably would not have occurred. The mass line should not be a matter of interpretation; it should be firm and easy to extract from the FRFs. Then the measurement can be considered as successful. Increasing the resolution could improve the readout, but this feature affects other parameters, such as the record time. Then, an appropriate excitation would be required.

Two following issues were not included in this thesis, but are thought to be important and worth mentioning. Firstly, a simplified model of complete vehicle could be created and modelled, to obtain FRFs and compare them with those from the measurements. Secondly, it would be advantageous to conduct the mass line analysis, without introducing the mass. It would require calculating force equations along with momentum equations, using the least square method. This would result in obtaining more values at once. Since the mass is needed to derive the COG coordinates, which are used for translation of the reference frame and then the mass calculation; both would be derived at the same time. This also applies to the inertia tensor.
7. Conclusions

7.1 Rottne AB cabin

First of all, the measurements provided useful data and information about the cabin. Especially the inertia tensor and the location of the centre of gravity are data of high value. In case when the suspension system or its automatic control system will be intended to be developed; inertia properties will be of great help. Basing on the observations made during the conducted tests, it is advised to consider the implementation of an active suspension system.

OMA provided some characteristic motions, i.e. modes that, displayed, allowed comparison of motion of the cabin with the chassis motion. It was noticeable in the visualisations of each and every test, that high frequencies were not carried by the suspension to the cabin. This proves that the solutions applied in order to eliminate high frequency vibrations work well. On the other hand, rigid body modes are strong, and what is even worse they could not be extracted from cross-powers. In none of the modes in low frequencies from all tests, bouncing, tilting or rotation could be seen. However it is possible that the displayed motion was a combination of a few rigid body modes.

Some of the problems could be caused by behaviour of point 3 on the cab (front left), that was more active than any other cab’s point, which is seen in all modes in low frequencies. It is possible that due to some reason this corner was not as stiff as the others. The alternative option is that, despite LMS did not signal any problems, one or more accelerometers were broken and gave too strong responses. It is, as always, possible that channels were mistaken, despite precision of the set-up as well as later check.

Further, the obtained FRFs and cross-powers constitute a data-base that could be useful in further design and development.

7.2 Method

It can be stated that the mass line calculations results are satisfying. The accuracy is not as good as it was expected and desired, but this was caused by deteriorated mass lines and FRFs. The best example is the mass calculations. With \( m_{scale} = 1319\, kg \), the resultant value is \( m_{FRF} = 1074\, kg \), that gives about 20% error.

The advantage of the method is that it uses the multiple time least square method, that averages the results well; even if few of the FRFs are unreliable, the result does remain reliable and close to the real value.
Based on data and results obtained from OMA, a conclusion can be made that there is a need to carry out a larger number of tests. This could help to improve the reliability of the measurements as well as precision of determination of the resonance frequencies. It is worth considering conducting test drives in different conditions in order to check and compare the object’s behaviour under various sources of excitation (road surfaces). A wide test spectrum should give comprehensive results. If responses show considerable differences, it would mean that the surface is too irregular for OMA test.

The results from different tests points out that it is better to keep low speed; test series could start even from a few percent of maximum velocity. Another case is the time of measurement, as prolongation of the measurement should pay back in more clear responses in low frequencies.

7.3 LMS

Despite rough interface, LMS® unit was a very useful and reliable tool. The Modal Testing toolbox step-by-step leads through the test setup, which is very helpful in on-site conditions. The input basket allows picking desired data set for the analysis that later on can be processed in various toolboxes.

The possibility of exporting measurement records to MATLAB® files (*.mat) gives a chance to process data in any desired way. However, this does not applies to all data and some of them can be exported only to a universal file (*.uv); that turned out to be troublesome.
References


Bibliography

Appendices

Appendix 1: LMS Print screens
Appendix 2: MATLAB code
Appendix 3: Hammer test FRFs
Appendix 4: Operational Modal Analysis Cross-powers
APPENDIX 1

Figure 28 Correlation matrix tests 1 and 3

Figure 29 Correlation matrix tests 1 and 4

Figure 30 Correlation matrix tests 2 and 3
Figure 31 Correlation matrix tests 3 and 4

Figure 32 Correlation matrix test 3

Figure 33 Correlation matrix test 4
Appendix 1: 3
Jan Grzeszczak
Michał Płygawko
Figure 36 Synthesis test 5
APPENDIX 2

