Analytical Modeling of Iron Losses for a PM Traction Machine

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By

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Abstract

Permanent magnet (PM) machines offer several advantages in traction applications such as high efficiency and high torque per volume ratio. The iron losses in these machines are estimated mostly with empirical laws taken from other types of machines or with finite element simulations (FEM). In the first part of this thesis the objective is to define an accurate analytical model for the stator yoke, teeth and rotor of a PM motor which should work well enough for all operating point (different loads and frequency).

This analytical model is found using an iterative process. After building a loss matrix and flux matrix based on FEM simulations, it is possible to curve fit each of the lines or the rows of the matrix in order to achieve the best fitting for every operating point. This is a very new approach; it was shown that it gives the possibility, even with a very limited number of FEM simulations, to achieve an accurate estimation of the losses.

The second part of this report focuses on optimizing this analytical method, comparing it with other possibilities, analyzing limits and advantages. Special attention is also given to the effects of the losses on the temperatures in different parts of the machine. In the last part of the thesis, the analytical model is used to test a new control strategy. Its goal is to reduce the total losses of the motor and optimize the ratio between torque and total losses for a given driving cycle.

Keywords: PM synchronous motor, Iron losses, Driving Cycle, Curve Fitting, Interpolation, Air-gap Flux, MTPA and MTPW
Sammanfattning

Permanentmagnetiserade (PM) maskiner erbjuder flera fördelar i traktion applikationer såsom hög verkningsgrad och högt vridmoment per volym. Föruster modeller är viktiga för att kunna dra njuta av maskinens fördelar. Järn föruster i dessa maskiner beräknas ofta med empiriska lagar tagits från andra typer av maskiner eller med finita element simuleringar (FEM). För att börja med utvecklas en analytisk modell för järn föruster i stator rygg, och tänder samt rotorn i en PM motor som bör fungera tillräckligt bra för alla arbetspunkten (olika belastning och frekvens).

Denna analytiska modellen byggs med en iterativ process. En förlust matris och magnetisk luftgapsflöde matris baserade på FEM simuleringar tas fram. Sedan väljs varje linje eller rader i matrisen för att optimera två parametrar i en analytisk uttryck för att uppnå den bäst passande förlust estimering för varje arbetspunkt. Detta är en mycket ny metod och i det här fallet ger en tillräcklig bra uppskattning av förlusterna även med ett mycket begränsat antal FEM simuleringar.

Den andra delen av rapporten fokuserar på förbättring av denna analysmetod, jämförelse med andra möjligheter, och analys av begränsningar och fördelar. Särskild uppmärksamhet ges också till påverkan av förlusterna på temperaturerna i olika delar av maskinen. I den sista delen av avhandlingen används den analytiska modellen för att testa en ny kontrollstrategi. Målet är att minska de totala förlusterna i motorn och optimera förhållandet mellan vridmoment och de totala förlusterna för en given körcykel.

Sökord: Permanentmagnetiserade synkronmotor, järn föruster, körcykel, kurvanpassning, magnetiskt luftgapflöde.
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1. Introduction
Permanent magnet (PM) synchronous motors are getting more and more used in traction applications. The main reason is the higher efficiency compared to induction motor in the same volume. Nowadays efficiency is becoming a very important parameter, since the electric energy demand is increasing and consequently electricity is more expensive.

The cooling of the machine and all the issues connected to it are of primary importance. The rotor is typically the part of the motor that is most difficult to cool. The rotor has theoretically no losses since it is magnetized with magnets (no copper wire current means no Joule losses) and rotates synchronously with the rotating field. Therefore the cooling of the whole motor can be considered less critical than for induction machine.

Typically more than 60% of the motor failures are due to bearings. As a rule of thumb decreasing the temperature of the bearings by 15 °C would double their lifetime. Most of the heat that is transferred to the bearings comes from the shaft which is directly connected to the rotor. Consequently having theoretically no losses in the rotor is a really relevant advantage for the lifetime of the motor. This is also one of the reasons that can lead to opt for the PM machine instead of induction motor.

Up till now, most of the analytical models for prediction of iron losses for PM machines are taken from empirical laws, and many of these are derived for induction machines.

The aim of this work is to find an accurate analytical model for the iron losses in the stator yoke, teeth and rotor of a PM motor and to study some control strategy in order to minimize the total losses of the machine. By analytical model is meant that if the iron losses are known for one starting operating point, at a certain frequency and flux, the iron losses of all the other operating points with different frequency and flux can be evaluated by mathematical expressions.

An accurate estimation of the iron losses is needed both for the control and for a correct estimation of the temperatures given by the thermal model. This can lead to a better use of the motor.
Fig. 1-1 shows ECO[^1] the new high efficiency technology for trains, developed by Bombardier Transportation.

The developed iron loss models are based on a comparison with finite element method (FEM) simulation estimation of the losses. This is already a strong limitation. Furthermore, the machine is assumed to be fed by purely sinusoidal currents. This means that all the losses due to inverter switching are not considered.

Chapter 2 is a brief overview on the equations and methods used conventionally for evaluating the iron losses in AC machines. The description is focusing on the one implemented in the FEM program Flux [2]. Chapter 3 is an introduction to PM motors with some of the main concepts being introduced. Chapter 4 shows the geometry of the investigated motor and the first model (called starting model). The analysis and discussion of the method used in this master thesis start in this chapter as well. In Chapter 5, the analytical model is explained in details and applied to the stator tooth iron losses. In Chapter 6, the same method is applied to the stator yoke and the rotor. In Chapter 7, the analytical model is tested on a different geometry and with different matrix dimensions. Chapter 8 shows the effect of an error in the evaluation of iron losses on the temperatures in the different parts of the motor through a sensitivity analysis. In Chapter 9, a control strategy that optimizes the total losses utilizing the analytical model developed in the previous chapter is studied. Chapter 10 contains the analysis of the obtained results and the conclusions.
2. Literature review on iron loss models in AC machines

The aim of this chapter is to give an overview about the models used for predicting iron losses and to focus on the ones implemented in the FEM software used in the simulations. A good overview about the available iron loss models and a comparison between certain models for analytical and numerical machine design methods is given in [3].

The base physical phenomenon behind losses in ferromagnetic materials is Joule heating [4]. Every time a change in the magnetization occurs there is a movement of domain walls, this creates eddy currents which turn into Joule heating. Therefore, it is important to keep in mind that to separate the iron losses for example in eddy current losses, hysteresis losses and excess losses is just an empirical approach. This is done to try to separate the different physical influences and relate them to frequency and flux density.

2.1. Overview of iron loss models

During a machine design process in order to predict the iron losses one can choose among a wide range of different iron loss models for electrical machines. It is important also to take into account the effect of the assembly and manufacturing process that can influence the iron losses.

Fig. 2-1 gives an overview of the most often used methods for determining iron losses [3].

![Fig. 2-1 Model approaches to determine iron losses in electrical machines [3]](image)

The first group of models is based on Steinmetz equation [5] which is

\[ p_{Fe} = C_{SE} f^\alpha \frac{B^\beta}{f} \]

Where \( C_{SE} \), \( \alpha \) and \( \beta \) are three coefficients determined by fitting the equation to the measurement data. The main problem of this equation is that it assumes sinusoidal waveforms for the flux density. Some
modifications of the Steinmetz equation are used to estimate iron losses for non-sinusoidal flux densities. $\bar{B}$ is the peak value of the flux density.

The second group is based on the separation of the different physical phenomena that cause the losses. The first model was presented by Jordan in [6] and separates the iron losses in eddy current and hysteresis losses.

\[
p_{Fe} = p_{hyst} + p_{ec} = C_{hyst} f \bar{B}^2 + C_{ec} f^2 \bar{B}^2
\]

A further improvement to the approach was given by Bertotti in [7] [8] [9] [10] which introduced an additional loss term $p_{exc}$ to take into account a different contribution of iron losses called excess losses or anomalous losses and to give a physical description of the loss factor $C_{exc}$.

\[
p_{Fe} = p_{hyst} + p_{ec} + p_{exc} = C_{hyst} f \bar{B}^2 + C_{ec} f^2 \bar{B}^2 + C_{exc} f^{1.5} \bar{B}^{1.5}
\]

Where

\[
C_{exc} = \sqrt{S V_0 \sigma G}
\]

$S$ is the cross sectional area of the lamination sample, $G$ is a dimensionless coefficient and $\sigma$ is the electric conductivity of the lamination. $V_0$ takes into account the grain size and the statistical distribution of the local coercive fields.

