Seminar Paper No. 199

OPTIMAL SUBSIDIES TO DECLINING INDUSTRIES:
EFFICIENCY AND EQUITY CONSIDERATIONS

by
Harry Flam, Torsten Persson
and
Lars E.O. Svensson

INSTITUTE FOR INTERNATIONAL ECONOMIC STUDIES

University of Stockholm
Seminar Paper No. 199

OPTIMAL SUBSIDIES TO DECLINING INDUSTRIES:
EFFICIENCY AND EQUITY CONSIDERATIONS

by
Harry Flam, Torsten Persson
and
Lars E.O. Svensson

Seminar Papers are preliminary material circulated to stimulate discussion and critical comment.


Institute for International Economic Studies
S-106 91 Stockholm
Sweden
OPTIMAL SUBSIDIES TO DECLINING INDUSTRIES:
EFFICIENCY AND EQUITY CONSIDERATIONS

by

Harry Flam, Torsten Persson and Lars E.O. Svensson
Institute for International Economic Studies
University of Stockholm

This paper considers equity vs. efficiency in a small economy that subsidizes an industry facing falling world market prices. Subsidies keep up output in the short run when wages and factors are rigid. But once introduced subsidies become permanent, because of pressures from vested interests. This creates misallocation of resources in the long run. An optimal efficiency subsidy balances the short-run gains and long-run losses. It should be raised when prices fall if there is full employment initially and lowered if there is unemployment. An optimal distribution subsidy, which aims at maintaining the existing income distribution, should always be raised.

Correspondence to: Torsten Persson
Institute for International Economic Studies
S-106 91 STOCKHOLM
Sweden
OPTIMAL SUBSIDIES TO DECLINING INDUSTRIES:
EFFICIENCY AND EQUITY CONSIDERATIONS

by

Harry Flam, Torsten Persson and Lars E.O. Svensson

1. Introduction

The last decade has been a period of substantial changes in world market prices and, it seems, in the comparative advantage of many countries. In many industrialized countries adjustment to these changes has been rather slow and blocked by rigid wage rates and immobile factors of production. Governments, facing risks of large-scale unemployment and losses of production, have often reacted by giving massive subsidies to the most adversely affected industries.\(^1\)

The motives behind this reaction are of course many and varied. Aside from economic efficiency, a concern for the income of those directly threatened by unemployment seems to be a main motive, however. The extent and the duration of subsidization is usually not determined at the government's own discretion, but is rather the outcome of explicit or implicit bargaining with labor unions, political constituencies and not seldom capital owners as well. How strong the government's position is in the bargaining process depends on the degree of unionization in the affected industries, their geographical concentration, the date for the next elections and a host of other factors.
In this paper we abstract from much of the political complexity of giving subsidies to declining industries. The bargaining takes place only in the minds of cabinet ministers and is exclusively concerned with the trade-off between efficiency and equity. Outside interests do enter into the analysis in a crucial way, however. In the real world these interests have often successfully lobbied for the extension of what originally was intended as temporary subsidies. In order to highlight this lobbying activity and its effects in the analysis we shall assume that the government is free to introduce and set the level of subsidies, but that once introduced subsidies become permanent.

With short-run rigidity of wages, a temporary subsidy that maintains a sector's employment and incomes after a fall in the price of its output may be motivated both from efficiency and income distribution considerations. A permanent subsidy, on the other hand, prevents efficient reallocation of resources in the longer run, thus creating a conflict between short-run gains in output and income and long-run efficiency losses. It is this conflict that is the topic of the present paper.

We study a simple two period model of an economy with two export sectors. In the first period, the short run, labor is sector specific and completely immobile between the sectors. In the second period, the long run, labor is perfectly mobile between sectors. This is a simple way to represent the idea that the cost of moving from one sector to another is lower if there is time to plan, to get the necessary retraining, etc. Wages are predetermined and hence rigid in the short run, whereas they are flexible in the long run. It might be argued that in the real world wages are somewhat rigid also in the long run, especially if wage earners have learned to expect
that excessive real wages will be met by subsidies to prevent unemployment. Such mechanisms may be at work, but we take long run wage flexibility to be predominant.

A permanent fall in the price of the output of one of the sectors creates unemployment and income losses for its labor in the short run. In the longer run, some of its labor moves to the other sector and full employment is restored at equilibrium wages. A permanent subsidy to the sector subject to the price fall will increase the sector’s employment and income in the short run, but creates a distortion in the long run; there is too much labor in the subsidized sector. One would, of course, expect a further distortion by a misallocation of investment (at least if the continued subsidization was foreseen). But to make our point, one misallocated factor is really sufficient, so we simplify the analysis by assuming fixed sector-specific capital stocks in both periods.

We determine the optimal permanent subsidy, first with regard to pure efficiency considerations, then with regard to pure income distribution considerations. The optimal efficiency subsidy has the following characteristics: When the subsidy is increased from a situation of no subsidies, the short-run gain in output is a first order effect, whereas the distortion in the long run is of second order. For small unemployment the optimum may therefore be a corner solution where full employment is restored at a subsidy level that causes smaller marginal efficiency losses in the long run than the marginal gains in the short run. If so, a further fall in the output price of the subsidized sector implies a compensating increase in the subsidy. However, for an interior solution where the marginal short-run gain is equal to the marginal long-run loss
at less than full employment, we can show that, under reasonable assumptions, the optimal response to a further price fall is to decrease the subsidy, thus aggravating the short-run unemployment.

