Mathematics can be simplified if teachers will increasingly focus on reaching students’ mathematical awareness.

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Abstract
Mathematics is perceived as a difficult subject to many students internationally and Sweden ranks among EU/OECD countries that perform poorly, cited in the TIMSS 2007 assessments. The aim of this essay is to investigate the causes of the poor performance in Mathematics in many of Swedish Secondary schools and it will contribute to the solutions of this problem. The framework of this essay takes into consideration issues on; school curriculum, instruction of knowledge by teachers, construction of knowledge by students, their interactions and experiences in the situation of learning. Teaching methods have been seen as the starting point upon which students can be invited to carry out communications, reasoning and arguments in mathematics. This can be useful in developing capabilities of solving mathematical problems as recommended in Secondary School regulations book. The interviewed teachers’ experiences and approaches in selecting teaching methods have been interpreted to correspond with students’ involvement in learning mathematics. The essay has found out that there is a positive effect on understanding mathematics if teachers can select teaching methods that suits a specific object of learning. This however, has left one disturbing question for further researchers to answer; to whether it is enough for highly motivated and hard working students to study mathematics based on memorized wisdom (ideas) as it does not promote mathematical awareness.

Keywords
Mathematics, capabilities, awareness, space of learning and teaching methods
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1. Introduction
Mathematics is known to be difficult to many students all over the world. This claim has attracted many researchers in education to investigate the cause as many journal and articles are published. It is some of these research articles which inspired this investigation to analyze why mathematics is difficult to Swedish’ secondary school students. According to the article from Dagen Nyheter (DN) published on 2011-02-05 it says that performance in mathematics and science subjects has declined based on the study by TIMSS (Trend in International Mathematics and Science Studies) in 2007. This study expresses its concern in such way that “examining the assessments in depth, comparing their features, frameworks, and items, can provide specific information on how they are similar and different and allow us to understand how they complement one another” (TIMSS 2007 s 14). The assessments means even the findings as compared to PISA(Program for International Students Assessment) whereby the declining performance at the primary and secondary school levels from EU/OECD countries was also reported (TIMSS, 2007 cited in DN, 2011). It follows that the problem of poor performance in mathematics still persists in Sweden and should therefore be researched.

The Swedish School Inspection team took a survey about the declined performance in mathematics in secondary schools. A similar survey was done on primary schools before and the results were that teaching of mathematics has not been varied enough to a level required by the curriculum (School Inspections Report 2010: 13). It follows that teachers fail in their methods of teaching by doing mechanical calculations and limiting the content knowledge to students by selecting those tasks which teachers feel comfortable to work with.

It appears in DN that Swedish students attempt only those tasks which they feel comfortable with and leave out those they find difficult. *Svenska elever klarar de uppgifter de är vana vid, men har svårigheter att använda sina kunskaper i nya situationer (som ett TIMSS-prov).* *Kunskapsluckorna kan vara svåra att upptäcka för den enskilde läraren, som anpassar sina
prov efter undervisningen (DN) published on 2011-02-05. That is to say, Swedish students have problem in answering mathematical problems which they are unfamiliar with. This investigation responds to the schools inspection findings and tries to dig into how both teachers and students can play in the knowledge exchange to lessen the perceived difficulty in mathematics. The aim of this essay is to investigate on teachers’ approach in teaching mathematics to secondary school students and the students’ involvement in learning mathematics.

1.1 Purpose
This assay takes up the challenge to investigate why mathematics is known to be difficult to many secondary school students in Sweden. It is mostly based on variation theory where the space of learning constitutes shared responsibilities between the teacher and the students (Marton et. al. 2004); the following objective questions are based on in searching for the answer to this problem:

- How do teachers select relevant methods of teaching that maximize understanding of intended lessons’ content firstly to themselves and secondly to students, when planning lessons? Do they follow the school curriculum when setting goals and preparing tasks needed by the students?
- How much interaction is there in relation to when students are capable of learning new knowledge? Are students experiences take into consideration in situations when they’re experiencing new knowledge in the space of learning?

It is hypothesized that well selected teaching method suitable for a specific aspect of the object of learning has the potential to improve students’ achievements in mathematics.

2. Theoretical framework

2.1 Complexities
The background of teaching and learning of mathematics together with its complexities is the focus of this section. A survey undertaken by the Swedish’ schools inspection finds teachers’ limited knowledge on the content of instruction as stated in the school curriculum to be partly responsible for students’ poor achievements in mathematics: “a big problem for that matter as teachers lack understanding of the syllabus,” (cited Skolinspektionsrapport 2010:13). Furthermore, some teachers lack the requisite content knowledge in some areas of
mathematics according to the report. The lack of content knowledge of some mathematics teachers has longed been argued as Anderson cites Freudenthal (1973 in Haggarty 2002, s.28) that “soon mathematics will be taught in lower grades of the schools of quite, a few countries by people who do not even know what mathematics is.” In the same vein Ball et al. (2001 cited in Baumert et al.2009, s.138) add that if teachers don’t have enough understanding of mathematical content they fail to deliver that content students need, something that can never be replaced by pedagogical skills alone. According to the School Inspections report teachers’ lack of content knowledge and the pedagogical skills are considerable challenges as students were not inspired to learn mathematics in many of the lessons the inspectors observed. Mathematics teachers have an important responsibility of helping learners to see mathematics as elegant and to create the possibility to succeed in learning it (Haggarty, 2002). The author adds that teachers’ beliefs play crucial role in the way they handle mathematics. For instance one might believe that it is enough to explain bite-size pieces of mathematics to students and let them go on with exercises at length until they develop a routine of answering tasks; whereas another may believe in creating varying situations under which students can explore and construct knowledge out of their experiences. It follows that the former uses relatively close questions to test students understanding on what has been taught; whereas the latter ask relatively open questions thereby enabling students to produce solutions based on their own understanding of the problems: the instructional style required by the Swedish curriculum.

Contrary to the requirement of the Swedish curriculum, the Skolinspektionsrapport (2010:13 s, 15 - 19) discovered that 85% of the schools visited do not consider the type of teaching which is based on exploration and problem solving criteria as stated in the syllabus. The findings of the schools inspection survey suggest that mathematics’ teachers encourage students to memorize concepts and procedures instead of comprehension of the concepts and procedures. Teachers believe in simplifying work to slow learners by focusing on elaboration and carrying out mechanical calculation without considering activities that can awaken students’ awareness. The kind of awareness means the underlying mathematical facts and properties of the object of learning. This method of teaching based on such a belief can only be short lived solution and possibly resulting into students’ failure to ever understand mathematics (Skolinspektionsrapport, 2010). That means, it is not enough for teachers to explain mathematical concepts and carry out mechanical calculations but they also should awaken students’ consciousness of the concepts; perhaps by taking students through practical mathematical problems. Richard Noss (cited in Haggarty 2002 s, 45) states that “it is hard to
understand mathematical ideas until you have used it, until you have seen its connection with other mathematical ideas and possible application.” Perhaps, the teachers lack the requisite content knowledge (so he memorizes concepts and procedures to teach) and/or the pedagogical skills: either way the need for teachers to perform is unquestionably clear; however, the role of students is largely ignored so far. Both teachers and students have roles to play to alleviate the difficulty in mathematics and hence improve performance.