In order to obtain higher accuracy in the prediction of iron losses, it is possible to use some mathematical hysteresis models. This can be done if measurements of full hysteresis curve are available for the investigated material. Some classical hysteresis models are the ones studied by Preisach [11] [12] and Jiles/Atherton [13]. There are also several improvements of these iron loss models in literature that can be applied to steel sheets and even to complete electrical machines.

The range of iron loss models available for electrical machines is very wide; they are designed for different purposes and differ in several aspects. The models that are best suited for fast and rough estimation of iron losses are the Steinmetz equations and the loss separation model. They are the ones typically integrated in FEM simulations since the flux density can be determined for each element of the mesh in the post-processing.

In this master thesis, FEM simulations are used to calculate iron losses and to compare them with the analytical model to be developed. Therefore it is important to know and describe how this FEM simulator works with iron losses.
2.2. Iron loss determination in FEM simulations

The FEM program utilized to run electromagnetic simulations is Flux [2]. The computation of magnetic losses in Flux is dealt by means of the formulas of Bertotti [14].

The computation of the iron losses in steady state AC magnetic applications is done by means of the volume density of average power

\[
dp_{Fe} = k_f \left[ k_{hyst} f B^2 + \frac{\pi \sigma d^2}{6} f^2 B^2 + k_{exc} f^{1.5} B^{1.5} \right]
\]

Where

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_{hyst})</td>
<td>coefficient of losses by hysteresis</td>
<td>W s T(^{-2}) m(^{-3})</td>
</tr>
<tr>
<td>(k_{exc})</td>
<td>coefficient of losses in excess</td>
<td>S m(^{-1})</td>
</tr>
<tr>
<td>(\sigma)</td>
<td>conductivity of the material</td>
<td>W(T s(^{-1})-1.5) m(^{-3})</td>
</tr>
<tr>
<td>(d)</td>
<td>thickness of the lamination</td>
<td>m</td>
</tr>
<tr>
<td>(f)</td>
<td>frequency</td>
<td>Hz</td>
</tr>
<tr>
<td>(\dot{B})</td>
<td>peak value of the flux density</td>
<td>T</td>
</tr>
<tr>
<td>(k_f)</td>
<td>coefficient of filling</td>
<td>-</td>
</tr>
</tbody>
</table>

\(k_{hyst}\) and \(k_{exc}\) are typically obtained from manufacturer’s data sheet.

The average power dissipated in a volume region is written as [14]

\[
P_{Fe} = \iiint_{\text{reg}} dp_{Fe} \, dv
\]

The computation of the iron losses in transient magnetic applications is done by means of the volume density of the instantaneous power

\[
dp_{Fe}(t) = k_f \left[ k_{hyst} f B^2 + \frac{\pi \sigma d^2}{6} \left( \frac{dB}{dt}(t) \right)^2 + k_{exc} \left( \frac{dB}{dt}(t) \right)^{1.5} \right]
\]

The volume density average power over a period is

\[
dp_{Fe, av} = \frac{1}{T} \int_{0}^{T} dp_{Fe}(t) \, dt
\]

That becomes
The average power dissipated in a volume region can be written as in equation 2.6.

\begin{equation}
    dp_{\text{Fe,av}} = k_{\text{hyst}} f \dot{B}^2 k_f + k_f \frac{1}{T} \int_0^T \left[ \frac{\pi \sigma d^2}{6} \left( \frac{dB}{dt} (t) \right)^2 + k_{\text{exc}} \left( \frac{dB}{dt} (t) \right)^{1.5} \right] dt
\end{equation}

The average power dissipated in a volume region can be written as in equation 2.6.
3. **PM synchronous machines**

Historically all the high power industrial or traction motors were induction motors or DC motors with excitation windings. Permanent magnets were typically used for small applications because it was difficult to find a material capable of retaining a high-strength field. Only with the recent advances in material technology that allowed the creation of permanent magnets with high energy density, such as SmCo or NdFeb magnets, it has been possible to develop compact high-power motors without field coils.

The main advantages that PM machines offer in traction application are high torque per volume ratio. Theoretically no losses in the rotor lead to a higher efficiency of the motor.

Fig. 3-1 shows a PM motor of the MITRAC series by Bombardier Transportation.

![PM traction motor, MITRAC designed by Bombardier](image)

3.1. **dq-axis representation**

The best way to describe a PM machine with equations is the dq-axis representation in a rotating coordinate system. The coordinate system is rotating with the stator flux and therefore is synchronous to the rotor itself in the case of a synchronous motor.

The direct axis (d-axis) lies in the direction of the flux, while the quadrature axis (q-axis) is 90 electrical degrees ahead counter clockwise.

The following matrixes are used to transform all the three phase stator related quantities to a rotor related two phase rotating representation (also known as the Clarke and Park transformations) [15]:

\[
\begin{pmatrix}
  u_d \\
  u_q
\end{pmatrix}
= \begin{pmatrix}
  u_a \\
  u_b \\
  u_c
\end{pmatrix}^T * \frac{2}{3}
\begin{bmatrix}
  \cos(\omega t) & -\sin(\omega t) \\
  \cos\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t - \frac{2\pi}{3}\right) \\
  \cos\left(\omega t + \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right)
\end{bmatrix}
\]
\[(3.2)\]

\[
\begin{pmatrix}
  i_d \\
  i_q \\
\end{pmatrix}^T =
\begin{pmatrix}
  i_a \\
  i_b \\
  i_c \\
\end{pmatrix}^T \times \frac{2}{3}
\begin{pmatrix}
  \cos(\omega t) & -\sin(\omega t) \\
  \cos\left(\omega t - \frac{2\pi}{3}\right) & -\sin\left(\omega t - \frac{2\pi}{3}\right) \\
  \cos\left(\omega t + \frac{2\pi}{3}\right) & -\sin\left(\omega t + \frac{2\pi}{3}\right) \\
\end{pmatrix}
\]

The factor \(\frac{2}{3}\) allows the amplitude of the current and voltage vectors in the new coordinate system to be the same for the three phase representation; this is called current and voltage invariant method.

The Park transform is valid with the following assumptions:

- No magnetic saturation
- Stator windings with a sinusoidal distribution
- No space harmonics in the magneto-motive force distribution

### 3.2. Dynamic equations

A complete dynamic set of equations that describes the PM synchronous machine is [16]:

\[(3.3)\]

\[u_d = R_s i_d + \frac{d\Psi_d}{dt} - \frac{p}{2} \omega_r \Psi_q\]

\[(3.4)\]

\[u_q = R_s i_q + \frac{d\Psi_q}{dt} + \frac{p}{2} \omega_r \Psi_d\]

\[(3.5)\] \[\Psi_d = L_d i_d + \Psi_m\]

\[(3.6)\] \[\Psi_q = L_q i_q\]

\[(3.7)\]

\[T = \frac{3}{2} \frac{p}{2} \left(\Psi_d i_q - \Psi_q i_d\right)\]

\[(3.8)\]

\[T = J \frac{d\omega_r}{dt} + T_r\]

### 3.3. T-\(\omega\) Diagram

Field-weakening operation is extremely important for PM synchronous motors. The machine is fed with a negative d-axis current in order to de-flux the machine at high speeds. A negative d-current creates a flux in the air-gap that opposes the flux created by the PM’s. This is a way to deal with a limited voltage level applied by the inverter in order to reach higher speeds.