With regard to pure income distribution considerations, more precisely a conservative welfare function that aims at maintaining income shares at any cost, it is shown that the optimal distribution policy always restores full employment and income of the sector's labor. A further price fall for the sector's output calls for an increase in the subsidy that completely compensates for the price fall. Thus, (for the interior solution for the optimal efficiency subsidy) there is a clear conflict between efficiency and distribution considerations.\(^3\)

The paper is organized as follows. Section 2 presents the basic model. Section 3 derives some comparative static results for a change in the relative price of export goods. Section 4 discusses the optimal efficiency subsidy. Section 5 derives the effects on the income distribution of a relative price change, and Section 6 derives the optimal distribution subsidy. Finally, Section 7 offers some concluding remarks.

2. **The model**

The model we use is a simple two-sector, two-factor trade model. There are two time periods. It is assumed that one factor, labor, is sector-specific in the first period but that it can move freely between sectors in the second period. The other factor, capital, is sector-specific in both periods.

Two goods are produced and exported in each period, \(x\)-goods and \(z\)-goods, in the quantities \(x^t\) and \(z^t\); \(t = 1,2\) are time period
superscripts. The prices of these two goods in terms of an imported consumption good $p^t$ and $q^t$ are given from the world market.

Total factor endowments in each period are fixed. Since capital is specific to each sector in both periods it can be kept in the background. The supplies of labor in the first period in the $x$- and $z$-sector are $\bar{z}^1$ and $\bar{n}^1$ respectively, and the total supply of labor in the second period is $L^2$. Labor need not be fully employed in the first period, i.e. $\bar{z}^1 \leq \bar{z}^1$ and $\bar{n}^1 \leq \bar{n}^1$. There is always full employment of labor in the second period however,

$$\bar{z}^2 + \bar{n}^2 = L^2.$$ (1)

There are time invariant production functions, homogenous of degree one in capital and labor, viz.

$$x^t = f(k^t)$$ (2)

and

$$z^t = g(n^t).$$ (3)

Both have a positive first derivative, and, because of the fixed capital stock, a negative second derivative.

The profit maximizing conditions are

$$w^t_x = p^t f'_k(k^t)$$ (4)

and

$$w^t_z = q^t g'_n(n^t),$$ (5)

where $w^t_x$ and $w^t_z$ are the wage rates, and subscripts $k$ and $n$ denote partial derivatives. In the first period it is assumed that the
rigid wage rate is the same in both sectors, i.e. \( w^1_x = w^1_z = w^1 \). Consequently, employment has to adjust so that conditions (4) and (5) are fulfilled, and output quantities are then given by the production functions (2) and (3). In the second period the common wage rate, \( w^2 \), is flexible. Output quantities, employment levels and the wage rate are then determined by equations (1)-(5) simultaneously.

Consumption consists of only one good, \( c^t \), which is wholly imported. Its price is set at unity in both periods. We assume that consumers have access to perfect international credit market where any amount can be borrowed or lent at a given real interest rate \( r \). Preferences can be represented by a well-behaved aggregate utility function \( u = U(c^1, c^2) \), defined over consumption only. Consumers maximize utility subject to a wealth constraint \( c^1 + \delta c^2 = y \), where \( y \) is wealth and \( \delta \) is a constant discount factor, defined by \( \delta = 1/(1+r) \). This yields demand functions for the consumption goods in the usual way. Total wealth is given by

\[
y = p^1 x^1 + q^1 z^1 + \delta p^2 x^2 + \delta q^2 z^2;
\]

the present value of output in the two periods.

We can now describe the full equilibrium in this economy. We note that the model is recursive in the sense that production is determined independently of demand. Hence, given the technology, the exogenous world market prices \( p^1 \) and \( q^1 \), and the rigid wage \( w^1 \), we get \( x^1, z^1, x^1 \) and \( n^1 \) from equations (2)-(5), and given the technology, world market prices \( p^2 \) and \( q^2 \), and the fixed endowment of labor \( L^2 \), we obtain \( x^2, z^2, x^2, n^2 \) and \( w^2 \) from equations (1)-(5).
Output quantities together with world market prices give total wealth in equation (6). Consumption levels are then determined by wealth and preferences.

We note that there is a monotone relation between utility and wealth. The indirect utility function is \( u = V(\delta, y) \). Since \( \delta \) is constant and \( V_y > 0 \), utility is strictly increasing in wealth. If preferences were to incorporate labor-leisure considerations as well, the effects of employment changes per se would obviously also have to be included when determining welfare changes. This would not change any of our qualitative conclusions, however (see also Footnote 9 below).