Furthermore, the grade that students get is a product of the efforts of both the teacher and the students. Thus, working to help teachers improve on their content knowledge and instructive approach for better standards in mathematics is not enough; it is equally important to find out if students participate (get involved) actively in both classroom and out of classroom mathematical activities. Sherin (2002 cited in Walshaw & Glenda, 2008) notes “that in classroom exchange of ideas, typically, teachers negotiate between three areas of knowledge: their understanding of the subject matter, their perception of the curriculum materials, and their personal theories of students learning.” It is interesting Sherin (2002) implicitly describes teaching and learning as “exchange” yet left the negotiation of the other party (student) blank.

2.2 Curriculum
Firstly, the curriculum is treated largely important as far as teaching mathematics is concern in Swedish secondary school. It is the view of this essay to make curriculum part of the framework for the investigation. Secondly, variation theory will be used as well for the analysis. The assumption in consideration is that students can easily learn mathematics if teachers succeed in varying the mathematical content and open discussion to students to experience learning new tasks.

According to school inspections report on its recommendation to secondary school mathematics teachers, students must have objectively directed and strongly developed designed work that will improve teaching of mathematics in many of the visited schools (Skolinspektionsrapport 2010: 13. s 8). There are two types of objectives that Swedish curriculum has: ’mål att uppnå’ which means attainable objectives, and ’mål att sträva mot’ which means desirable objectives. It appears in the school report that attainable goals describe that mathematical content intend for students to learn. Meanwhile such goals aim at the specific object of learning that students shall achieve by the end of every teaching moment, the desirable goals aim at the general mathematical concepts and their applications that
students shall develop. The desirable objectives are described as something meant for picking up the underlying qualities towards teaching. Those objectives will be based on to evaluate students acquired knowledge. It follows that easily can teachers only concentrate on attainable objectives without paying attention to the desirable objectives because the former has to do with the specific content. It is, however, emphasized in the report as well as the secondary school regulation book (Werner 2009/2010, s.235) that desirable objectives are highly recommended in teaching mathematics. It implies that to a greater extended shall desirable objectives be treated as objectives for capabilities which students need in situations such as problem solving, deliberate reasoning, computational skills, relationships between abstract and concrete and communication using mathematical language(ideas). According to Lars these learning capabilities can be developed in groups as individuals reflect on their acquired knowledge. It points out that understanding, analyzing the solution procedures, weighing the results and drawing conclusions are the foundations of secondary school mathematics.

2.3 Instruction
Here we present the nature of instructions that teachers need in their interactions with the students. In an article about curriculum (Hewitt cited in Haggarty 2002, s. 46-60) explains the complexities related to teaching and learning mathematics. Increasingly are the instructions teachers need to make students learn require not mere informing students without considering whether it’s necessary to do so or not. In his explanation, Hewitt makes distinctions between what is arbitrary and what is not, what is necessary and what is not, as teachers select between mathematical facts and non mathematical facts. He emphasizes on the one hand that students need to develop awareness of what is necessary(properties that can be worked out with the help of already acquired knowledge) whereas on the other they need to be informed of what is arbitrary(such as labels, symbols or names and conventions) in mathematical context. He further states that students need not to memorize facts instead with teachers’ success in selecting activities that match the lesson content; students can be able to find out what is necessary be aware of. Hewitt points out those mathematical properties which require only selected activities suitable for the students to develop knowledge about without being told. On the one hand it explains those facts which can be discovered by students through exploration of knowledge. Whereas on the other hand it explains that, “if they are mathematical ideas which happen not to be facts then supposedly they can be just names and convention whereby students need to be informed about”. For example something can be a man´s invention whereby there is no way of showing it out other than telling it to student and this is what he
calls arbitrary. When teachers select their methods of teaching based on just explanations simply they are encouraging students to memorize without awareness; “for many students mathematics can become an arbitrary aggregation of useless subtleties, or they will lose test of it, or they will look upon it as an amusing games,” asserts (Poincare(undated) cited Hewitt, ibid.).

Dave Tout (cited in Clarke et. al. 2004, s 463) adds that focus on teaching method that allow students tasks to aim at carrying out investigation or involving in projects, mathematically this enables students to solve problems in real life. It means that “the mathematics skills that are taught arise out of the tasks that are being investigated.” This creates some sort of interaction among individuals, small groups and whole class which may be different to conventional textbook approach.

2.4 Construction
Here I will present the way student make sense of the knowledge in their learning. The curriculum urges teachers to help students develop capabilities necessary to interpret their knowledge as mentioned above according to Lars Werner (2009/2010). Knowledge is not something that we take in from outside ourselves but something we construct out of our experiences in connection to our surrounding Jaworski (cited in Haggarty 2002, s 68 – 82). She explains what is meant by students interaction in such a way that “as people in the group interact, talk with and across each other, challenge, argue, disagree, ask questions after explanations it can be as if knowledge grows with the group; as if knowledge is located somewhere in the group space rather than in individuals heads”. It follows in her explanations that “out of interaction from a group individuals construct knowledge as a product of interaction.”

When construction of knowledge is seen from Vygotsky’s (cited in David C. Leonard 2002, s37) perspective, it’s out of the social interactions that individuals can learn whereas from Piaget’s (cited in David C. Leonard 2002, s 37 - 38) perspective it’s out of the adults interference with the child’s development that can hinder a child from learning. According to Piaget when the child is prematurely informed of something that he/she would have been capable of discovering, the possibility of him/her understanding it gets closed. It states that in terms of mathematics the ability to generalize and to express unambiguously in abstract form brings about students learning as variety of skills and their applications can get connected to each other. It goes on to say that mathematical ideas will get connected to each other much as
something mathematical will be separated from daily practices. This implies that students are able to use the mathematical ideas by performing those selected activities in interactive way. As example given by Jaworski children need to develop concept when they can see something in the nature of varying dimensions: “to gain such understand children need opportunity to participate in activity through which they can encounter these mathematical ideas and come to know their meaning.” She points out that through interaction in acquiring knowledge they create a culture of mathematical learning using mathematical ideas thereby strengthening their understanding of mathematics in social and cultural context. This defines constructivism theory partly being when individuals cognitively come into terms with their social and culture world to perform activities that bring knowledge to them.

### 2.5 Variation theory

In this section variation theory is presented as a framework that is believed to be effective in improving students’ learning of mathematics. Marton et al. (2004) assume that learning depends on what should be learnt in a given situation under specified conditions and to why something should be learnt in varying situations. It points out that teachers are supposed to help learners in developing certain capabilities. This explains the kind of variation that directs both teachers and students to critical features that can be experienced on a specific object of learning in the shared space of learning (Marton et al. 2004).