It is possible to plot the maximum torque as a function of the speed of the machine into a torque-speed diagram.
The maximum torque can be delivered from the machine until the point in which the maximum power is reached. The speed corresponding to that point is referred as the base speed of the motor $n_r$. The speed range between 0 to $n_r$ is called constant torque speed range (CTSR). If a higher speed needs to be reached it is necessary to reduce the torque in order to keep the power constant to the maximum value. The speed range between $n_r$ to $n_{\text{max}}$ is called constant power speed range (CPSR). Fig. 3-2 shows the CPSR and CTSR for an ideal motor.

### 3.4. Types of permanent magnet machines

Fig. 3-3 shows four main types of PM machines. The surface mounted PM offers a simple mechanical construction but poor robustness. Another simple mechanical construction but with a higher robustness is
offered by the surface inset PM motor. This solution has a slightly higher saliency compared to the surface mounted motor. The interior permanent magnet (IPM) synchronous machine is typically used for high speed applications due to the high mechanical robustness. The drawback is that the manufacturing is more complex compared to the other solutions.

The motor that is analyzed in this master thesis is an IPM motor, the geometry and the main characteristics are shown in the next chapter.

3.5. Driving cycles

A series of data points representing the speed and torque versus time of a vehicle is called driving cycle. Driving cycles are produced to assess the performance of vehicle’s motor in various ways. In this master thesis, they are utilized widely, for thermal simulations and to calculate average total losses, iron losses and make comparisons between different analytical models.

Since the most common and useful representation of operating points of the motor is the d-q current plane it becomes very interesting to show which sectors of this plane are being used and which are not with a certain driving cycle.

Depending on the control strategy adopted the same driving cycle can lead to different operating points. Typically an optimal curve is implemented in the control software in order to make the motor work as close as possible to it. A very common control strategy is the maximum torque per ampere (MTPA), more about how to obtain such curve is shown in chapter 9.

The frequency (which is directly proportional to the speed) versus time plot of one of the driving cycles that is utilized widely in this thesis is shown in Fig. 3-4. This is a driving cycle for a regional train.

![Fig. 3-4 Frequency as a function of time for one of the utilized driving cycles](image)
4. Investigated geometry

The motor studied is an IPM synchronous motor with two pole pairs. It has two magnet layers (see Fig. 4-1).

4.1. Motor data

The investigated motor data is showed in Table 2.

<table>
<thead>
<tr>
<th>Cooling</th>
<th>Air cooled self-ventilated</th>
</tr>
</thead>
<tbody>
<tr>
<td>Type of winding</td>
<td>Form winding</td>
</tr>
<tr>
<td>Number of poles</td>
<td>4</td>
</tr>
<tr>
<td>Number of slots</td>
<td>36</td>
</tr>
<tr>
<td>Nominal speed</td>
<td>5500 rpm</td>
</tr>
<tr>
<td>Max current, peak</td>
<td>700 A</td>
</tr>
<tr>
<td>Outer stator diameter</td>
<td>460 mm</td>
</tr>
<tr>
<td>Starting torque</td>
<td>3050 Nm</td>
</tr>
</tbody>
</table>

Table 2 Investigated motor data

The geometry of the analyzed motor is shown in Fig. 4-1.

4.2. Initial analytical model

In the initial analytical model there are three different analytical models for the three main parts of the machine: the stator yoke, the stator tooth and the rotor. These different areas considered for calculating the losses are shown in Fig. 4-2.
The objective of this analytical model is to derive in the most accurate way the iron losses of the motor knowing the flux maps, the currents, the geometry of the machine and the iron losses calculated with FEM simulations at only one single operating point. It is necessary to have the value (or a good approximation) of the flux for every operating point. This means that it is possible to create flux matrixes (maps) for both the flux in the d and in the q direction with rows and columns corresponding to Id and Iq axis.

One of the important characteristics of this analytical model is that it has to be as simple as possible, since it has to be implemented both in the control and for a thermal analysis of the motor.

In the given model there are two variables, the airgap flux and the frequency. The airgap flux for each operating point $\Psi_\sigma$ is calculated as follows:

\[
\Psi_\sigma = |\Psi_d + j\Psi_q - L_\sigma(i_d + j i_q)|
\]

Fig. 4-3 shows the flux in the airgap for different $i_d$ and $i_q$. The second plot with respect to $i_q$ shows clearly the non-linear characteristic of the machine. Non-linearity created by flux saturation in this case is the reason why it is very difficult to find an accurate analytical model.
Equations 4.2, 4.3 and 4.4 are the three parts of the initial analytical model. This is an empirical model inspired by the one used for induction motors.

Iron losses in stator teeth:

\[
P_{fe,th} = PPFE \cdot \frac{100 + f}{100 + f_n} \cdot \frac{f}{f_n} \cdot \frac{\Psi_\delta}{\Psi_{\delta R}}
\]  

Iron losses in rotor:
Iron losses in stator yoke:

\[ P_{fe,\text{rot}} = PPFERO \times \left( \frac{f}{f_n} \right)^2 \times \frac{\Psi_\delta}{\Psi_{\delta r}} \]

(4.3)

(4.4)

\[ P_{fe,\text{yoke}} = PPFER \times \frac{100 + f}{100 + f_n} \times \frac{f}{f_n} \times \left( \frac{\Psi_\delta}{\Psi_{\delta r}} \right)^2 \]

All these equations have a term with a quadratic dependency from the frequency. The equations corresponding to the stator have also a term with a linear dependency to the frequency. Since the quadratic dependency to the frequency is typical of eddy current losses and the linear dependency of hysteresis losses, it is possible to conclude that in the model used the rotor hysteresis losses are neglected. This can be reasonable because the iron losses in the rotor are mainly caused by the flux harmonics due to the slotting effect which are at a frequency multiple of the fundamental.

A very important step to achieve an accurate estimation of the iron losses is choosing the reference operating point. From the losses at this point, all the others will be calculated. It defines the value of the loss coefficients PPFET, PPFER and PPFERO. The values of the iron losses, airgap flux and leakage inductance for the reference operating point are presented in Table 3.

This reference point is chosen such that it can be considered in the middle of the operating area of some typical trains driving cycles.

The chosen reference operating point is:

\[ i_{d_r} = -150 \, A, \quad i_{q_r} = 150 \, A, \quad f_n = 90 \, Hz \]

(4.5)

For this operating point the coefficients can be calculated running a FEM simulation. The values are given in Table 3.

<table>
<thead>
<tr>
<th>( \Psi_{\delta r} )</th>
<th>2.91</th>
<th>Wb</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPFET</td>
<td>1710</td>
<td>W</td>
</tr>
<tr>
<td>PPFER</td>
<td>2320</td>
<td>W</td>
</tr>
<tr>
<td>PPFERO</td>
<td>197</td>
<td>W</td>
</tr>
<tr>
<td>( L_{\text{sigm}} )</td>
<td>160.584</td>
<td>( \mu H )</td>
</tr>
</tbody>
</table>

Table 3 Values of the coefficients calculated for the reference operating point

To have a global view on how the analytical model is performing it would be necessary to build a full map of iron losses in all the possible operating points of the machine. Since there are a large number of possible
operating points, in order to have an acceptable view, 225 simulations are run. This means that a 15x15 matrix can be built for each of the iron losses in the stator teeth, stator yoke and rotor.

The chosen frequencies are: 50 Hz, 90Hz which is the nominal frequency of the motor and 175 Hz which is the frequency of the machine operating at the maximum speed. The 225 simulations for each frequency are chosen such that: $I_d$ goes from 0 to -700 A, and $I_q$ goes from 0 to 700 A.

Fig. 4-4 shows the comparison between the initial analytical model and the FEM simulations. The losses are shown only for the nominal frequency.

From Fig. 4-4 it is clear that this model needs to be improved. The losses in the yoke are well predicted while in the other parts of the machine there are significant differences. The main problem in dealing with the dependency to the flux is saturation which is not considered properly in the initial model.

Fig. 4-4 shows the comparison between the initial analytical model and the FEM simulations. The losses are shown only for the nominal frequency.