3. Effects of price changes

We will not consider the effects of a change in relative world market prices between the export goods. More precisely we assume that the price of x-goods is lowered and that of z-goods is raised. For simplicity we assume here and in the following that the rates of change are equal, i.e.

\[
\hat{p}^1 = \hat{p}^2 = \hat{p} = - \hat{q} = - \hat{q}^1 = - \hat{q}^2. \tag{7}
\]

In period 1 labor is specific to each sector and in fixed supply. In general we have that

\[
\bar{w}^1 \leq p^1 f^1_x (\xi^1) \quad [\lambda^1 \geq \xi^1] \tag{8}
\]

and

\[
\bar{w}^1 \leq q^1 g^1_n (n^1) \quad [n^1 < n^1], \tag{9}
\]

i.e. below full employment labor will be paid its marginal product, but at full employment the wage rate will be less than or equal to
the value marginal product. But we shall assume that the uniform rigid wage rate is such that before the change in the relative price between the export goods, labor in the z-sector and firms in the x-sector are not rationed in their sectoral labor markets.

Then logarithmic differentiation of (8) and (9), taking (7) into account, yields

\[ x^1 = \gamma_x p^1 < 0 \]  \hspace{1cm} (10)

and

\[ n^1 = 0, \]  \hspace{1cm} (11)

where the labor demand elasticity \( \gamma_x^1 \) is positive. We find that the rate of change in employment is negative in the x-sector. Employment in the z-sector must remain constant when the output price rises, since there is full employment initially.

Hence, output of z-goods is also constant. The rate of change in production of x-goods is obtained by logarithmic differentiation of equation (2) and substitution of equation (10):

\[ x^1 = \theta_x^1 \gamma_x^1 p^1 < 0. \]  \hspace{1cm} (12)

The symbol \( \theta_x^1 \) denotes labor's share of the value of output of x-goods - and, due to the homogeneity assumption, also its cost share. The product \( \theta_x^1 \gamma_x^1 \) is the partial output elasticity of x-goods; since \( p^1 \) is negative \( x^1 \) must also be negative.

In period 2 labor is mobile between sectors and the uniform wage rate is flexible. We solve for the change in the wage rate by substituting equations (4) and (5) into equation (1), all expressed in rates of change, and use (7) to obtain
\[ \omega^2 = \eta_x^2 p_x + \eta_z^2 q_z = (\eta_x - \eta_z)^2 p, \]  

(13)

The wage elasticities, \( \eta_x^2 \) and \( \eta_z^2 \), are defined as

\[ 0 < \eta_x^2 = \frac{\lambda^{2 \times x}}{\lambda^{2 \times x} + \lambda^{2 \times z}} < 1 \quad \text{and} \quad \eta_z^2 = 1 - \eta_x^2, \]  

(14)

where the sectoral labor shares, \( \lambda^t_x \) and \( \lambda^t_z \), \( t = 1, 2 \), are defined as

\[ 0 < \lambda^t_x = \frac{L^t_x}{L^t} < 1 \quad \text{and} \quad \lambda^t_z = 1 - \lambda^t_x. \]

It can be shown that output of \( x \)-goods, whose price has fallen, will fall, and that output of \( z \)-goods will rise. Consequently, labor is reallocated from the \( x \)- to the \( z \)-sector. The movement of the wage rate as given by (13) is ambiguous, however, a well-known result from the specific-factor model (see e.g. Jones (1971)).

To facilitate the analysis in later sections we will now make some simplifying assumptions. Among other things these assumptions suffice to give a determinate change in the wage rate. First, we assume that the value of output in the two sectors are equal in the absence of changes in relative prices and subsidies, i.e.

\[ \frac{1}{\alpha_x} = \frac{1}{\alpha_z} = \frac{2}{\alpha_x} = \frac{2}{\alpha_z} = \frac{1}{2}, \]  

(15)

where \( \alpha^t_i, i = x, z \), denotes the share of the \( i \)-sector in the value of output in period \( t \).

Next, we assume that the time invariant production technology in both sectors is Cobb-Douglas. Hence, the common elasticity of substitution between capital and labor in the two sectors, \( \sigma^t_x \) and \( \sigma^t_z \), is equal to unity. We make the assumption that \( x \)-goods are labor intensive, in the sense that the time invariant cost shares
for labor $\theta_x$ and $\theta_z$ relate as

$$\theta_x > \theta_z$$  \hspace{1cm} (16)

Since the demand elasticity for labor equals the elasticity of substitution divided by one minus the cost share for labor, i.e. $\gamma = \sigma/(1-\sigma)$, it follows that the demand elasticity for labor is higher in the $x$-sector,

$$\gamma_x > \gamma_z = \gamma_z$$  \hspace{1cm} (17)

Also, by (15) and (16) the initial labor shares satisfy

$$\lambda_x > \lambda_z$$.

From (14) we now have that $\eta^2_x$ is greater than $\eta^2_z$. Using assumption (7) in (13) we then get the result that the wage rate in period 2 will fall.