According to Marton et al. (2004) knowledge can be acquire out of something that is being learnt, known as the object of learning. It describes a capability which has two aspects namely a general aspect and a specific aspect. Firstly, when an act of learning is carried out such as through remembering, discerning, interpreting and so on that means the general aspect has be experienced. Secondly, for the general aspects to take place there must be a thing or subject on which the acts of learning are carried out and that is called the specific aspect. This implies that the specific aspect explicitly can be observed whereas the general aspects implicitly can be observed. Thus general aspects are indirect objects of learning whereas specific aspects are direct object of learning. It follows that teachers should focus on both direct and indirect object of learning whereas learners normally focus on direct object of learning. This means that student pay much attention on the subject or topic in question. It further states that as teachers get aware of both indirect and direct objects of learning; this effect creates something called the intended object of learning. This can be seen in teachers’ instructions and actions. It points out that how teachers design activities to make it possible for learners to come into awareness with the object of learning creates something else known as the enacted object of
learning (Marton et al. 2004, s 4). It says that the enacted object of learning defines what is possible to learn in the actual setting. It is however, not necessarily that the way teachers plan students’ activities towards the specific object of learning shall be understood as intended. It can instead be seen differently as students focus on the critical aspects of the object of learning. This brings us to something that students actually learn and it’s called the lived object of learning. In other words it implies that students get aware of those critical features of the specific object that are possible for them to discern. It further states that learning can only take place if students successfully discern the object of learning.

2.5.1 The way of experiencing knowledge

This section presents how learners see and act differently in given situations of learning. The term experience is defined as the way awareness is structured and organized in a specific moment (Gustavsson; 2008 s 157). This can as well be understood as (Marton et. al.. 2004, s 5) state that that knowledge we might try to exploit depends to how we make sense of the situation. This explains that we tend to perceive situations based on our previous experiences as we try to understand a particular situation. It follows that we fail to see a particular situation objectively instead we act in relation to that what we want to achieve. This is what is defined as acting in a powerful way. It states that learners can act in the powerful way based on the capabilities developed in seeing and experiencing the critical features of the object of learning.

According to Marton et.al.2004 mathematically there’re expressions which describe the part-whole relations such as $a + b = c$; meaning that two parts namely ($a$ and $b$) make a whole namely $c$. If any of the three terms of the expression is not given, learners can find it out based on how much of the parts’ side has been take away in order to be equal to the whole. Learners can see this in a way that the whole will decrease to the size of the missing part. Therefore this comes to the students’ awareness that by distinguishing between what the parts and the whole are from what they are not. This emphasizes that “seeing the problem as a part – whole relation enables the learner to act in a powerful way,” (Marton et.al.2004 s 7). When learners see and experience certain features of the learning object without being informed what they are, it implies that they have perceived more than one feature at once. This explains that the aspects that are discerned have been attended to simultaneously. Marton et al. (2004) say that we would be seeing and experiencing those critical feature in the same way but we don’t. This explains the difference we have in our previous experiences on which we depend to connect what we want to discern. The variation that develops in seeing and experiencing a particular
aspect helps us to attend and focus on that we find critical in the process of acquiring new experiences. There are dimensions of variation that make discernment possible in the learning environment.

The assumption is that, variation enables the learners to experience the features that are critical for a particular object of learning at the same time for the development of a certain capability; this explains the dimensions of variation. The features with any aspect need to be discerned in order to find those corresponding dimensions of variation with the aspect. The features in this case mean attributes or aspects that can be used to define a specific object of learning. For instance when dealing with a problem in mathematics, the procedure and strategies involved in helping learners to develop capabilities of solving the problem require varying dimensions. Every example given would mean different feature of a specific aspect and those features are seen critical because they are meant to be discerned by the learners.

The dimensions of variation are classified (Marton et.al. 2004, s16 – 17) as follows;

- **Contrast** – where one experience of something is compared with another in order to distinguish the two. It means that a learner has to understand what something is and what it’s not as he/she makes comparison between situations.

- **Generalization** – where one experiences the varying appearances of something; it can be in size, color, and context. It means that we want take the irrelevant features of an aspect away from what is required.

- **Separation** - keeping one aspect invariant in order to separate it from those aspects that vary. That implies that learners have to see difference in outcome if one variable remain constant or fixed.

- **Fusion** - where several critical aspects has to be experienced simultaneously by the learner. This suggests that in life more than one aspect of something can vary at the same time causing us to experience more than one critical feature simultaneously.

The space of learning is constituted through interactions between all actors of a particular situation where learning takes place. It is seen in a classroom environment as students and a teacher jointly constitute a space of learning. When students use their experience in seeing the particular pattern of variation as presented by the teacher they manage to come up with their own ideas in response to the problem surrounding the object of learning. It further states that jointly the actors constitute a space of learning when the teacher takes on students’ response seriously and opens up for inquiry and further discussion (Marton et.al. 2004, s 35 - 39).
example one study was done on two classes dealing with the same object of learning. It followed that hardly could the researcher hear of any comment about what was possible to learn in one class but was not in the other. Instead most comments concerned; whole class teaching, teacher center or students centered, IT an so on. According to the authors the study never considered the space of learning in the view of to how the object of learning was dealt with and whether the communication and interaction were in the direction of varying dimensions. Finally it stated that, “if we knew what students should learn, and if we knew the condition necessary for them to learn, then we would reasonably work towards creating those necessary conditions (ibid.).” The assumption in this case is that “differences in the way in which conditions of learning may be undoubtedly constrain or facilitate the space of learning is necessary for the development of a certain capability.” The capability means here that the particular way of seeing the object of learning in relation to the dimensions of variation will enable the learners to experience varying aspects simultaneously.

2.6 Teaching mathematics involving problem-solving approach as compared to textbook approach

Here I will present suggestions as studied by different people from different articles. Firstly, Mary Barns´ article known as magical moments in mathematics (cited in Haggary, 2002, s 83 – 98) defines magic moment in comparison to the sense of ‘Aha’ among other expression based on the studies that create sensational feeling in the process of the problem solving. Empirically, Barns observed a mathematics class in a high school in Melbourne where such moments appeared evident, “Naidra’s remarks had aroused my curiosity about magical moments,” she said. Barns noticed that the teaching approach used was well structured to fit a description of its key feature (compare with Marton & Tsui 2004). Further it was observed that students worked with different tasks in groups based on their personalities and mathematical capabilities. With a joint effort in a collaborative way groups were supposed to work through the problem and resolve it. The role of the teacher was to monitor progress made without involving himself / herself in the process of seeking solutions to the problems. Instead he/she would create questions which may lead students into discovery of what is necessary. It pointed out that “magical moments would be less likely to occur in expository teaching where work on problems is preceded by carefully structured preparatory explanations and guided practices,” (compare with School Inspections’ Report 2010: 13). That implied that students would be, less active, lazier, and less innovative of which they might lose focus and forget their goals. One of the students experienced that a word or
something by a fellow student in the course of interaction was enough to offer a click that would awake or add to another students’ ideas.

Secondly is another article known as the open and closed approaches by Jo Boaler cited Haggart (2004, s 99 – 112). Boaler carried out a case studied in two secondary school in UK on students’ experiences and achievements in different teaching environments. One school was called Amber hill and another is Phoenix Park, both were at similar level at the time when Boaler began her investigation.