%Jan Grzeszczak                                       Vaxjo, 2014
%Michał Płygawko
%Mechanical Engineering – Master Programme, Master Thesis

%Code aim:
%Calculations of centre of gravity and inertia properties of Rottne Cabin
%using mass line analysis on FRFs from hammer test

clear all, clc, close all
NoV=2049;   %number of values
increment=0.5; %FRF resolution
Fd=0:0.5:1024; %Frequency domain vector
% - Uploading hits in first point
load('Test1-p1pX.mat');P1X=FRF.y_values.values';P1X=FRFpreparationT1(P1X);
load('Test1-p1mY.mat');P1Y=FRF.y_values.values';P1Y=FRFpreparationT1(P1Y);
load('Test1-p1pZ.mat');P1Z=FRF.y_values.values';P1Z=FRFpreparationT1(P1Z);

% - Uploading hits in second point
load('Test1-p2mX.mat');P2X=FRF.y_values.values';P2X=FRFpreparationT1(P2X);
load('Test1-p2mY.mat');P2Y=FRF.y_values.values';P2Y=FRFpreparationT1(P2Y);
load('Test1-p2pZ.mat');P2Z=FRF.y_values.values';P2Z=FRFpreparationT1(P2Z);

% - Uploading hits in third point
load('Test1-p3mX.mat');P3X=FRF.y_values.values';P3X=FRFpreparationT1(P3X);
load('Test1-p3mY.mat');P3Y=FRF.y_values.values';P3Y=FRFpreparationT1(P3Y);
load('Test1-p3pZ.mat');P3Z=FRF.y_values.values';P3Z=FRFpreparationT1(P3Z);

% - Uploading hits in fourth point
load('Test1-p4pX.mat');P4X=FRF.y_values.values';P4X=FRFpreparationT1(P4X);
load('Test1-p4mY.mat');P4Y=FRF.y_values.values';P4Y=FRFpreparationT1(P4Y);
load('Test1-p4pZ.mat');P4Z=FRF.y_values.values';P4Z=FRFpreparationT1(P4Z);

% - Uploading hits in fifth point
load('Test1-p5pX.mat');P5X=FRF.y_values.values';P5X=FRFpreparationT1(P5X);
load('Test1-p5mY.mat');P5Y=FRF.y_values.values';P5Y=FRFpreparationT1(P5Y);
load('Test1-p5mZ.mat');P5Z=FRF.y_values.values';P5Z=FRFpreparationT1(P5Z);

% - Uploading hits in sixth point
load('Test1-p6mX.mat');P6X=FRF.y_values.values';P6X=FRFpreparationT1(P6X);
load('Test1-p6mY.mat');P6Y=FRF.y_values.values';P6Y=FRFpreparationT1(P6Y);
load('Test1-p6mZ.mat');P6Z=FRF.y_values.values';P6Z=FRFpreparationT1(P6Z);

% TEST 2 - complementary!
% - Uploading hits in first point
load('Test2-p1pX.mat');P1X2=FRF.y_values.values';P1X2=FRFpreparationT2(P1X2);
P1X2([19:24],:)=P1X2([I3:18],:);
load('Test2-p1mY.mat');P1Y2=FRF.y_values.values';P1Y2=FRFpreparationT2(P1Y2);
P1Y2([19:24],:)=P1Y2([I3:18],:);
load('Test2-p1pZ.mat');P1Z2=FRF.y_values.values';P1Z2=FRFpreparationT2(P1Z2);
P1Z2([19:24],:)=P1Z2([I3:18],:);

% - Uploading hits in second point
load('Test2-p2pX.mat');P2X2=FRF.y_values.values';P2X2=FRFpreparationT2(P2X2);
P2X([19:24],:) = P2X2([13:18],:);
load('Test2-p2mY.mat');P2Y2 = FRF.y_values.values';P2Y2 = FRFpreparationT2(P2Y2);
P2Y([19:24],:) = P2Y2([13:18],:);
load('Test2-p2pZ.mat');P2Z2 = FRF.y_values.values';P2Z2 = FRFpreparationT2(P2Z2);
P2Z([19:24],:) = P2Z2([13:18],:);