Finally, we turn to the change in total wealth following the output price changes. Logarithmic differentiation of equation (12) yields

$$\dot{y} = \alpha_t(\alpha_x^1 p^1 + \alpha_z^1 q^1) + \alpha^2_t(\alpha_x^2 p^2 + \alpha_z^2 q^2) + \alpha^1_t a^1 x^1$$,  \hspace{1cm} (13)

where $\alpha_t$ is the share of the present value of output in period $t$ in total wealth. The first two terms on the RHS are the familiar terms of trade effects on wealth and welfare. Under our assumptions (15) and (7) about equal output value shares and equal price rates of change they sum to zero. The remaining third term is the negative effect of unemployment in the $x$-sector in period 1. Hence, total wealth is reduced.
4. The optimal efficiency subsidy

Consider now a situation where the government has decided to correct the inefficiency of having unemployed labor in the first period following a change in the relative prices of exports. Unemployment occurs in the x-sector because the wage rate is rigid in the first period. We assume that the government is interested in efficiency and not in correcting the distributional consequences of a change in export prices. (We return to distributional objectives of the government in Section 6.) The policy instrument that we will consider is a production subsidy in the x-sector. We shall assume that the subsidies are financed by equal, proportional and, since factor supplies are fixed, non-distortive taxes on all incomes.

Denote the rate of subsidization in period \( t \) by \( s^t \), \( t = 1, 2 \). Then the description of the production side of the model in Sections 2 and 3 is still valid, provided that we substitute \( (1 + s^t)p^t \) for \( p^t \) everywhere. Welfare is still monotonic in wealth at world market prices.

Assume that subsidies are increased in both periods: \( ds^1 \) and \( ds^2 > 0 \). That will increase production in the x-sector in both periods. The effect on the economy's wealth at world market prices is obtained by differentiating (6), which gives

\[
dy = p_1^1 dx^1 + \delta (p_2^2 dx^2 + q_2^2 dz^2)
\]

However, we know that for movements along the transformation curve in period 2, \( (1 + s^2)p_2^2 dx^2 + q_2^2 dz^2 = 0 \). Therefore we can rewrite the above expression as

\[
\begin{align*}
dy &= p_1^1 dx^1 - \delta s^2 p_2^2 dx^2 \\
&= (19)
\end{align*}
\]
In other words, subsidies will increase wealth at world market prices by an employment effect in the first period. The increase is equal to the value of the increase in production of good $x$ at world market prices. However, the increased production of good $x$ in period 2 at distorted domestic prices, where $s^2 p^2$ measures the distortion, will decrease income at world prices and thereby wealth. It follows immediately that any temporary subsidies, i.e. subsidies only in period 1, always are welfare-increasing, since there will only be the positive employment effect in period 1.

As discussed in the introduction, we are interested in subsidies which become permanent. In the following we therefore assume that subsidies, and changes in subsidies, in period 1 remain in period 2, that is $s^1 = s^2 = s$ and $ds^1 = ds^2 = ds$. In this case there is obviously a trade-off between the employment effects and the distortion.

When the subsidy is optimally chosen subject to this constraint a small change in it should not increase wealth, that is the RHS of (19) should be equal to zero. Let us express this condition in relative changes

$$y = a^1 a^1_{xx} - a^2 a^2_{xx} s = 0,$$

or

$$s = \frac{a^1 a^1_{xx}}{a^2 a^2_{xx}} = \frac{p^1}{\delta p^2} \frac{x^1}{x^2}.$$

Taking the endogenous wage response in period 2 into account and drawing on the Cobb-Douglas assumption, this expression may be rewritten as
\begin{equation}
    s = \frac{1}{\frac{p^1}{\delta p^2 x^2} \frac{x^2}{x^2} \frac{1}{(1 - \eta_x^2)}} \tag{20}
\end{equation}

We can now define the optimal rate of subsidization. To simplify the expression, we set \( p^1 = p^2 = p \) without loss of generality. Then, if we denote the optimal subsidization rate by \( s^* \), we have

\begin{equation}
    s^* \leq \frac{1}{\frac{x^1}{\delta x^2} \frac{1}{(1 - \eta_x^2)}} \cdot [\xi^1 \leq \xi^1] \tag{21}
\end{equation}

The inequality arises because it is never optimal to subsidize more than what is necessary to maintain full employment in the x-sector.

Now, consider an economy where the rate of subsidization is optimally chosen and let the relative export prices change according to (7). That would lower employment in the x-sector, ceteris paribus. What is the optimal response of the government?

There are two possible situations. Either there is full employment initially in the x-sector, and \( s^* \) fulfills (21) with strict inequality. Then it is clearly optimal to maintain full employment in the x-sector by increasing the rate of subsidization. This is done by increasing the subsidies so that the price facing domestic producers is kept constant. Formally,

\begin{equation}
    \hat{s} = \frac{1 + s^*}{s^*} \tag{22}
\end{equation}

In an economy with less than full employment initially, where condition (21) is fulfilled with equality, it will in general be optimal to decrease the subsidization rate, however.
In the Appendix, where we rely on the assumptions stated in the end of the former section, we derive the following relation between $\hat{s}^*$ and $\hat{p}$

$$
\hat{s}^* = \frac{[2n_x^2 + 2n_y n_z (4n_x^2 n_z^2 + n_x^2 - n_z^2)]}{[1 - \frac{s}{1 + \frac{1}{s} n_x^2 (1 + 2n_y n_z (2 + n_z^2 - n_x^2))}] \hat{p}} \cdot (23)
$$

It can be shown that the coefficient on $\hat{p}$ in general is positive. For reasonable subsidy levels this holds independent of whether the $x$-sector or the $z$-sector is labor intensive, unless the two sectors are very different in terms of their relative factor intensities.