Amber Hill School happens to be with highly motivated students and hard working though according to Boaler mathematic was generally boring and tedious for them. They received closed teaching approach where a teacher went through examples from the textbook followed by students’ exercises. In those lessons that were observed students in that school showed repeatedly motivation and hard work without any complaints or disruption when doing exercises. One student however expressed her frustration: “mathematics would be more interesting if we had more practical or group work; we have had group activity once a year,” she added. It followed that similar comments/ responses were received from the students. Boaler’s interpretation is that since there isn’t any discussion during lessons students cannot know when situations are mathematically similar. All what students know has do to with a rule and formula as they appear in their textbooks which implies that students can get confused if they come across let’s say simple and obvious mathematical task that requires no rule or formula to solve. That can even suggest that students receive mathematical knowledge mechanically with the heap of memorized rule and formula that easily can be forgotten at a time one would need to apply them. In a different situation outside rules and formula students of Amber Hill were accustomed to using cue to solve problems. It was observed that in a situation where a mathematical problem diverted from what appeared in the textbook students would be forced to give up because they had not learnt how to interpret the problem in a given context mathematically. In Boaler’s conclusive remarks, he stated that teachers at amber Hill were dedicated and effective at teaching textbook by rote of which students were limited on developing knowledge that is usable outside the textbook situations.

Phoenix Park students differ largely in the nature of teaching they receive as compared to those of Amber hill. According to Boaler, Phoenix Park students do a lot of mathematics on their own and they are working in groups of mixed abilities. It is however stated that they don’t use as much time allocated to a lesson as Amber Hill students do. About 95percent of
Amber Hill averagely stays at work up the end of each lesson while 67 percent of Phoenix Park students do. This emphasizes the amount of freedom they enjoy when learning mathematics. Teachers happen not to have structured lessons apart from prepared open - close project questions that cover three weeks’ time to finish. When asked about how they find mathematic lessons: “students talked about relaxed atmosphere, emphasis on understanding, and need to explain methods,” he asserted. It appears that more students express the degree of responsibility themselves have on their studies and the amount of capabilities they can develop. The teacher was there when they needed him/her mainly when they would like to use some mathematics that looked unfamiliar to them.

The two schools where involved in a progressive test. One task was an open closed type and another in a short kind of written test. The results from the short test are slightly similar though Phoenix Park achieved slightly higher than Amber Hill. When it came to applied mathematics Amber Hill students scored slightly lower because they failed to think beyond textbook as the tasks required mathematical thinking relevant to a given context. The tradition of teaching mathematics at Amber Hill based on closed tasks offered no opportunity to students seeing mathematics in everyday situations which could be the opposite of what students of Phoenix Park experience.

Thirdly in article known as Landscapes of investigation by Ole Scovsmose (cited in Haggarty 2004, s 115 – 127) presents a method of teaching that encourages students to explore knowledge through questioning and explaining. It states that, “a landscape of investigation invites students to formulate questions and to look for explanations.” It follows in the explanation that the teacher invites students to respond on his/her questions about a mathematical content (compare with specific features according to Marton et. al. 2004).

It begins with the teacher’s presentation and invitation to the students. It’s possible that students can mumble or stay silent before any clear acceptance for teacher’s call takes place, later on by accepting the invitation with a word like ‘Yes, but what if…?’ This implies a sign of exploration from the students. It further follows that the teacher poses another question in form of ‘why is it that…?’ It states that, “the teacher’s why is it that…?” provides a challenge, and the students ‘Yes why is it that…?’, means that they are up to the challenge in searching for explanations. Scovsmose points out that as students increasingly involve themselves in the processes of exploration and explanation will mean the landscape investigation has come into use. The process is supposed to make students invent new
knowledge. It follows that, “a landscape only becomes a landscape of investigation if students
do accept the invitation.” This means that the nature of invitation has to be mathematically
descriptive to suit the landscape of investigation (compare with critical feature, Marton et al.
2004).

2.7 Summary
The assumption is that the roles both teachers and students have on the complexity of teaching
and learning mathematics can be worked out when teachers are able to interpret the
curriculum in the objective form as stated earlier in this essay. The space of learning is
supposes to be shared between the teacher and the students in conditions favorable for
meaningful interaction. The instructions from the teacher are meant to create a discussion of
which individual experiences can converge into invention of new knowledge. The
constructivism theory from both Vygotsky and Piaget’s perspectives would be seen useful in
forming a mathematical culture necessary for students to learn to construct mathematical
ideas in their daily life.

3. Method
Firstly, it was by exploration of articles from the internet which were corresponding to my
question of research where relevant information was taken. Since the source of the claim that
mathematics is difficult to many students in Sweden is internationally connected, so does
much of sources of literature used in this research. The essay aims at trying to answer the
questions about poor mathematical knowledge students have and in which way it can be
improved. Exploratory research tends to explain the course of the event and relations between
different situations (Olsson & Sörensen 2007, s 29 - 30). It implies that we research on
connections in causes of a problem. We gather as much information as possible until one gap
of knowledge can be cover with the newly invented knowledge.

Secondly, empirically qualitative inquiries were made through wider and deeper interviews.
Teachers of mathematics from two secondary schools in Södermanland’s county in Sweden
participated in this research by expressing their views based on the experience they have in
teaching mathematic during the interview. The interviewees have to feel important and able to
talk straight to the point (Olsson & Sörensen 2007, s 80). That means the dialogue between
the interviewee and the interviewer must create a good climate in form of co-operation that
makes it possible to get good information from the interviewee. The standardized form of
interview was used here, according to (Olsson & Sörensen 2007, s 81) the order of questions
ought to be well planned without any possibility of variation in situations from one interviewee to another. This order is supposed to be followed in the course of the interview because it’s important to stay on course as one interview may dominate with unwanted information.

It’s necessary, however to consider that the aim of the interview is to describe and understand the main theme which the interviewee experiences and attaches him/herself to. Based on phenomenographic method, the interview was analyzed. According to (Olsson & Sörensen 2007, s 106), it states that phenomenography is interested in our understandings, notions and ideas for our apprehension of phenomena in the surrounding world.

3.1 Themes
The interviewed teachers’ response referring to the essay’s question of research whose central themes are; curriculum interpretation in relation to lesson plan, problem solving in relation to students activities, interactions in relation to teaching method and co operation among mathematics teachers in relation to sharing and gaining knowledge are presented in this section. The qualitative inquiry via interview tends to gather so rich and unbiased descriptions that are of relevant themes to the interviewee’s living world (Olsson & Sörensen 2007, s 81 - 82). By the phenomenographic approach the understanding of reality must take place through a comparison made upon different people’s (interviewees) understanding of the same aspect.

3.2 Preparation
It started with making a sketch of a phenomena under which description of categories (themes) were made based on the theoretical framework. Phenomena in this case implied the selection of teaching methods used in mathematics whereby teachers’ response in reality would provide answers to the investigation.