% - Uploading hits in third point
load('Test2-p3mX.mat');P3X2 = FRF.y_values.values';P3X2 = FRFpreparationT2(P3X2);
P3X([19:24],:) = P3X2([13:18],:);
load('Test2-p3mY.mat');P3Y2 = FRF.y_values.values';P3Y2 = FRFpreparationT2(P3Y2);
P3Y([19:24],:) = P3Y2([13:18],:);
load('Test2-p3pZ.mat');P3Z2 = FRF.y_values.values';P3Z2 = FRFpreparationT2(P3Z2);
P3Z([19:24],:) = P3Z2([13:18],:);

% - Uploading hits in fourth point
load('Test2-p4pX.mat');P4X2 = FRF.y_values.values';P4X2 = FRFpreparationT2(P4X2);
P4X([19:24],:) = P4X2([13:18],:);
load('Test2-p4mY.mat');P4Y2 = FRF.y_values.values';P4Y2 = FRFpreparationT2(P4Y2);
P4Y([19:24],:) = P4Y2([13:18],:);
load('Test2-p4pZ.mat');P4Z2 = FRF.y_values.values';P4Z2 = FRFpreparationT2(P4Z2);
P4Z([19:24],:) = P4Z2([13:18],:);

%Pick frequency band:
F1 = 20
F2 = 30
%transformation into place in matrix:
Fbot = 1 + F1 / increment;
Ftop = 1 + F2 / increment;
NoC = Ftop - Fbot + 1;  % Number of Cells - band width in vector

%LMS - let's try....
coord = [0.000 0.445 0.0
         1.888 0.445 0.0
         1.888 -0.445 0.0
         0.000 -0.445 0.0
         0.536 0.547 2.018
         1.651 0.547 2.018
         1.651 -0.547 2.018
         0.536 -0.547 2.018]

%Global R
for j = 1:8
    R([3*j-2:3*j],:) = [1 0 0 0 coord(j,3) -coord(j,2);
                       0 1 0 -coord(j,3) 0 coord(j,1);
                       0 0 1 coord(j,2) -coord(j,1) 0];
end
w = (R' * R); W = inv(w);

%local Rs
R1 = [1 0 0 0 coord(1,3) -coord(1,2);
      0 1 0 -coord(1,3) 0 coord(1,1);
      0 0 1 coord(1,2) -coord(1,1) 0];
R2 = [1 0 0 0 coord(2,3) -coord(2,2);
      0 1 0 -coord(2,3) 0 coord(2,1);
      0 0 1 coord(2,2) -coord(2,1) 0];
R3=[1 0 0 coord(3,3) -coord(3,2); 0 1 0 coord(3,3) 0 coord(3,1); 0 0 1 coord(3,2) -coord(3,1) 0]';
R4=[1 0 0 coord(4,3) -coord(4,2); 0 1 0 coord(4,3) 0 coord(4,1); 0 0 1 coord(4,2) -coord(4,1) 0]';

%Storing everything in 3D matrices
FRF=zeros(24,NoV,1);Er=zeros(6,1);
FRF(:,:,1)=P1X;  E1=[1;0;0];   E1r=R1*E1;   Er(:,1)=E1r;
FRF(:,:,2)=P1Y;  E2=[0;-1;0];  E2r=R1*E2;   Er(:,2)=E2r;
FRF(:,:,3)=P1Z;  E3=[0;0;1];   E3r=R1*E3;   Er(:,3)=E3r;
FRF(:,:,4)=P2X;  E4=[-1;0;0];  E4r=R2*E4;   Er(:,4)=E4r;
FRF(:,:,5)=P2Y;  E5=[0;0;1];   E5r=R2*E5;   Er(:,5)=E5r;
FRF(:,:,6)=P2Z;  E6=[0;0;1];   E6r=R2*E6;   Er(:,6)=E6r;
FRF(:,:,7)=P3X;  E7=[-1;0;0];  E7r=R3*E7;   Er(:,7)=E7r;
FRF(:,:,8)=P3Y;  E8=[0;0;1];   E8r=R3*E8;   Er(:,8)=E8r;
FRF(:,:,9)=P3Z;  E9=[0;0;1];   E9r=R3*E9;   Er(:,9)=E9r;
FRF(:,:,10)=P4X; E10=[1;0;0];  E10r=R4*E10; E10r(:,10)=E10r;
FRF(:,:,11)=P4Y; E11=[0;-1;0];  E11r=R4*E11; Er(:,11)=E11r;
FRF(:,:,12)=P4Z; E12=[0;0;1];  E12r=R4*E12; Er(:,12)=E12r;