An intuitive explanation for this result is as follows. At the outset our first-order condition (19) guarantees that the subsidy is such that the gain in period 1 balances the loss in period 2. If $p$ falls with the subsidy kept constant, the marginal gain of increasing employment has fallen by more than the marginal distortion cost from producing too much $x$ in period 2; $\delta s^2 p^2$ is (in most cases) a fraction of $p^1$. Therefore in general the production of $x$ has to be reduced in both periods to restore the optimum. This implies that the subsidy has to be lowered. The reason for the caveat is that when $p$ and $q$ change, the whole general equilibrium is upset. In particular, the elasticities involved in our condition (21) will change. Therefore the proposed intuition does not hold without qualifications (cf. the Appendix and Footnote 9).

To summarize, the response of a purely efficiency oriented government to a change in relative export prices depends on the initial situation in the industry that is adversely affected. If the initial optimal subsidy is associated with full employment in that industry it is efficient to increase the permanent subsidy by an amount that keeps the domestic price constant. If, on the other
hand, the initial situation is one of unemployment, it is better to lower the subsidy so that the domestic price falls even more. It should be noted that the gain in efficiency in this case involves more unemployment in the x-sector in period 1 than what would have been the outcome with unchanged subsidies. 10

5. The distribution of welfare and wealth

In this section we shall discuss the wealth and income distribution between workers and capitalists in the economy in the absence of subsidies. By our assumption about the utility function in Section 2 the distribution of wealth among the groups also represents the distribution of welfare. 11

We distinguish between workers in the two sectors and capitalists. The wealth of workers in the two sectors is given by

\[ y^t_i = y^t_x + \delta y^t_x = (\omega^t_x + 1) + \delta (\omega^t_x) \]

and

\[ y^t_n = y^t_z + \delta y^t_z = (\omega^t_z + 1) + \delta (\omega^t_z) \]

where \( y^t_i \) denotes income to workers in sector \( i \) (\( i = x, n \)) in period \( t \) (\( t = 1, 2 \)). Note that \( y^t_x \) refers to the present value of income to workers who are associated with the x-sector in period 1, and that some of them may work in the z-sector in period 2. This is why the second period income in (25) is \( \omega^2_z \) rather than \( \omega^2_x \). Analogously, \( y^t_n \) refers to the present value of income to workers associated with the z-sector in period 1. Capitalists' wealth is given by

\[ \pi = \pi^1 + \delta \pi^2 = (\pi^1_x + \pi^1_z) + \delta (\pi^2_x + \pi^2_z), \]
where \( \pi^t_j \) (\( j = x, z \)) is profits in sector \( j \) in period \( t \), and 
\[
\pi^t = (\pi^t_x + \pi^t_z)
\]
is total profits in period \( t \). For simplicity we will assume that the sectoral capital stocks are equal to unity, so that profits equal rental rates on capital.

Let us now look at incomes in period 1. The rate of change in income to workers in sector \( x \) in the first period is
\[
\check{\gamma}_x^1 = \check{w}_x^1 + \check{z}_x^1 = \check{z}_x^1 = \gamma_x (\check{p}_x^1 - \check{w}_x^1) = \gamma_x \hat{p} < 0,
\] (27)
since \( \check{w}_x^1 = 0 \). The income to workers in sector \( z \) does not change,
\[
\check{\gamma}_z^1 = \check{w}_z^1 + \check{z}_z^1 = 0.
\] (28)

As for capitalists, period 1 profits change by the relative price change over the profit shares, i.e. we have
\[
\check{\pi}_x^1 = (\check{p}_x^1 - \theta_x \check{w}_x^1)/(1 - \theta_x) = \hat{p}/(1 - \theta_x) < 0
\] and
\[
\check{\pi}_z^1 = (\check{q}_z^1 - \theta_z \check{w}_z^1)/(1 - \theta_z) = -\hat{p}/(1 - \theta_x) > 0.
\] (29)

This follows from the standard result that \( \dot{p}_j = (1 - \theta_j)\pi_j + \theta_j w_j \), the relative price change is equal to the cost share weighted relative factor price changes.

Since the relative change in total period 1 profits is the share weighted relative changes of profits in the two sectors, we can write
\[
\check{\pi}^1 = \frac{\alpha_x^1 (1 - \theta_x) \pi_x^1 + \alpha_z^1 (1 - \theta_z) \pi_z^1}{\alpha_x^1 (1 - \theta_x^1) + \alpha_z^1 (1 - \theta_z^1)},
\] (30)
since $\alpha_x^1(1 - \theta_x)/[\alpha_x^1(1 - \theta_x) + \alpha_z^1(1 - \theta_z)]$ is the share of profits in sector x in total period 1 profits. It follows from (29) and (15) that total period 1 profits do not change,

$$\hat{\pi}^1 = 0.$$  

The fact that total period 1 profits do not change illustrates a general result: If wages are rigid, the change in profits equals the terms of trade change, whereas the change in the wage bill equals the employment effect. By our assumption of equal output value shares in (15), the period 1 terms of trade effect is zero and, hence, so is the change in profits.