From Fig. 4-4 it is clear that this model needs to be improved. The losses in the yoke are well predicted while in the other parts of the machine there are significant differences. The main problem in dealing with the dependency to the flux is saturation which is not considered properly in the initial model.

Fig. 4-4 Iron losses obtained with analytical models from induction motor and iron losses obtained with a FEM simulation for different parts of the machine at 90 Hz
5. Analytical modeling of iron losses in stator tooth of a PM machine

In this part the focus is shifted on the stator teeth iron losses. The losses at a given frequency obtained from FEM being dependent on $i_d$, $i_q$, it would be necessary to do a surface fitting to develop a suitable analytical model. However, since this is a complex tool and it is hard to set the starting equations in order to make the tool work, a multiple curve fitting is chosen instead.

5.1. Curve fitting of losses in stator tooth with $i_q$ constant (MODEL 1)

As said in the previous chapter a 15x15 matrix is built containing the iron losses in the stator teeth calculated running 15x15=225 FEM simulations at 90 Hz. Each row of the matrix has a different $i_q$ value starting from the 1st equal to 0 A to the 15th equal to 700 A. Each column of the matrix has a different $i_d$ value starting from the 1st equal to 0 A to the 15th equal to -700 A as shown in Fig. 5-1. The value of 700 A is the maximum current for the motor.

The curve fitting general equation used is (the reference point and the corresponding coefficients are kept as in chapter 4):

$$ y = PPFEt * a * x^b $$

Where
For each of these curve fittings the coefficients \(a\) and \(b\) are calculated. This means that for each value of \(i_d\) there is a different value of the two coefficients \(a\) and \(b\).

According to Bertotti’s equation of iron losses assuming the excess losses can be neglected, the iron losses depend on the frequency with a linear and a quadratic term that correspond respectively to hysteresis losses and eddy current losses.

The model used is:

\[
(5.3) \quad P_{fe,th} = PPFET \cdot a \cdot \left( \frac{100 + f}{100 + f_n} \right) \cdot \left( \frac{\Psi_\delta}{\Psi_{\delta_r}} \right)^b
\]

The values of \(a\) and \(b\) calculated as a function of \(i_d\) are shown in Fig. 5-2.

Fig. 5-2 shows that the reference operating point being chosen is fairly good since the value of \(a\) is close to 1.
The fundamental flux in the airgap decreases with increasing negative $i_d$ as shown in Fig. 4-3. A negative flux exponent $b$ means that the iron losses in the stator teeth are increasing even though the fundamental flux density is decreasing.

To explain this negative exponent, it is necessary to analyze what happens in the stator tooth when the value of $i_d$ is increasing (i.e. more negative). This is done running a FEM simulation at different values of $i_d$ keeping $i_q$ constant. The flux density waveform in the middle of the stator tooth is plotted for two different values of $i_d$, the first one is run with $i_d = -150$ A and the second with $i_d = -550$ A with $i_q$ constant to 150A. This is shown in Fig. 5-3, it is interesting to look at the spectrum of the two waveforms. The fundamental decreases with increasing $i_d$ as expected from Fig. 4-3 but the harmonic distortion is increased.

The increase of high order harmonics as the 13$^{th}$ and the 15$^{th}$ but also the lower ones is the reason why the losses are increasing even though the machine is getting de-fluxed.

Another important consideration is that the considered current values are in a much wider range than the practical ones. In the driving cycles provided for this motor, the Id current would never reach the value of 500 A because the magnets could get damaged in such case. This is clearly shown in Fig. 4-3, the flux from the magnets is totally canceled for a value of $i_d$ around -500 A.

5.2. Curve fitting of losses in stator tooth with $i_d$ constant (MODEL 2)

The same process can be done for constant values of $i_d$. This means that the curve fitting is done for each of the 15 columns of the matrix shown in Fig. 5-1. The curve fitting general equation and the model for iron losses in the stator tooth are the same as in section 5.1. The values of $a$ and $b$ calculated as a function of $i_d$ are shown in Fig. 5-4.
Fig. 5-4 shows that the choice of the reference operating point is fairly good since the value of \( a \) is close to 1 as in the previous curve fitting. This time the flux exponent starts as positive and is decreasing with increasing negative \( i_d \). The fundamental flux in the airgap increases with increasing \( i_q \) as shown in Fig. 4-3. This explains the positive flux exponent for low values of \( i_d \), because in this case the iron losses in the stator tooth are increasing if the flux density fundamental is increasing as shown in Fig. 5-5.

Fig. 5-5 Comparison between flux density in the middle of the stator tooth with \( i_q = 150 \text{ A} \) (left) and with \( i_q = 550 \text{ A} \) (right) \( i_d \) is constant to -150A
5.3. Analytic model of fitting coefficients

It is not possible or anyway it is very unpractical to run simulations and curve fit the map losses for each different motor and every possible operating point to curve fit loss maps. To avoid this, an analytical model of the variation of a and b coefficients calculated in the previous two chapters is looked for. As said previously one of the objectives is to have it as simple as possible, therefore the lowest general equation order that approximates the coefficients should be chosen.

Fig. 5-6 shows the analytical model found for the coefficients shown in Fig. 5-2. To fit properly these coefficients a cubic equation has to be used for the b coefficient and to approximate well the a coefficient even a higher order equation has to be used, a 5th order general equation. This is a big restriction for MODEL 1 since it would make the final analytical model for iron losses in the stator tooth complex.

Fig. 5-6 Analytical models of the fitting coefficients for different values of $i_q$
The general equations used to fit the coefficients are:

For the b coefficient

\[ b(x) = p_1 * x^3 + p_2 * x^2 + p_3 * x + p_4 \]  

where

\[ x = \left( \frac{i_q}{i_{max}} \right) \]  

and

\[ i_{max} = 700 A \]

\[ p_1 = 22.09 \quad p_2 = 41.31 \quad p_3 = 20.66 \quad p_4 = -0.1962 \]

For the a coefficient

\[ a(x) = k_1 * x^5 + k_2 * x^4 + k_3 * x^3 + k_4 * x^2 + k_5 * x + k_6 \]

where

\[ k_1 = 45.38 \quad k_2 = 137.2 \quad k_3 = 147.5 \quad k_4 = 63.99 \quad k_5 = 7.124 \quad k_6 = 0.4771 \]

Fig. 5-7 shows the analytical model found for the coefficients shown in Fig. 5-4. In MODEL 2 the plots of the two coefficients are much simpler and the order of the general equation used to approximate them can be lowered to a linear model for coefficient a and a quadratic model for coefficient b. This is a really big advantage of MODEL 2 because it reduces the complexity of the final equation used for the iron losses in the stator teeth.
The general equations used to fit the coefficients are:

For the $b$ coefficient

$$b(x) = p_1 \cdot x^2 + p_2 \cdot x + p_3$$

where

$$x = \left( \frac{i_d}{i_{\text{max}}} \right)$$

and
\[ p_1 = 2.860 \quad p_2 = 5.037 \quad p_3 = 2.105 \]

For the \( a \) coefficient

\[(5.10) \quad a(x) = k_1 \cdot x + k_2 \]

where

\[ k_1 = -0.697 \quad k_2 = 0.578 \]

Now that an analytical model is defined, the iron loss model in the stator tooth would be:

\[(5.11) \quad P_{fe,th} = PPFET \cdot a(x) \cdot \frac{100 + f}{100 + f_n} \cdot \frac{f}{f_n} \cdot \left( \frac{\Psi_s}{\Psi_{Sr}} \right)^{b(x)} \]

5.4. Stator tooth iron losses with the new models

The loss percentage error can be calculated and used to evaluate how close to the FEM and the analytical models are to each other. It is defined as follows:

\[(5.12) \quad \Delta P_{fe} \% = \frac{P_{fe,an} - P_{fe,FEM}}{P_{fe,FEM}} \times 100 \]