3.3 Actual interviews
Teachers were reminded of the purpose of the interview because I felt it was not enough that they were already informed by email I sent inviting them as participants in this research. The interview questions are written in English (see appendix) but we agreed between us that one can respond in Swedish. However all questions were presented in English and responses were mainly in Swedish. In other words both languages were used where necessary during the interview. It implied that some translation was made in producing results, similarly Swedish literature was translated in some of the references used in this essay. They were four
interviews altogether taking averagely around half an hour each. The response went according to the plan, I mean answering question as the structure stands; low structuring implies the interviewee can interpret questions freely depending on communication tendencies, experiences and beliefs that she/has (Olsson & Sörensen 2007, s 80). The emphasis was put on the descriptive aspect of the interview, mainly on ones experience and practice. All interviews were recorded which made it easier for the interviewer to concentrate on the dialogue aspect as it is always considered necessary.

3.4 Sample
This essay uses two sampling techniques: simple random and purposeful sampling techniques to gather the necessary data. Langham (1992) discusses a simple random sampling as a probability sampling technique which gives equal chance to individual respondents (in this case schools) being selected thereby producing a defensible estimates, and purposeful sampling is necessary where specific respondents (in this case only mathematics A teachers) were contacted because the data required cannot be obtained from other respondents (Maxwell at al 1997). First, a sample of secondary schools was randomly selected from the lists of secondary schools in the Södermanland’s county; and then purposeful sampling was used to contact teachers of mathematics course A in the selected schools to gather the data through interviews. Teacher A and B are from one school whereas teachers C and D comes from another school. The head teachers of the selected schools were contacted to discuss the research and to establish contact with the teachers to schedule the interviews. Teachers were contacted and information about the interview and students’ questionnaire was passed on to them through a letter of consent via email. The next stage was the interviews followed by answering the questionnaire.

3.5 Procedures of analysis
The aim of this investigation is to study what teachers of mathematics think/do about the difficulty students face in learning mathematics. Based on contextual analysis interviews where transcribed analysis was done according to respondents’ experiences as differences and similarities were interpreted. Out of the phenomenographical perception each of these mathematics teachers understands the phenomena individually and it’s upon such understanding that the analytical procedure is based. The interviews were transcribed following the categories of themes related to the underlying concept about the underlying theory of this research. Categories according to (Olsson & Sörensen 2007, s 108) expresses the researchers own understanding of the phenomena in his/her surrounding world. That
implies as far as contextual analysis is concern, its analysis is described in terms of major parts and major relations together with the meaning they have (ibid. s108). Interviews’ transcriptions were written on computer as saved document that has been used to read back any time for this analysis. By use of categorized themes the major parts could easily be identified (citation) because the document has similar major parts as themes and major relations in both similarities and differences in respondents’ experiences. The questionnaire completed by students was analyzed quantitatively by rating respondents’ choices for every item of measurement instrument. The last item was meant for every student briefly to express what form of interaction he/she does as a part of contribution during mathematics lesson (see the appendix). In other words how do students experience the space of learning (Matron et. al..2004). The rating scale used here for analysis measures the level of involvement towards learning mathematics ranges from being very strongly involved to hardly any involvement by the students. The ratings were converted into scores for each category of involvement (Muijs 2004, s 45 -50; s 88) in a range of (4 to 1) points. The average scores were worked out from every class of students corresponds to every teacher who was involved in the interview respectively. Since every student has a proposed grade aiming at by the end of the course, the analysis on their performance has put into consideration both actual grades so far scored from internal tests and the proposed grades. It implies that the three categories of tests namely; involvement test, actual test and proposed (focused) test were useful in measuring the students’ achievements.

3.6 Ethics
The interviewed teachers’ identities were protected upon their respect and integrity on conditions of anonymity (Olsson & Sörensen 2007, s 54 – 59). It follows that a well conducted research project must observe the ethical guidelines of the Swedish Council of Scientific research. Teachers were informed about the purpose of this research, the method of interview and when it will take place. Further they could “freely participate in the interview with all the rights to cancel it at any time without any consequences (Olsson & Sörensen 2007, s 56). The letters of consent by e mail were written seeking the personal contacts with teachers and expressing politely a kind of invitation on their participation in the project.

3.7 Validity
Like in any other areas of research, validity measures that we want to know though it is not all that simple in educational research (Muijs 2004 s 65). It means that many of the educational concepts are regarded as abstract variables that cannot be measured directly. Instead we use
some test in form of a questionnaire whereby each question represents these concepts that are indirectly measurable. Content validity calls upon the righteousness of the content that is designed in the questionnaire (Muijs 2004 s 66). This implies that those questions have to correspond with the underlying theory. This investigation measures the students’ role in learning mathematics as they respond to the questionnaire. The underlying theory means here that students must be actively involved in constructing mathematical knowledge based on the teaching methods the teacher uses. In other words the validity of my researcher is measured by the level of involvement in which students can actively participate in constructing mathematical knowledge. It follows that, “the better we know about our subject and how the concepts we are using are theoretically defined, the better will be able to design that is content valid,” (Muijs 2004 s 66). On the other hand qualitatively has the consistence found in the information gathered from the teachers’ interview been used as a measure of validity. If there is consistency between the reality and interpretation man talks about validity (Olsson & Sörensen 2007, s 66).

### 3.8 Reliability

This is another key concept that is useful in determining the quality of our measurement instruments (Muijs 2004 s 71). It is stated that there’s always some amount of error in our statistical measurements which can be offset by use of reliable instruments. Reliability is therefore referred to as “the extent to which test score are free of measurement error,” (Muijs 2004 s 71). It follows that random errors need to be worked out in the process of developing the instruments for avoiding ambiguity in our statements; otherwise our test can be less reliable. This suggests that low reliability leads to untrustworthy results as we may fail to come up with clear research findings (Muijs 2004 s 72). This can as well mean that for any item that seems faulty to the test can be left out. It is stated that when we use more than one item, individual errors that respondents can make in answering a single item offsets each other when a single measurement for all items is used (Muijs 2004 s 74). The single measurement in this investigation has been used as several items of the instrument (questionnaire) measuring one concept of students’ involvement in constructing mathematical knowledge. It appears that, “In general more items means higher reliability,” (ibid. p 74). In addition to that, one measurement instrument to four respondents (teachers) from each of the two visited schools in form of open structured interview and another to four classes of the same schools in form of a questionnaire were used. That means that information from each interview goes with corresponding response from the questionnaire of a respective class.
3. Results and Analysis

In this section the teachers’ experiences are put together in each context such that similarities as well as differences can easily be identified. Firstly tables show the highlighted transcriptions made from the interview which are used as results for analysis. Secondly tables represent statistics quantitatively generated from respondents to the questionnaire which were used to measure the students’ achievement in mathematics.