%--------- calculations for each hit -----------
%mass=1319  %mass of cabin

%----------- Least square calculations of COG ---------------
%----------- for all hits and each spectral line: -------------
F=zeros(3,1);K=zeros(3,3);
jump=3*NoC;
for m=1:6
    FRFr(:,:,m)=W*R'*FRF(:,[fbot:ftop],m);
    for j=1:NoC
        F([3*j-2+jump*(m-1):3*j+jump*(m-1)])=Er([1:3],m)-mass*FRFr([1:3],j,m);
        K([3*j-2+jump*(m-1):3*j+jump*(m-1)],:)=[
            0  -FRFr(6,j,m)  0]
        FRFr(4,j,m);
        -FRFr(5,j,m)  FRFr(4,j,m)
    end
end
COG=(K'*K)^-1*K'*F

%------------- Calculation of inertia properties -------------------
% Change of reference frame into COG - recalculation of position of each
% point (saved in the same matrix), transformation matrices and excitations
for p=1:size(coord,1)
    coord(p,:)=coord(p,:)-COG;
end
coord

%Global R
for j=1:8
    R([3*j-2:3*j],:)=
        [1 0 0 coord(j,3) -coord(j,2);
         0 1 0 -coord(j,3) 0 coord(j,1);
         0 0 1 coord(j,2) -coord(j,1) 0];
end
w=(R'*R);W=inv(w);

%local Rs
R1=[1 0 0 0 coord(1,3) -coord(1,2); 0 1 0 -coord(1,3) 0 coord(1,1); 0 0 1 coord(1,2) -coord(1,1) 0 0]';
R2=[1 0 0 0 coord(2,3) -coord(2,2); 0 1 0 -coord(2,3) 0 coord(2,1); 0 0 1 coord(2,2) -coord(2,1) 0 0]';
R3=[1 0 0 0 coord(3,3) -coord(3,2); 0 1 0 -coord(3,3) 0 coord(3,1); 0 0 1 coord(3,2) -coord(3,1) 0 0]';
R4=[1 0 0 0 coord(4,3) -coord(4,2); 0 1 0 -coord(4,3) 0 coord(4,1); 0 0 1 coord(4,2) -coord(4,1) 0 0]';

%Storing everything in 3D matrices
Er=zeros(6,1);
E1r=R1*E1;   Er(:,1)=E1r;
E2r=R1*E2;   Er(:,2)=E2r;
E3r=R1*E3;   Er(:,3)=E3r;
E4r=R2*E4;   Er(:,4)=E4r;
E5r=R2*E5;   Er(:,5)=E5r;
E6r=R2*E6;   Er(:,6)=E6r;
E7r=R3*E7;   Er(:,7)=E7r;
E8r=R3*E8;   Er(:,8)=E8r;
E9r=R3*E9;   Er(:,9)=E9r;
E10r=R4*E10; Er(:,10)=E10r;
E11r=R4*E11; Er(:,11)=E11r;
E12r=R4*E12; Er(:,12)=E12r;

%----------- Least square calculations of IP -------------
%----------- for all hits and each spectral line: -----------
F=zeros(3,1);K=zeros(3,6);
for m=1:6
    FRFr(:,:,m)=W*R'*FRF(:,[fbot:ftop],m);
    for j=1:NoC
      F([3*j-2+jump*(m-1):3*j+jump*(m-1)])= Er([4:6],m);
      K([3*j-2+jump*(m-1):3*j+jump*(m-1)],:)=FRFr(4,j,m) 0 0 -FRFr(5,j,m) 0 0 -FRFr(6,j,m) 0 0 0 0 0 0;
    end
end
IP=(K'*K)^(-1)*K'*F