By using MODEL 1 at the three different values of frequency the loss percentage error is obtained. Fig. 5-8, Fig. 5-9 and Fig. 5-10 show respectively the loss percentage error and the losses calculated both with the analytical model and with the FEM simulation at 50Hz at 90Hz and at 175Hz. The improvement compared to the starting model is remarkable. From the plots it is possible to confirm that the dependency from the frequency is reliable. The loss percentage error plot with the color scale is a fast and easy way to understand which operating points on the current plane are more or less critical for the model (for critical is meant a high absolute value of the loss percentage error).
Fig. 5-8 Loss percentage error and iron losses in the stator tooth calculated both with the analytical model and with the FEM simulation at 50Hz of MODEL 1

Fig. 5-9 Loss percentage error and iron losses in the stator tooth calculated both with the analytical model and with the FEM simulation at 90Hz of MODEL 1

Fig. 5-10 Loss percentage error and iron losses in the stator tooth calculated both with the analytical model and with the FEM simulation at 175Hz of MODEL 1

Fig. 5-11, Fig. 5-12 and Fig. 5-13 show respectively the loss percentage error and the losses calculated both with the analytical model and with the FEM simulation at 50Hz at 90Hz and at 175Hz using MODEL 2.
Both the models are fitting the losses much better than the initial model, but it is now interesting to understand which one should be used and why.
5.5. Comparison between MODEL 1 and MODEL 2 for evaluating iron losses in stator tooth

In order to evaluate which of the two models is better the driving cycle shown in chapter 3 is considered, both at full DC-link voltage 1650 V, named CY1, and with reduced DC-link voltage 1250 V, CY2. It is interesting to check both possibilities because with low voltage the operating points tend to “leave” the maximum torque per ampere (MTPA) curve at a lower speed.

It is necessary for each time step of the driving cycle to check frequency, fluxes and currents and calculate the iron losses. Therefore an interpolation between different frequencies is needed.

The simulations and the models are run for four different frequencies: 0, 50, 90, 175 Hz. (at 0 Hz the losses in the motor are supposed to be 0).

The function interpolates four different loss maps (one for each frequency) for positive and negative $i_q$ (-700 A to 700 A) and negative $i_d$ (-700 A to 0 A). The loss map for negative values of $i_q$ is exactly the symmetrical with respect to $i_q =0$ to the positive part. For each point of the driving cycle the speed, as well as the values of $i_d$ and $i_q$ are checked in order to interpolate for both current and frequency and find the loss percentage error between the FEM simulation and the model used. The mean loss error and the mean electrical frequency of the motor are also calculated for each driving cycle.

A comparison between linear and cubic interpolation is shown in Fig. 5-14. Since the difference is very small while the computation time is much longer (more than 10 times longer) for the cubic interpolation, a linear interpolation is prefered. The method used is a 3D linear interpolation, i.e. a linear interpolation for each variable $i_d$ and $i_q$ and $f$. 
Once the interpolation is done it is possible to plot the losses for each time step of the driving cycle for both models.

The results of these interpolations are shown in Appendix A. Fig.A.1, Fig.A.2, Fig.A.3 and Fig.A.4 show the results of this interpolation applied to MODEL 1 for the two different driving cycles and Fig.A.5, Fig.A.6, Fig.A.7 and Fig.A.8 show the same thing for MODEL 2.

If a global look is given to Appendix A it can be said that both the models are quite accurate on average, MODEL 1 is slightly more accurate than MODEL 2 on the whole driving cycle for both CY1 with full voltage and CY2 with reduced voltage 1250 V. However, it is also important to notice that MODEL 1 has some operating points in which the loss percentage error is above 80% while MODEL 2 is always under 20%, so it can be concluded that MODEL 2 is more reliable and less cycle dependent than MODEL 1.

It is also really interesting to represent in the d-q current plane an iso-value map of the loss percentage error and the total current showing also operating points for both driving cycles. The operating points of the driving cycles are split between the three different frequencies, from 0 to 70 Hz the points are plotted in the 50 Hz figure, from 70 to 130 Hz the points are plotted in the 90 Hz figure and for frequencies above 130 Hz they are plotted in the 175 Hz. This is done in order to view where the operating points are on the d-q current plane for different frequencies, but also to evaluate if some of the operating points lay where the loss percentage error is high.
The iso-value maps are shown in Appendix B. Fig.B.1, Fig.B.2 and Fig.B.3 show respectively the iso-value maps for 50, 90 and 175 Hz for MODEL 1. Fig.B.4, Fig.B.5 and Fig.B.6 show respectively the iso-value maps for 50, 90 and 175 Hz for MODEL 2.

Appendix B gives a useful view of how the operating points of the driving cycle are situated in the d-q current plane, and from these plots it can be seen that the areas of the d-q current plane with high values of loss percentage error are “almost empty”. This is especially true for MODEL 2. That is why the second model can be considered more reliable.

From the considerations done so far MODEL 2 is chosen as the best one and the two main reasons are: more reliable and simpler model of the equations.
Analytical modeling of iron losses in rotor and stator yoke of a PM machine

The same procedure used in chapter 2 can be used to fix an analytical model both for the rotor and the yoke iron losses.

6.1. Rotor fitting with MODEL 2

From similar considerations from the previous chapter, more reliable and much simpler equations due to easier curve fitting of the coefficients, it has been chosen to apply MODEL 2 also to the rotor.

Fig. 6-1 shows the analytical models of the fitting coefficients for different values of $i_d$.

The general equations used to fit the coefficients are:
For the b coefficient

\[ b(x) = p_1 * x^2 + p_2 * x + p_3 \]

where

\[ x = \left( \frac{i_d}{i_{\text{max}}} \right) \]

and

\[ p_1 = 1.462 \quad p_2 = 4.077 \quad p_3 = 2.288 \]

For the a coefficient

\[ a(x) = k_1 * x + k_2 \]

where

\[ k_1 = -0.397 \quad k_2 = 0.287 \]

Now that an analytical model is defined, the iron loss model in the rotor would be:

\[ P_{fe,\text{rot}} = P_{PFERO} * a(x) * \left( \frac{f}{f_n} \right)^2 * \left( \frac{\Psi_\delta}{\Psi_{\delta r}} \right)^b(x) \]

Fig. 6-2, Fig. 6-3 and Fig. 6-4 show respectively the loss percentage error and the losses calculated both with the analytical model and with the FEM simulation at 50Hz at 90Hz and at 175Hz.

**Fig. 6-2** Loss percentage error and iron losses in the rotor calculated both with the analytical model and with the FEM simulation at 90Hz of MODEL 2
Fig. 6-3 Loss percentage error and iron losses in the rotor calculated both with the analytical model and with the FEM simulation at 50Hz of MODEL 2

Fig. 6-4 Loss percentage error and iron losses in the rotor calculated both with the analytical model and with the FEM simulation at 175Hz of MODEL 2

The dependency to the frequency is the major discrepancy in this model, the fact that at 50 Hz the losses are almost everywhere underestimated and at 175 Hz overestimated suggests a lower grade dependency to the frequency than the model that has been chosen.

6.2. Stator yoke fitting with MODEL 2

The exact same model can be applied to the stator yoke even though the starting model of the yoke is working well.

For the same reasons as above it has been chosen to apply MODEL 2 also to the stator yoke.