4.1 Curriculum interpretation in relation to lesson plan

<table>
<thead>
<tr>
<th>Teacher A</th>
<th>Teacher B</th>
<th>Teacher C</th>
<th>Teacher D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Selects activities according to individual capabilities and his lesson plans are based on students’ abilities. He aims at achievable goals by teaching the basic mathematical concepts.</td>
<td>Considers the prior knowledge students have in his lesson plan and simply cannot follow what curriculum says as “it may require a lesson plan to every student,” he added.</td>
<td>Considers a strategy that suits individual students based on his experience but not on the curriculum. He said that one central strategy for all learners is not easy to find, but he tries to find a connection among possible strategies.</td>
<td>Considers curriculum mainly in respect to attainable objectives. When preparing her lessons she pays attention to variations: going through exercise, working in group and having class discussion.</td>
</tr>
</tbody>
</table>

The similarity in the context of curriculum and lesson plan from three teachers (A, B and C) is that they pay attention mainly to attainable goals (mål att uppnå) except for teacher D who is slightly different in that she uses three variants in her lesson plans. This is highly recommended in both School Inspection report and School regulations book. Individual teaching as mainly used by the three teachers (A, B and C) only does the so called ´björn tjänst` meaning disservice to students because the approach is not sustainable. This method does not promote the objectives of capabilities which students need to develop mathematically.

Capabilities like communication, reasoning and argumentation seem to be ignored instead this method can increasingly make students memorize mathematical knowledge without understanding it. In order to reduced the burden of memorizing mathematics from students Werner (2009/2010) refers to working as a group in exercising the objectives of capabilities something which in turn brings about desirable objectives. There are desirable objectives that
Swedish curriculum recommends to all teachers of mathematics because individuals can easily reflect on the knowledge acquired as they continue solving problems.

4.2 Problem solving in relation to students activities

<table>
<thead>
<tr>
<th>Teacher A</th>
<th>Teacher B</th>
<th>Teacher C</th>
<th>Teacher D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Works mostly from the textbook because it has a great deal of ready prepared tasks. “Weak students often need guidance to build a foundation that makes it possible for them to carry on with problem solving,” he said.</td>
<td>“Problem-solving approach does not suit students who luck basics in mathematics,” he added that if you lack the tool you cannot simply fix it. He follows textbook because it covers the syllabus nicely.</td>
<td>If time allows, he would once a week carry out problem solving though students prefer working from textbook. He says that students feel more comfortable there than working with problem solving tasks.</td>
<td>“Students believe that problem solving takes much time and they prefer working from the textbook”. They feel more comfortable in working with textbook because there’re ready prepared examples to refer to at anytime. She goes on to say that students easily forget the previous knowledge as they fail to understand concepts.</td>
</tr>
</tbody>
</table>

It is evident from these teachers that students prefer working from textbooks to elsewhere and there are no efforts made by teachers to change this practice as it fails to improve students’ achievements in mathematics. According to teacher D textbooks make students forget easily because they learn mechanically by referring to ready prepared examples from the textbook. Whereas teachers A and B have no problem with textbook because it’s like readymade outfit so to speak in the sense that textbook has a great deal of ready prepared tasks according to teacher A, in the same respect, teacher B said textbook covers the syllabus nicely. The instructions required of any mathematics teacher are to help students become aware of mathematical concepts as argued by Hewitt (2002). It implies that facts and properties embedded in mathematical concepts and ideas need well selected and guided students’ activities that leads them to invent knowledge without being told. In that way students can develop objectives of capabilities in the process of inventing knowledge in situations such as problem solving, deliberate reasoning, computational skills, relationships between abstract and concrete, and communication using mathematical language (ideas) (Werner, 2009/2010). On the other hand, Marton et.al.
(2004) express this as helping student seeing and experiencing the object of learning in a powerful way. In the same vein, this aspect of seeing and experiencing the object of learning in a powerful way is treated as the capabilities a student use in handling novel situations.

4.3 Interactions in relation to teaching method

<table>
<thead>
<tr>
<th>Teacher A</th>
<th>Teacher B</th>
<th>Teacher C</th>
<th>Teacher D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Talks only what he does but not what students do. It seems he does a lot of telling/informing than discussing with students or leading them into awareness for what is necessary (facts/properties).</td>
<td>Talks about things students do in normal life and relate them back to mathematics. Student can be asked to cut whole into part. “Most of the teaching is teacher center based just in form of lectures,” he said.</td>
<td>Talks a lot of what he does for the students but not what students do in the classroom environment. He says interaction does not work for him because students believe that it’s the teacher that should do the job of teaching them.</td>
<td>Talks mainly about what students do. Students who find mathematics difficult fail to join in discussion by avoiding teacher D. They are not willing to show their work to the teacher instead they let other group members solve problems for them.</td>
</tr>
</tbody>
</table>

It seems that in almost all teaching methods used above; teacher A, B and C can be classified as teacher - centered because they are telling us how much they talk to students but not how much they share with students in the space of learning: which has no possibility of showing us some good in their methods according to Marton et el (2004). The applicable teaching method would rather emphasize the kind of tasks that students encounter of which according to Dave (2004) students can develop their mathematical skills in real life. Teacher D seems to be student centered though she fails to make discussion healthy where free riding among students can be avoided and social – cultural aspect beneath the constructivism theory can be considered useful. Both Piaget and Vygotsky realize the need of interaction coming from the social – cultural perspective as individuals construct knowledge out of that. In terms of mathematics the ability to generalize and to express unambiguously in abstract form brings about students learning as variety of skills and their applications can get connected to each other and mathematical ideas will get connected to each other much as something mathematical will be separated from daily practice (Jaworski as cited in Haggarty 2002).

We can also consider Marton et. al. 2004 in respect to the conditions surrounding the space of learning: The shared space of learning must be defined in the view of how the object of
learning can be dealt with and whether the communication and interaction goes in the
direction of varying dimensions. This comes about with the conditions of learning that
undoubtedly constrain or facilitate the space of learning necessary for the development of a
certain capability. This implication would lead learners to seeing the object of learning in
relation to the dimensions of variation that will enable them to experience varying aspects
simultaneously. We would recommend to teacher D that it’s not enough to have class
discussion without well guided instruction in a varying dimension where possibly students
can experience the critical features of the object of learning.

4.4 Co operation among mathematics teachers in relation to sharing and
gaining knowledge

<table>
<thead>
<tr>
<th>Teacher A</th>
<th>Teacher B</th>
<th>Teacher C</th>
<th>Teacher D</th>
</tr>
</thead>
<tbody>
<tr>
<td>They’ve got a joint office, common textbooks, one lesson study different lesson designs such that it would be a fair play to students</td>
<td>They design similar tasks accordingly ranging from simple to difficult ones. Students are free to work with tasks that match with their individual abilities.</td>
<td>Man can only find another who is the closest, besides there is email system of contacting each other where necessary. “We have one mathematical internal test so that it will be fair judgment when giving grades to students”.</td>
<td>We’ve tried to work together for one common goal of improving students’ capabilities but it didn’t work. We have mathematics teachers sitting as a team and where possible we can easily get help from each other.</td>
</tr>
</tbody>
</table>

Cooperation in this case is seen as something which focuses on a joint internal test so that students will have a fair judgment in their grades. It fails to show to us how jointly teachers prepare students for these internal tests. If we look back out of the study by Boaler (2002) on schools (Amber Hill & Phoenix Park School) there is a connection here because teachers of Amber Hill were dedicated and students were motivated and hard working, but they fail to deliver good result out of it. We can see that teachers who participated in this research can be regarded as hard working because even they look at time as something not enough for lessons. Like teachers of Amber Hill, the interviewed teachers have not got good results from their students. Amber Hill students learnt mathematics mechanically which caused them never to relate mathematical ideas to any context outside the textbook (compare with School Inspections Report, 2010: 13). It means that students of all the four teachers that were interviewed hardly can escape what Amber Hill students have experienced.
If students can receive lessons designed to take care of variations found in a given specific object of learning in the nature of exploration it will be considered useful as interaction takes place. Phoenix Park students are just a good example that if teachers work toward desirable objectives students can score highly in mathematic. It says that these students can see and experience mathematics in a powerful way because formulas and cues only matters in a given context but not in relation to textbooks (compare with the conditions of the space of learning Marton et. al., 2004).