%----------- Least square calculations of mass -------------
%----------- for all hits and each spectral line: -----------
F=zeros(3,1);K=zeros(3,3);
for m=1:6
    FRFr(:,:,m)=W*R'*FRF(:,[fbot:ftop],m);
    for j=1:NoC
      F([3*j-2+jump*(m-1):3*j+jump*(m-1)])= Er([1:3],m);
      K([3*j-2+jump*(m-1):3*j+jump*(m-1)],:)=FRFr(1,j,m) 0 0 0 0 0 0 0 0;
    end
end
massv=(K'*K)^(-1)*K'*F
mass=mean(massv)
function [ FRF ] = FRFpreparationT1( FRF )
%Jan Grzeszczak Vaxjo, 2014
% Michal Plygawko
% Mechanical Engineering - Master Programme, Master Thesis

% Function aim:
% Prepare FRFs from hammer test 1 to direct use in the main code

% a) change in FRF order from alphabetical back to channel
% 1 chair
EV=FRF(1,:);
FRF([1:18],:)=FRF([2:19],:);
FRF(19,:)=EV;
% 2 point 4
EV=FRF(10,:);
FRF(10,:)=FRF(11,:);
FRF(11,:)=EV;
% 3 point 5
EV=FRF(15,:);
FRF([14:15],:)=FRF([13:14],:);
FRF(13,:)=EV;

% b) normalise values to unity, not gravity (from g/N to ms^-2/N)
FRF=FRF*9.81;

% c) include opposite values of FRFs measured in -x, -y and -z directions
FRF(1,:)=-FRF(1,:);
FRF(2,:)=-FRF(2,:);
FRF(3,:)=-FRF(3,:);
FRF(5,:)=-FRF(5,:);
FRF(6,:)=-FRF(6,:);
FRF(9,:)=-FRF(9,:);
FRF(10,:)=-FRF(10,:);
FRF(12,:)=-FRF(12,:);
FRF(13,:)=-FRF(13,:);

% d) deal with imaginary parts according to theory
FRF=sign(real(FRF)).*abs(FRF);
end
function [ FRF ] = FRFpreparationT2( FRF )
%Jan Grzeszczak
%Michał Plygawko
%Mechanical Engineering - Master Programme, Master Thesis

%Function aim:
%Prepare FRFs from hammer test 2 to direct use in the main code

%a) change in FRF order from alphabetical back to channel
%1 chair
EV=FRF(1,:);
FRF([1:18],:)=FRF([2:19],:);
FRF(19,:)=EV;
%2 point4
EV=FRF(10,:);
FRF(10,:)=FRF(11,:);
FRF(11,:)=EV;
%3 point7
EV=FRF(14,:);
FRF(14,:)=FRF(15,:);
FRF(15,:)=EV;
%4 point8
EV=FRF(16,:);
FRF([16:17],:)=FRF([17:18],:);
FRF(18,:)=EV;

%b) normalise values to unity, not gravity (from g/N to ms^-2/N)
FRF=FRF*9.81;

c) include opposit values of FRFs measured in -x, -y and -z directions
FRF(1,:)=-FRF(1,:);
FRF(2,:)=-FRF(2,:);
FRF(3,:)=-FRF(3,:);
FRF(5,:)=-FRF(5,:);
FRF(6,:)=-FRF(6,:);
FRF(9,:)=-FRF(9,:);
FRF(10,:)=-FRF(10,:);
FRF(12,:)=-FRF(12,:);
FRF(14,:)=-FRF(14,:);
FRF(16,:)=-FRF(16,:);
FRF(17,:)=-FRF(17,:);

d) deal with imaginary parts according to theory
FRF=sign(real(FRF)).*abs(FRF);
end
<table>
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<th>Accelerance, ms</th>
<th>$	ext{rad} / 	ext{s}$</th>
<th>$\text{m} / \text{s}^2$</th>
<th>$\text{N} / \text{m}$</th>
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<td>10</td>
<td>0</td>
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</tr>
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</table>

**FRF P5/Hit DOF8**

**FRF P6/Hit DOF8**

**FRF P7/Hit DOF8**

**FRF P8/Hit DOF8**

**FRF P5/Hit DOF9**

**FRF P6/Hit DOF9**

**FRF P7/Hit DOF9**

**FRF P8/Hit DOF9**

Appendix 3: 6
Jan Grzeszczak
Michał Płygawko
Appendix 3: 8
Jan Grzeszczak
Michał Pływawko
APPENDIX 4

Appendix 4: 1
Jan Grzeszczak
Michał Płygawko
Appendix 4: 3
Jan Grzeszczak
Michał Płygawko
Appendix 4: 5
Jan Grzeszczak
Michał Płygawko