Summarizing, in period 1 sector x workers loose, whereas sector z workers' and capitalists' incomes are both unchanged, i.e.,

$$\hat{y}_x^1 < y_n^1 = \hat{\pi} = 0.$$  \hspace{1cm} (31)

For period 2, with full employment, the relative change of workers' income equals the relative change in wages, hence

$$\hat{y}_x^2 = y_n^2 = \hat{w} = \eta_x^2\hat{\omega} + \eta_z^2 \hat{\omega} = (\eta_x^2 - \eta_z^2)\hat{\omega}.$$  

By our assumptions in section 3, the wage rate in period 2 will fall and, hence, also the income of workers:

$$\hat{y}_x^2 = y_n^2 = \hat{\omega} < 0,$$

By (15) and (7) total income in period 2 remains constant. Therefore, we know that capitalists' income rises,

$$\hat{\pi}^2 > 0.$$
Summing up the period 2 income changes, we have

\[ \hat{y}_2^2 = \hat{y}_n^2 < 0 < \hat{\pi}^2. \]  \hspace{1cm} (32)

The relative changes in each group's wealth are weighted averages of the relative change in income in each period; the weights are the share in wealth of each period's (present value) income. Putting our results (31) and (32) together in that way, we get

\[ \hat{y}_n < \hat{y}_n < 0 < \hat{\pi}. \]  \hspace{1cm} (33)

Capitalists gain, both groups of workers loose, those in sector x most.

6. The optimal distribution subsidy

Subsidies were used in Section 4 to increase efficiency, i.e. to maximize national wealth, following a change in relative export prices. The social welfare function being maximized did not put any restriction on income redistribution effects of subsidies. In this section we shall examine the distributional consequences of changes in both export prices and subsidies, and discuss subsidies which are optimal with regard to the wealth distribution rather than to efficiency.

Assuming that subsidies are financed by a proportional tax with the same tax rate for all kinds of incomes in both periods, the before-tax income distribution will be the same as the after-tax one. Hence it is sufficient to discuss before-tax incomes only. With the same subsidy rate \( s \) on x-goods, domestic prices of x-goods are given by \( (1 + s)p^t = Sp^t \) and the corresponding
relative changes by \( \hat{S} + \hat{p}^t \). It follows that we get the changes in before-tax incomes by simply substituting \( \hat{S} + \hat{p}^t \) for \( \hat{p}^t \) in the expressions (27), (29) and (33). Letting all income and wealth variables in this section be before-tax, we hence get, with (28), (29) and (30),

\[
\hat{y}_x^1 = \gamma_x(\hat{S} + \hat{p}),
\]

\[
\hat{y}_n^1 = 0,
\]

\[
\hat{\pi}_x^1 = (\hat{S} + \hat{p})/(1 - \theta_x),
\]

\[
\hat{\pi}_z^1 = -\hat{p}/(1 - \theta_z),
\]

\[
\hat{\pi}^1 = \hat{S}/[(1 - \theta_x) + (1 - \theta_z)]
\]

\[
\hat{y}_x^2 = \hat{y}_n^2 = \hat{\omega}^2 = \eta_x^2(\hat{S} + \hat{p}) - \eta_z^2 \hat{p}, \quad \text{and}
\]

\[
\hat{\pi}^2 = [\hat{S} - (\theta_x + \theta_z)\hat{\omega}^2]/[(1 - \theta_x) + (1 - \theta_z)].
\]

(In deriving the change in total profits in the two periods, we assume that (15) holds for the inclusive-of-subsidy production shares.) We see that in period 1 sector x workers and capitalists improve their relative positions because of the subsidy.

Let us next discuss how subsidies would be chosen to be optimal with regard to the income distribution. Suppose we have a weighted Rawlsian social welfare function, expressed in before-tax wealths as

\[
W(\hat{y}_x, \hat{y}_n, \hat{\pi}) = \min [\beta_x \hat{y}_x, \beta_n \hat{y}_n, \beta_\pi \hat{\pi}],
\]

The positive constant weights \( \beta_x \), \( \beta_n \) and \( \beta_\pi \) are such that before the change in export prices and subsidies, the wealth distribution
is optimal and

\[ \beta^*_y y^*_y = \beta^*_n y^*_n = \beta^*_\pi. \]

This welfare function and the choice of weights imply that after a change in export prices, subsidies should be adjusted so as to restore income shares to their previous values. If not all three income shares can be restored, at least the shares between the two relatively deteriorating groups should be equalized. This follows since the relative change in social welfare will be

\[ \hat{W} = \min [\hat{y}_y, \hat{y}_n, \hat{\pi}]. \]