4.5 Teachers final words on complexity of teaching and learning mathematics

<table>
<thead>
<tr>
<th>Teacher A</th>
<th>Teacher B</th>
<th>Teacher C</th>
<th>Teacher D</th>
</tr>
</thead>
<tbody>
<tr>
<td>Students who just joined secondary school have got poor knowledge today than it was before (10 years back). Teacher A has no independent information to why it is so. The school has allocated more time to mathematics mainly for course A to reduce the existing gap.</td>
<td>“I find it much harder today than it was nearly six year ago because students don’t easily understand me as they used to”. It requires much more time to explain one thing to a student nowadays. He thinks that students don’t want to do the homework today, they don’t carry book home with them. It is hard to teach mathematics to students today because they have less and less knowledge with them.</td>
<td>Those students who believe in themselves take mathematics serious and usually find mathematics easy, whereas those who have negative opinion about mathematics believe that mathematics is a talent. If it does not exist in any of the student’s parents, often will such students drop the interest for mathematics and it always remains difficult, he concluded</td>
<td>Students must feel that mathematics is possible and one can successfully learn it. Students must take studies serious by carrying the burden and find a way of get it going in the positive direction. They must always be in charge of their studies.</td>
</tr>
</tbody>
</table>

It seems that all the teachers interviewed agree that students have failed to take up the role they need to play in solving this problem. On one hand students do not have enough knowledge before joining secondary school. On the other hand they are less committed to their studies due to some reasons like: failure to do homework, being pessimistic that mathematics is a talent and failure to confront mathematics head on. We can see how much teachers simply put the blame on students but not in any way to themselves. Teacher C said “I don’t have any problem with mathematics accept a bunch of some students, and I have about
two minutes to attend to every student which is not enough at all.” He added that it is up to the students to work out how they want to learn mathematics and that’s why he does not have one strategy that works for everybody.

4.5.1 Level of students’ involvement in learning and constructing mathematical knowledge

<table>
<thead>
<tr>
<th>Scores</th>
<th>Class for teacher A (n = 11)</th>
<th>Class for teacher B (n = 12)</th>
<th>Class for teacher C (n = 19)</th>
<th>Class for teacher D (n = 29)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Very Strongly</td>
<td>6.3%</td>
<td>45.7%</td>
<td>19%</td>
<td>14.3%</td>
</tr>
<tr>
<td>Strongly</td>
<td>26.3%</td>
<td>35.5%</td>
<td>40%</td>
<td>25.5%</td>
</tr>
<tr>
<td>Less strongly</td>
<td>30.5%</td>
<td>4.6%</td>
<td>25.1%</td>
<td>37.5%</td>
</tr>
<tr>
<td>Hardly any</td>
<td>36.9%</td>
<td>14%</td>
<td>15.9%</td>
<td>23.1%</td>
</tr>
</tbody>
</table>

Class B has outstanding scores because its students follow a different program called international baccalaureate (IB). According to teachers A and B, IB students are more committed to their studies than the rest who follow social studies program (SP). Classes A, C and D have a decreasing trend of involvement as the table above shows. Nonetheless, a student from the SP group says, “I have never really been a great fun of mathematics, I don’t understand anything even though I try, but I fight on,” she concluded. This student and the rest who share the same vision stand the risk of joining their friends who appear at the bottom of the table above if teachers don’t change in their teaching methods. Students work from the textbook mostly and to some of them this has become a routine without any reflection on what they are doing. It is through teachers’ effort to help student begin seeing mathematics as elegant subject (Haggerty 2002). It states that the beliefs teachers have play a big role in the way students learn mathematics. In a situation like this, teachers believe that SP students are worse off than IB students and this stands a risk of failing to help them as they are supposed to. Teachers need to inspire their students not just by talking to them. Explanations alone will not help them educate their awareness because for many students, mathematics can become an arbitrary aggregation of useless symbols (Poincare, n.d. in Hewitt, 2002). Many students’ responses were received such as “not so much” from the one of the items of measurement instrument (see the appendix); “how much do you feel that mathematics is important in your life?” The responses implied the absence of involvement in learning mathematics. In order to reverse this decreasing tendency of involvement in learning mathematics among students,
teachers must take heed of what has been said to them that explanation must be followed by students’ exploration, Skovsmose (cited in Haggerty, 2002). It points out that teachers need to invite students to the landscape of investigation. There are general aspects which must be discerned when students act on a specific aspect such that they can possibly develop their capabilities from the intended object of learning (Marton et. al. 2004). That means unless students’ experiences are taken seriously by the teachers in every situation surrounding the space of learning, the nature of interaction for that matter can lead to reduced participation from the students.

4.5.2 Students’ actual and proposed achievements

<table>
<thead>
<tr>
<th>Scores</th>
<th>Class for teacher A</th>
<th>Class for teacher B</th>
<th>Class for teacher C</th>
<th>Class for teacher D</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Actual</td>
<td>proposed</td>
<td>Actual</td>
<td>proposed</td>
</tr>
<tr>
<td>MVG (pass with an A)</td>
<td>0 %</td>
<td>9.1%</td>
<td>38.5%</td>
<td>46.1%</td>
</tr>
<tr>
<td>VG pass with a B</td>
<td>18.2%</td>
<td>9.1%</td>
<td>38.5%</td>
<td>38.5%</td>
</tr>
<tr>
<td>G pass with a C</td>
<td>72.7%</td>
<td>81.8%</td>
<td>23%</td>
<td>15.4%</td>
</tr>
<tr>
<td>IG failed</td>
<td>9.1%</td>
<td>0%</td>
<td>0%</td>
<td>0%</td>
</tr>
</tbody>
</table>

There are two types of scores in each case namely actual scores and proposed scores. The former were derived from the internal tests that students did whereas the later was base on what students aimed at from the course. The purpose of having both scores is for us to see how students are working towards their goals in mathematics. Firstly, the scores of involvement were treated as the effect of the methods of teaching that were used whereas the actual scores were treated as the students’ achievement. Secondly, if we put scores of involvement against the actual scores we shall be able to conjecture that teacher’s selection of suitable teaching methods for a specific object of learning can result into better performances in mathematics.