Fig. 6-1 Fig. 6-5 shows the analytical models of the fitting coefficients for different values of $i_d$. 
The general equations used to fit the coefficients are:

For the $b$ coefficient

\begin{equation}
    b(x) = p_1 \cdot x^2 + p_2 \cdot x + p_3
\end{equation}

where

\begin{equation}
    x = \left( \frac{i_d}{i_{\text{max}}} \right)
\end{equation}

Fig. 6-5 Analytical models of the stator yoke fitting coefficients for different values of $i_d$
and

\[ p_1 = -0.609 \quad p_2 = 0.575 \quad p_3 = 1.906 \]

For the \( a \) coefficient

\begin{equation}
(6.7) \quad a(x) = k_1 \cdot x + k_2
\end{equation}

where

\[ k_1 = -0.354 \quad k_2 = 0.965 \]

Now that an analytical model is defined, the iron loss model in the rotor would be:

\begin{equation}
(6.8) \quad P_{fe, yoke} = PPFR \cdot a(x) \cdot \left( \frac{f}{f_n} \right)^2 \cdot \left( \frac{\psi_\delta}{\psi_{\delta_R}} \right)^b(x)
\end{equation}

Fig. 6-6, Fig. 6-7 and Fig. 6-8 show respectively the loss percentage error and the losses calculated both with the analytical model and with the FEM simulation at 50Hz at 90Hz and at 175Hz.
As in the case of the rotor also here there is a small error in the dependency to the frequency. Anyway it is a minor problem since the loss percentage error in the utilized region of the current plane is still very small, the drawback of increasing the complexity of the analytical model by changing also the frequency exponent makes the correction of this error not worth it.
7. **Analytical model tested on a different geometry and lower limit of matrix size**

The final objective of this work is to find a general model for PM machines. So far the models of the different parts of the machine are an optimized analytical model that describes quite accurately the iron losses for a given geometry. Therefore, the models are tested with modified motor geometries.

7.1. **Test on different geometry**

The main objective of this chapter is to test the different models found in the previous chapters with new geometries.

The main reason to increase the complexity of the iron loss models was to take into account flux density distortion created by the slot leakage at flux-weakening and non-linear effects as saturation. The first choice in modifying the geometry is to increase the stator tooth width (more slot leakage) and decrease the stator yoke height. The stator slot area is kept constant.

The new geometry is shown in Fig. 7-1.

![Fig. 7-1 Modification of the geometry](image)

The model that is used for the stator tooth iron losses is MODEL 2 and the results for 90 Hz are shown in Fig. 7-2 and Fig. 7-3. All the losses and the coefficients $a$, $b$ are recalculated at 90 Hz for both the yoke and the tooth. The rotor geometry is unchanged. The two figures show that the analytical model is still fitting properly for this new geometry even though the flux saturation is very high in the yoke.

This is not a proof that the method works independent from the geometry, since many approximations were done considering the initial geometry. It could be interesting to see what the limit of validity for those assumptions is. At this stage the model has still the potential to be used for a wide range of machines.
7.2. Lower limit of matrix size

Simulating 225 times on FEM for each needed frequency to obtain a 15x15 matrix a procedure that takes a lot of time and creates a lot of data; this could also be a limit.

It is interesting to try to decrease the size of the matrix used to calculate the coefficients for the analytical model. To do this the same method is used with a 8x8 and 4x4 matrix.

The main problem of reducing the size of the matrix is that the approximation of the coefficients could become critical, i.e. the number of points could be too low to make an accurate and significant curve fitting of a and b.

Fig. 7-4 and Fig. 7-5 show how the two coefficients are affected by the matrix reduction in the case of the stator tooth iron loss analytical model at 90 Hz.
In Fig. 7-4 and Fig. 7-5 the coefficients corresponding to the 8x8 matrix give almost the same result as the 15x15 while the ones corresponding to the 4x4 matrix are slightly shifted. It is interesting to check how this difference in the coefficients affects the loss percentage error.
Fig. 7-6 Loss percentage error and iron losses in the stator tooth calculated both with the analytical model and with the FEM simulation at 90Hz of MODEL 2 with a 8x8 matrix

Fig. 7-7 Loss percentage error and iron losses in the stator tooth calculated both with the analytical model and with the FEM simulation at 90Hz of MODEL 2 with a 4x4 matrix

The error obtained with the 15x15 matrix is shown in Fig. 5-12 and as expected it is approximately the same as the one shown in Fig. 7-6 corresponding to the 8x8 matrix.

Comparing Fig. 7-6 and Fig. 7-7 it is possible to say that there is a difference in the loss percentage error but it is not significant. The result obtained with the 4x4 is still much better than the initial model and not far at all from a model obtained with a larger matrix. This is a very important result since it is possible to make such a model with just 16 FEM simulations and obtain an accurate enough estimation of the iron losses.
8. Sensitivity analysis

A sensitivity analysis is interesting to evaluate how much an error in the estimation of iron losses affects the temperature of the motor. This is done using the new analytical model presented in the previous chapters.

The new analytical model of the iron losses found in the previous chapter is implemented in a lumped parameter thermal model for traction motors. The thermal model is partly described in [18]. The temperatures in different parts of the motor, stator yoke and teeth, windings, magnets and bearing can be calculated for different driving cycles. For each requested torque, speed and DC-link voltage, the motor current, fluxes and losses are calculated. These parameters are used then to solve the thermal equations of the lumped parameter model and obtain the temperatures.

Four different thermal simulations are run. The first one is using the loss coefficients found for the reference operating point. The three other simulations are run changing each time only one of them, decreasing it by 20%. The driving cycle used is CY1 the one at full voltage utilized in the previous chapters.

\[
\Delta P_\% = 20\%
\]

<table>
<thead>
<tr>
<th>Coefficient</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPFE</td>
<td>1985</td>
<td>W</td>
</tr>
<tr>
<td>PPFE</td>
<td>2798</td>
<td>W</td>
</tr>
<tr>
<td>PPFE</td>
<td>200</td>
<td>W</td>
</tr>
</tbody>
</table>

Table 4 Reference operating point iron loss coefficients

The same thermal simulation is run reducing by 20% one of the reference operating point iron loss coefficients. It is then possible to calculate the temperature percentage error for each time step and for each part of the motor. The temperature percentage is defined as:

\[
\Delta T_\% = \frac{T_0 - T_1}{T_0} \times 100
\]

Where \(T_0\) is the temperature calculated for the original values of the iron loss coefficients and \(T_1\) is the one calculated for one of the coefficients with 20% variation.

8.1. Temperature differences

In this paragraph the effect of how much an error in the estimation of the iron losses in the different parts affect the temperatures of the different of the motor is studied for a given driving cycle.

The coefficients used for the stator tooth are shown in Table 5 and have the same values as for reference operating point except PPFE is decreased by 20%. This is a preliminary step to calculate later on the sensitivity coefficients for the different parts of the motor.
Fig. 8-1 shows $\Delta T = T_0 - T_1$, the stator and rotor temperature differences with 20% $\Delta P$ iron loss in stator tooth.

The plot on the left in Fig. 8-1 is showing the temperature difference for the three main parts of the stator, while the one on the right is showing the temperature error for the PMs and the bearings. The temperature difference in Fig. 8-1 at steady state is quite relevant, in the PMs the coil and the tooth is even above 5 °C. The second analysis is run with the coefficients shown in Table 6, these are the same coefficients of the reference operating point with the PPFER decreased by 20%.

Fig. 8-2 shows the stator and rotor temperature difference with 20% $\Delta P$ iron loss in stator yoke.

### Table 5 Coefficients used to evaluate the temperatures for an error of stator tooth iron losses

<table>
<thead>
<tr>
<th>PPFET</th>
<th>1588 W</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPFER</td>
<td>2798 W</td>
</tr>
<tr>
<td>PPFERO</td>
<td>200 W</td>
</tr>
</tbody>
</table>

### Table 6 Coefficients used to evaluate the temperatures for an error of stator yoke iron losses

<table>
<thead>
<tr>
<th>PPFET</th>
<th>1985 W</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPFER</td>
<td>2238 W</td>
</tr>
<tr>
<td>PPFERO</td>
<td>200 W</td>
</tr>
</tbody>
</table>
The third analysis is run using the loss coefficients in Table 7 with PPFERO decreased by 20%.

Table 7 Coefficients used to evaluate the temperatures for an error of rotor iron losses

<table>
<thead>
<tr>
<th>Loss Type</th>
<th>Value (W)</th>
</tr>
</thead>
<tbody>
<tr>
<td>PPFET</td>
<td>1985</td>
</tr>
<tr>
<td>PPFER</td>
<td>2798</td>
</tr>
<tr>
<td>PPFERO</td>
<td>160</td>
</tr>
</tbody>
</table>

Fig. 8-3 shows the stator and rotor temperature difference with 20% ΔP iron loss in stator rotor.