By the results in Section 5, a change in the relative price of export goods would yield

\[ \hat{W} = \hat{y}_y < \hat{y}_n = 0 < \hat{\pi}. \] (35)

Clearly we can improve social welfare by choosing the optimal relative change in subsidies so as to offset the change in the price of x-goods, i.e.,

\[ \hat{S} = -\hat{p} > 0. \] (36)

Then both x and z workers' incomes are unchanged in period 1, whereas they increase equally much in relative terms in period 2, i.e.,

\[ \hat{y}^1_y = \hat{y}^1_n = 0 \quad \text{and} \]
\[ \hat{y}^2_y = \hat{y}^2_n = \hat{\pi} = -\eta \hat{z}_p > 0. \]

Workers' and capitalists' before tax wealth will increase, although capitalists' relatively more. Then the relative change in the welfare
level is given by

\[ 0 < \dot{\hat{w}} = \dot{\hat{y}}_w = \dot{\hat{y}}_n < \ddot{\pi}. \quad (37) \]

In summary, given the above social welfare function, the optimal change in the subsidy is to completely offset the change in prices of \( x \)-goods. This holds constant the relative wealth distribution between workers, whereas the capitalists still improve their position relative to workers.

Comparing with the optimal efficiency subsidy in Section 4, we see that if there is full employment initially ((21) is fulfilled with inequality), there is no conflict between the optimal efficiency subsidy and the optimal distribution subsidy. The two should be adjusted in the same way following an export price change ((36) is equal to (22) by the definition of \( S \)). When there is unemployment ((21) is fulfilled with equality) there is a conflict. The optimal distribution subsidy should be raised but according to the discussion in Section 4, the optimal efficiency subsidy should be lowered. Adjusting the subsidy to maintain the wealth distribution deteriorates efficiency, adjusting it to maintain efficiency further aggravates the wealth distribution.

We have thus highlighted two polar forms of the government's objective function; either efficiency is maintained at whatever distribution cost, or vice versa. Obviously the more realistic case with a trade-off between the two objectives is also interesting. We have indeed tried to carry through the analysis with a more general social welfare function. But this was not very successful, since the problem became surprisingly hard to solve.
7. **Concluding remarks**

Stretching the results somewhat, they can be transformed into the following story. One sector in the economy is hit by a fall in the world market price of its output. In the absence of government intervention this will affect income distribution. The price fall will also create inefficiency, in the form of unemployment, because wage rates are rigid in the short run. The government seeks to prevent changes in income distribution. It knows that if it decides to subsidize the adversely affected sector for this reason it will, for reasons mentioned in the introduction, have to make the subsidy permanent. Nevertheless, because of the government's distributional goal, a subsidy is called for. The government also considers efficiency. Given the institutional framework and the constraint that subsidies become permanent, it finds that a subsidy is motivated also for reasons of efficiency. If the price fall is substantial, the subsidy is such that some employment is permitted in the short run; income distribution has had to give in somewhat to efficiency. Next, there is a new fall in the world market price. Equity considerations now call for a further increase in the subsidy. Efficiency considerations, on the other hand, call for a decrease if the subsidy already permits some unemployment. Hence, a genuine trade-off between equity and efficiency arises.

The lesson of the story is evident. No hard decisions have to be made to embark on the road of protecting declining industries. At first the ride is smooth and comfortable. Eventually the road divides into two, making hard decisions necessary. Whichever decision is taken, the ride proceeds at some sacrifice.
Footnotes

* We wish to thank participants in seminars at the Institute for International Economic Studies, as well as in the Stockholm Theory Workshop for helpful discussions, and J. Peter Neary and an anonymous referee for comments on an earlier draft. Each of the authors blames the other two for remaining flaws and obscurities.

1. The situation in Sweden is illustrative and by no means atypical. Subsidies to traditional export industries, such as mining of iron-ore, steel-making and shipbuilding, have been extensive in recent years and there is no end in sight. The textile industry, in decline for more than two decades, has been subsidized for almost as long. In the most heavily subsidized industry, shipbuilding, subsidies as a share of value-added were 69 per cent on average for the four fiscal years 1976/77 to 1979/80; in fiscal year 1978/79 they were 153 per cent and the subsidy per employee was 50,480 US dollars. The average ratio between subsidies and value-added for industry as a whole (ISIC 2+3) during the same period was 4.4 per cent. (All figures are from Hamilton's (1983) valuable study on public subsidies to the Swedish shipbuilding industry.)

2. Corden (1974) argues that actual trade policies of many countries can better be understood if it is thought that countries try to prevent "any significant absolute reductions in real incomes of any significant section of the community". This "conservative social welfare function" "... helps to explain the income maintenance motivation of so many tariffs in the past and the reluctance to reduce income maintenance tariffs even when it has become clear that the need for them is more than temporary ..." (p. 107).

3. Lapan (1976) has calculated the time path of the optimal efficiency subsidy for a two-sector model where labor is not instantaneously mobile between sectors. He assumes that the flow of labor between sectors is proportional to the level of unemployment in the declining sector. For long planning horizons (or small discount rates) he finds that some unemployment is desirable during
the early stages. When the time horizon is very short (or the discount rate is higher) there may be full employment in the short run. Lapan's analysis has recently been extended by Horen and Rieszman (1981), who calculate the optimal dynamic tariff in Lapan's model and then compare it to the optimal dynamic subsidy.