We can see that Class B students have greatly managed to keep up to their goals as the difference between the actual score and proposed scores is small. If we consider pass with grade C (G) to be the average score which corresponds with fairly or less involvement; it’s around 20% of students of classes A, C and D that managed to score above average and the
rest with the assumption that they are less strongly involved managed to score on average and below. Classes C and D have proposed score above average with 40% margin but the actual scores turned out to have decreased by 50%. This suggests that the teaching method by the teacher did not deliver good results to the students. In case of Class A, none of the students that proposed to pass with Grade A managed to get it instead they all dropped to the lower grade(s). It followed with the same class that 9% of those students who had proposed to pass with grade C instead they dropped to failures. We can again see from all the three classes A, C and D whose involvements were found to be less than class B, were even their achievements in mathematics far away from the proposed grades. Class B shows increasing scores from the involvement test and slightly similar to the achievement test though the later appears to be both constantly and increasingly formed. It shows already that teacher B uses lecture form as a teaching method which does not have much interaction in it. This seems to be inconsistent to see that his students have passed involvement test better than the rest. It does not mean that the teaching method used holds because even both stagnation and declination exist as we compare the actual and proposed scores of class B. It can be possible that some other factors must have contributed to the good scores received mainly from involvement test. It may be interesting to follow and see how far students of class B will go since they learn mathematics by rote based on textbook approach (Jo Boaler cited in Haggerty 2002).

5. Summary and Conclusion
Finally in this section the aim of this essay will be reviewed and conclusion will be drawn. Confirmation of the findings from this research in comparisons to earlier findings elsewhere will be made and some recommendation to further research will finally be made. From this essay we have tried to answer the question to why mathematics is difficult to many secondary school students in Sweden and in form of contribution to the problem, well selected teaching method have been found useful in increasing mathematical awareness among students.

Teacher’s knowledge about the curriculum and their commitment toward improvement of students’ mathematical knowledge was recommended by School inspections Report 2010: 13. From its survey teachers were found inadequate in their teaching methods which caused a decline in the way students perceive mathematics. Many of the visited schools focus mainly on attainable objectives in their teaching instead of the desirable objectives as it is emphasized by the Swedish curriculum. Teachers mostly use textbook approach as opposed to problem
solving approach where the later is believed to teach better mathematical skills than the
former (Dave Hewitt, Jo Boaler and Ole Skovsmose all cited in Haggarty 2002). This essay
shows that teachers seem to fail to involve students in constructing mathematical knowledge
by using textbook approach as a teaching method. The qualitative finding suggests that all the
interviewed teachers use textbook approach and much of individualized teaching as students
mainly work from textbooks. This suggests that students remain increasingly passive and
bored of mathematics lessons.

The results show that the level of involvement in learning mathematics has positive impact on
the performance/achievements in mathematics. We can assume that the results from this
research provide the answer to the problem in the sense that the underlying theory of
involvement that described here corresponds with the well selected teaching method to match
a specific object of learning. The results happen to agree with the findings in the School
Inspection Report where 85% of the visited schools are using textbook approach without
exploration and problem solving approach. It is also similar to findings by Jo Boaler cited in
Haggarty 2002 that students of Amber Hill in UK are motivated hard working but fail those
tasks involving contextualization even when there are supposed to be easy. This confirms to
us that the teaching method based on textbook approach has a negative impact on students’
performance in mathematics because it makes students less strongly or hardly involved in
learning mathematics.

The finding from this essay is that teacher’s methods of teaching are interpreted based on
students involvement. This means if interactions among students are taken seriously to the
extent that students can talk using mathematical ideas in given situations, they will be able to
construct mathematical knowledge through increased involvement. We can confirm the
results that teaching methods with increased involvement in a space of learning has the
positive effect on students’ understanding of mathematics. Thus conclusion can be drawn
based on the results that teaching methods that suits a specific aspect of learning can improve
students’ achievements/ performance in mathematics.

However, this does not imply that well selected teaching methods alone can completely be
answerable to the students’ improved performance in mathematics. This investigation has
shown some gaps that were found disturbing in case of class B students. Their results showed
positive relationship between involvement and achievement when teaching methods by the
teacher of class B students were not different from the other teachers. They were all using
textbook approach with less interactions, mostly teacher center and only two steps procedure (teacher’s explanation and doing exercises from the textbook).

We know that class B students are highly motivated and hard working just like Amber Hill students who fail in a mathematical contest. It was found out that they could not reason beyond textbook related task no matter how easy tasks were. The problem was said to have been caused by lack of seeing mathematics in the given context. They based on memorized ideas to solve mathematical problems which seemed to be unsustainable; such memorized ideas can be forgotten easily. We are left to wonder what has made class B to perform so well like that: could it have been the nature of internal tests such that they might have been so easy to them? How far can they go if they learn mathematics based on memorized wisdom without its awareness? This essay failed to come up with the true answer to these questions instead they would recommend this to further research. Specifically we would like to know if; it is enough for highly motivated and hard working students to study mathematics based on memorized wisdom (ideas) since it does not promote mathematical awareness.
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Tout Dave (2004), In Barbara Clarke et. al. (2004), international perspectives on learning and teaching mathematics, Göteborg University (p 457 – 472).

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Skolinspektionsnens rapport 2010:13 Undervisningen i matematik i gymnasieskolan.
Appendix: Preparation for the interview.

Mathematics is known to be difficult to students in Sweden as reported by (TIMSS, 2007 cited in DN, 2011).

- The objective questions are as follows: How do teachers select relevant methods of teaching that maximize understanding of intended lessons’ content firstly to themselves and secondly to students, when planning lessons? Do they follow the school curriculum when setting goals as well as tasks that students need to accomplish in their studies?
- How much interaction is there in relation to when students are capable of learning new knowledge? Are students experiences taken seriously in situations when they’re experiencing new knowledge in the space of learning?

It is hypothesized that well selected teaching method suitable for a specific aspect of the object of learning can improve students’ achievements in mathematics.

**Standardized Interview prepared for mathematics teachers with open questions.**

What matters most for you when designing/planning a lesson? Explain by giving examples where possible. (To what extend do you feel that school curriculum is so deterministic in your work?)

Which topic in your experience do students find most difficult in mathematics course A? How do you usually go about it? Let’s say as regards methods (pedagogical approach), procedures (content of knowledge) and students’ activities (complexity or level of interaction).

Do you follow one syllabus with other teachers with whom you share this course (matte A)? Where do similarities as well as differences lie? Can you explain why in each case?

When/how do you gain or share knowledge from one another particularly to simplify students’ learning of mathematics?
Appendix  Questionnaire prepare for students.

How do you find mathematics lessons?

Boring  Less interesting  Interesting  Very interesting

How do you like mathematics?

difficult  Very difficult  Fairly  Nicely/easily  Very much

Which grade do you mostly get from the local math’s tests?

IG  G  VG  MVG

What grade do you aim at by the end of this course?

IG  G  VG  MVG

What strategy/plan do you have for achieving your goal?

Teacher’s help  Friend(s)’ help  Parents’ help  Self help  doesn’t matter

How much time do you put in revising mathematics outside school timetable?

45mins / day  About 1 hour every another day  Several times a week  I don’t know

How much do you feel that mathematics is important in your life or studies?

Not so much  Good enough  Very much

Do you get that help you need from your teacher?

Yes  No

Explain your contribution during the mathematic lessons.