It is now possible to calculate the sensitivity coefficients.

8.2. Sensitivity coefficients

The sensitivity coefficients are calculated as:

\[ s = \frac{\Delta T}{\Delta P} \]
These coefficients are a good indicator of how much an error in one of the iron loss models effects the estimation of the temperatures in the different parts of the motor.

The values of $s$ shown in Table 8 are obtained using the mean $\Delta T_{\%}$ over one cycle after four cycles to reach the steady state.

<table>
<thead>
<tr>
<th>Loss error</th>
<th>Temp. error</th>
<th>Temp. error</th>
<th>Temp. error</th>
<th>Temp. error</th>
<th>T. error</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rotor</td>
<td>0,0112</td>
<td>0,0116</td>
<td>0,0106</td>
<td>0,0259</td>
<td>0,0153</td>
</tr>
<tr>
<td>Tooth</td>
<td>0,1759</td>
<td>0,1857</td>
<td>0,1697</td>
<td>0,1791</td>
<td>0,1526</td>
</tr>
<tr>
<td>Yoke</td>
<td>0,1299</td>
<td>0,1372</td>
<td>0,1555</td>
<td>0,1311</td>
<td>0,1257</td>
</tr>
</tbody>
</table>

The rotor losses are a small part of the losses considered. Even if there is a relatively high error in evaluating these losses the effect on the temperatures is ten times smaller than the same error in percentage in estimating the losses in the stator tooth or yoke.

It is always important to remember that the temperature in the magnets is one of the most critical because it can lead to damaging the PMs.
9. Maximum torque per watt control

One of the common control strategies utilized for PM machines is the maximum torque per ampere (MTPA) strategy. The principle is to always try to make the motor work with the maximum torque for each value of current.

9.1. MTPA

The MTPA curve can then be obtained from the definition, for each step of growing current amplitude the current angle that gives the maximum torque is taken. As said in chapter 4 the flux maps are known from the FEM simulations. This means that the matrices $\Psi_d$ and $\Psi_q$ as a function of $i_d$ and $i_q$ are known.

Since

$$i_d = -I \sin \theta$$

(9.1)

$$i_q = I \cos \theta$$

(9.2)

Where $\theta$ is the current angle. It is easy to obtain

$$\hat{i}_d = \begin{bmatrix}
i_{d=0,\theta=0} & \cdots & i_{d=0,\theta=\frac{\pi}{2}} \\
\vdots & \ddots & \vdots \\
i_{d=I_{max},\theta=0} & \cdots & i_{d=I_{max},\theta=\frac{\pi}{2}}
\end{bmatrix}$$

(9.3)

$$\hat{i}_q = \begin{bmatrix}
i_{q=0,\theta=0} & \cdots & i_{q=0,\theta=\frac{\pi}{2}} \\
\vdots & \ddots & \vdots \\
i_{q=I_{max},\theta=0} & \cdots & i_{q=I_{max},\theta=\frac{\pi}{2}}
\end{bmatrix}$$

(9.4)

It is then possible to obtain the two flux matrixes by linear interpolation from FEM simulation results:

$$\hat{\Psi}_d = \begin{bmatrix}
\Psi_{d=0,\theta=0} & \cdots & \Psi_{d=0,\theta=\frac{\pi}{2}} \\
\vdots & \ddots & \vdots \\
\Psi_{d=I_{max},\theta=0} & \cdots & \Psi_{d=I_{max},\theta=\frac{\pi}{2}}
\end{bmatrix}$$

(9.5)

$$\hat{\Psi}_q = \begin{bmatrix}
\Psi_{q=0,\theta=0} & \cdots & \Psi_{q=0,\theta=\frac{\pi}{2}} \\
\vdots & \ddots & \vdots \\
\Psi_{q=I_{max},\theta=0} & \cdots & \Psi_{q=I_{max},\theta=\frac{\pi}{2}}
\end{bmatrix}$$

(9.6)

Then the torque can be calculated ($p$ is the number of poles)
\[ T_{ij} = \frac{3}{2} p \frac{p}{2} \left( \Psi_{dij} i_{qij} - \Psi_{qij} i_{dij} \right) \]

It is possible to obtain the torque matrix by iterating the torque calculation for each element of the current and flux matrixes

\[
\tilde{T} = \begin{bmatrix}
T_{l=0, \theta=0} & \cdots & T_{l=0, \theta=\frac{\pi}{2}} \\
\vdots & \ddots & \vdots \\
T_{l=\text{max}, \theta=0} & \cdots & T_{l=\text{max}, \theta=\frac{\pi}{2}}
\end{bmatrix}
\]

Then the index of the maximum value element for each row of the torque matrix is taken. Using those indexes to check what are the corresponding values of \( I_d \) and \( I_q \) the MTPA curve can be obtained.

The results of these calculations are shown in Fig. 9-1.

![MTPA curve on current plane](image1)

![MTPA curve on flux plane](image2)

\textit{Fig. 9-1 MTPA curve on current and flux planes}

This method of control optimizes the ratio between torque and copper losses. Since in the PM motor the iron losses are a consistent part of the total losses it can make sense to try and optimize the ratio between torque and total losses. It is also interesting to see what happens if the optimization is done just with the iron losses and what the effects of these changes on the temperatures are.

\section*{9.2. MTPW for iron losses and total losses}

The iron losses of the different parts of the motor can be calculated with the analytical models (using MODEL 2) found in chapter 5 and 6. The total iron losses can be found as

\[
P_{fe} = P_{fe, \text{th}} + P_{fe, \text{yoke}} + P_{fe, \text{rot}}
\]
It is now possible to find the index of the maximum value element for each row of the ratio between the torque matrix and the iron loss matrix. The MTPW with respect just to iron losses (MTPWI) is obtained using those indexes to check what are the corresponding values of $I_d$ and $I_q$.

An auxiliary matrix $\tilde{H}$ is built where

\[
H_{ij} = \frac{T_{ij}}{P_{fe ij}}
\]

The index of the element with the maximum value for each row of $\tilde{H}$ is taken

\[
ind_i = \max(H_{i1}, ..., H_{ij}, ...)
\]

The plot of the values of $I_d$ and $I_q$ corresponding to the indexes found is the MTPWI curve.

The exact same process can be done with the total losses by adding the copper losses.

\[
P_{cu} = 3 \, R_{20^\circ}(1 + \alpha_{cu}(T - T_0)) \frac{i^2}{2}
\]

Where $T_0$ is 20°C, $T$ is the temperature of the motor and $R_{20^\circ}$ is the phase resistance for the machine at $T_0$

\[
R_{20^\circ} = 17.03 \, m\Omega
\]

And the temperature coefficient for copper [19]

\[
\alpha_{cu} = 0.0039 \, K^{-1}
\]

The copper losses theoretically are not affected by the current angle, the matrix built for copper losses is

\[
\tilde{P}_{cu} = \begin{bmatrix}
P_{cu l=0, \theta=0} & \cdots & P_{cu l=0, \theta=\pi} \\
\vdots & \ddots & \vdots \\
P_{cu l=\text{max}, \theta=0} & \cdots & P_{cu l=\text{max}, \theta=\pi}
\end{bmatrix}
\]

The elements of the auxiliary matrix $\tilde{H}'$ are calculated as

\[
H'_{ij} = \frac{T_{ij}}{P_{fe ij} + P_{cu ij}}
\]

By taking the index of the element with the maximum value for each row of $\tilde{H}'$
It is also very important to consider different frequencies since the iron losses are very frequency dependent. The frequencies considered are the same used in the previous chapters for this motor, 50 Hz, 90 Hz and 175 Hz.

Fig. 9-2 and Fig. 9-3 are showing the results for a temperature of the windings of 20°C while Fig. 9-4 and Fig. 9-5 for a temperature of 200°C.
Fig. 9-3 MTPW curves at 20°C on flux plane

Fig. 9-4 MTPW curves at 200°C on current plane
What comes out first from Fig. 9-2, Fig. 9-3, Fig. 9-4 and Fig. 9-5 is that the frequency does not affect the MTPWI. The reason is that every element $H_{ij}$ gets scaled in the same proportion when the frequency changes; this means that the maximum value for every row of the matrix is almost always the same for every frequency considered. The effect is relevant if the total losses are considered since the copper losses are not (in first approximation) affected by the frequency.

Fig. 9-6 and Fig. 9-7 are showing respectively the results of the simulations with MTPA and MTPW as a control strategy. The plots are showing the iron losses and the torque as a function of time but also the operating points on the current plane and also a plot that shows how the iron losses change with the total current. The average iron loss for the whole cycle is calculated. For the MTPA control strategy there is an average iron loss of

$$\overline{P_{fe, MTPA}} = 2943 \text{ W}$$
While with the MTPWI

$$\bar{P}_{fe,MTPWI} = 2686 \text{ W}$$

There is a loss saving of

$$\Delta\bar{P}_{fe} = \bar{P}_{fe,MTPA} - \bar{P}_{fe,MTPWI} = 257 \text{ W}$$

That in percentage would be

$$\Delta\bar{P}_{fe\%} = \frac{\bar{P}_{fe,MTPA} - \bar{P}_{fe,MTPWI}}{\bar{P}_{fe,MTPA}} \times 100 = 8.73\%$$

This result is showing that the control is working, but the goal is to achieve a better overall efficiency not just decrease the iron losses. This MTPWI is not taking at all into consideration the copper losses; this means that the total losses might be worse than with the MTPA (this will be confirmed in the temperature analysis in the next section of this chapter).

![Graphs showing iron losses, torque, and operating points](image)

Fig. 9-6 Iron losses with a MTPA control strategy
To obtain a decrease of the total losses the MTPW control strategy must be adopted instead. Fig. 9-8 and Fig. 9-9 are showing respectively the results of the simulations with MTPA and MTPW as a control strategy. The plots are showing the total losses and the torque as a function of time but also the operating points on the current plane and also a plot that shows how the total losses change with the total current. The average total loss for the whole cycle is calculated.
For the MTPA control strategy there is an average total loss of

\[ \bar{p}_{\text{Tot,MTPA}} = 9254 \, \text{W} \]

While with the MTPW
There is a total loss saving of

\[ \Delta \bar{P}_{\text{tot}} = \bar{P}_{\text{tot,MTPA}} - \bar{P}_{\text{tot,MTPW}} = 35 \text{ W} \]

That in percentage would be

\[ \Delta \bar{P}_{\text{tot,\%}} = \frac{\bar{P}_{\text{tot,MTPA}} - \bar{P}_{\text{tot,MTPW}}}{\bar{P}_{\text{tot,MTPA}}} \times 100 = 0.38\% \]

The new control strategy decreased the total losses but in a very small percentage. A much better result could be achieved if the optimal operating points could be adjusted for different frequencies. The MTPW control strategy would also give better results (compared with MTPA) if the iron losses get to be a more relevant part of the total losses.

It is now interesting to look at what is the effect on the temperatures in the motor of using MTPW compared to the MTPA. Fig. 9-10 shows the difference in temperatures between a thermal simulation run with MTPA and one run with MTPWI. Fig. 9-11 shows the difference in temperatures between a thermal simulation run with MTPA and one run with MTPW control strategy.

![Fig. 9-10 Stator and rotor temperature difference between MTPA and MTPWI control strategies](image)

Fig. 9-10 confirmed that optimizing just the iron losses is not the best choice for the overall efficiency and consequently also for the temperatures. The MTPA control strategy leads to lower total losses and therefore a lower temperature.
As expected Fig. 9-11 shows that the MTPW is slightly better than the MTPA when it comes to total losses and therefore to temperatures in the different parts of the motor. Since the difference in total losses is not a relevant percentage, the temperature difference is very small.
10. Conclusions and future work

10.1. Conclusions
This master thesis investigated a way to derive analytical models for iron losses based on a curve fitting process using FEM simulation results. The process to obtain an analytical model developed in this master thesis results to be very effective. As shown in Chapter 5 and Chapter 6, the developed analytical models are estimating properly the iron losses for stator tooth, stator yoke and rotor for the complete range of operating points. A very essential characteristic of the method is that the fitting equations are simple and easy to implement in the control and in a design optimization process.

It is interesting to point out that it is a very general process to obtain an analytical model for iron losses. That the exact same process works for the stator tooth, stator yoke and rotor without the need of any modification. Even though it has been just applied to an IPM machine with two different geometries it should work as well for all types of PM machines. No assumption is made about internal magnets during the whole process.

The main issue is that in order to apply this process and find an accurate analytical model a precise FEM description of the geometry and the materials is needed. Indeed, the whole process to obtain the analytical model for the iron losses is based on the assumption that the FEM simulations give accurate results.

It has been also shown that with a very small number of FEM simulations, it is still possible to obtain a very accurate analytical model. This means that even though the process implies the use of FEM simulations, it is not as time consuming as it may seem at first sight.

The sensitivity analysis chapter shows the effect of an error in the estimation of the iron losses on the temperatures of the stator and rotor. For the analyzed motor, iron losses in the stator tooth and in the yoke are the main contribution. This means that it is much more important to have a precise model there than in the rotor. The more accurate the loss prediction is the better the usage of the motor, which might imply a cost saving in the cooling system.

Once an adequate iron loss analytical model is available, it can be used to minimize the total losses. Chapter 9 shows how this can be implemented. The result of the MTPW control strategy is that there is a small percentage decrease in the total losses compared with the MTPA. The main advantage of this is that the loss saving is “for free”. By this is meant that the only change to be done is to implement the new optimal points found in the control software, which is costless. The loss saving between MTPW and MTPA is very small for this investigated motor but it might be better when it comes to different motors. If the same control strategy would be applied to a motor that has a higher percentage of iron losses on the total losses the loss saving would be more relevant.
10.2. Future work

The process to obtain the analytical model should be tested for a wide range of different PM machines. It would also be interesting to try it on synchronous reluctance machines.

A further development could be to change the voltage source from sinusoidal to PWM with different modulation patterns and analyze how the model works with harmonics due to the inverter. It could turn out that the model as it is cannot predict accurately the iron losses with a non-sinusoidal source. However it is quite likely that increasing the complexity with a higher order of the equations and more degrees of freedom would solve the issue.

Furthermore, the most important step to verify the reliability of the analytical model would be to be able to compare not only with FEM simulations but with measurements.

The MTPW optimal points in chapter 9 are obtained for the nominal frequency. This is a practical solution, but not the optimal one. The lowest total losses would be reached implementing a control that changes the optimal points with respect to the speed (frequency).
A. Appendix: Time-plots of driving cycles

Fig. A-1 Loss percentage error of the stator tooth iron losses using MODEL 1 for driving cycle CY1 (full cycle), the green line shows the average value.

Fig. A-2 Loss percentage error of the stator tooth iron losses using MODEL 1 for driving cycle CY1 (first 500 s), the green line shows the average value.
**Fig. A-3** Loss percentage error of the stator tooth iron losses using MODEL 1 for driving cycle CY2 at reduced voltage 1250 V (full cycle), the green line shows the average value.

**Fig. A-4** Loss percentage error of the stator tooth iron losses using MODEL 1 for driving cycle CY2 at reduced voltage 1250 V (first 500 s), the green line shows the average value.
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Fig. A-7Loss percentage error of the stator tooth iron losses using MODEL 2 for driving cycle CY2 at reduced voltage 1250 V (full cycle), the green line shows the average value.

Fig. A-8 Loss percentage error of the stator tooth iron losses using MODEL 2 for driving cycle CY2 at reduced voltage 1250 V (first 500 s), the green line shows the average value.
B. Appendix: Iso-value maps

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Fig. B-2 Iso-value plot of loss percentage error of the stator tooth iron losses at 90 Hz using MODEL 1

Fig. B-3 Iso-value plot of loss percentage error of the stator tooth iron losses at 175 Hz using MODEL 1

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