Our analysis differs from Lapan's in several aspects, most importantly in that we consider a permanent subsidy that is constant over time, whereas he considers a subsidy that can be freely adjusted over time; and in that we assume full mobility of labor in the long run regardless of the level of employment in the short run, whereas he, as mentioned, assumes that the migration of labor between sectors is proportional to the level of unemployment.

Diamond (1982) considers the choice between subsidies to a declining sector and adjustment assistance to the sector's workers, taking into account the loss in skill associated with moving as well as differences in individual workers' moving costs. He finds that, for a small fall in the price of the declining sector's output, subsidizing the sector to prevent migration is optimal in the sense of maximizing an additive social welfare function, whereas for a large fall in price, migration should be encouraged by adjustment assistance to workers. Diamond does not consider problems with permanent subsidies that remain in the long run.

4. The expression \( a \leq b, (c \leq d) \) denotes the complementary slackness conditions \( a \leq b, c \leq d, a < b \Rightarrow c = d \) and \( c < d \Rightarrow a = b \).

5. From the definition of \( \sigma \) we have \( \hat{\kappa} - \hat{k} = \sigma(\hat{r} - \hat{w}) \) and because of constant returns to scale \( \hat{r} = \hat{p}/(1 - \theta) - \theta\hat{w}/(1 - \theta) \). Substituting the latter expression into the former and setting \( \hat{k} = 0 \), we have \( \hat{\kappa} = \sigma(\hat{p} - \hat{w})/(1 - \theta) \).

6. We could alternatively have dealt with an employment subsidy. Since employment and production in the x-sector both depend on the product real wage only, there is, however, a one-to-one relation between the production and employment subsidies that is necessary for a given increase in employment.
7. To assume not only that the subsidy remains but also that it is constant may seem too restrictive. However, the point is that subsidies are permanent. Letting the level vary somewhat between periods would add complexity but no further understanding.

8. The relative change in the rate of subsidization \( \hat{s} \), is equal to a price change, also in relative terms, of \((s/(1+s))s\). Then we have, cf. Section 3,
\[
\hat{x}^1 = \theta_{yx}^1 (s/(1+s))\hat{s} \quad \text{and} \quad \hat{x}^2 = \theta_{yx}^2 [(s/(1+s))\hat{s} - \hat{w}^2] = \theta_{yx}^2 (1 - \eta_x^2) (s/(1+s))\hat{s},
\]
where as before \( \theta_{yx}^e \) is the (partial) elasticity of supply in the \( x \)-sector and \( \eta_x^2 \) is the elasticity of the wage with respect to changes in \( p \). With a Cobb-Douglas-technology the supply elasticity in both periods must be the same, however. Setting \( \theta_{yx}^1 = \theta_{yx}^2 \) in the expression for \( \hat{x}^1 \) and \( \hat{x}^2 \) and substituting them into the expression for \( s \) in the text, we obtain (20).

9. If the \( z \)-sector is labor intensive relative to the \( x \)-sector, \( \theta_z > \theta_x \), then \( \eta_z^2 > \eta_x^2 \). Let us assume that the two sectors are very different and set \( \theta_z = 2/3 \) and \( \theta_x = 1/3 \). This implies that
\[
\theta_{yx} = \theta_x/(1 - \theta_x) = \theta_x/(1 - \theta_x) = 1/2, \quad \text{and} \quad \theta_{yz} = \theta_z/(1 - \theta_z) = 2.
\]
Also \( \eta_x^2 = \lambda_x \phi_x/(\lambda_x \phi_x + \lambda_z \gamma_z) = \theta_{yx}/(\theta_{yx} + \theta_{yz}) = 1/2(1/2 + 2) = 2/10 \), and hence \( \eta_z^2 = 8/10 \).

Therefore the numerator in (23) is \( 0.42 > 0 \), and the denominator is \( 1 - \frac{s}{2+s} 0.2(1 + 0.5(2 + 0.8 - 0.2)) = 1 - \frac{s}{1+s} 0.46 > 0 \), since \( s/(1+s) < 1 \).

Assume, on the other hand, that the \( x \)-sector is labor intensive and set \( \theta_x = 2/3 \), and \( \theta_z = 1/3 \). This gives \( \theta_{yx} = 2, \eta_x^2 = 8/10 \), and \( \eta_z^2 = 2/10 \). The numerator in (23) is then \( 1.6 + 2(4 \cdot 0.16 + 0.8 - 0.2) = 3.08 > 0 \), and the denominator is \( 1 - \frac{s}{1+s} 0.8(1 + 2(2 + 0.2 - 0.8)) = 1 - \frac{s}{1+s} 3.04 \), which is positive unless \( s > 1/2.04 \approx 0.49 \).
10. One could also think of a third case when a corner solution to (21) is optimal before but not after the change in p. In this case it seems that the sign of $\hat{s}^*$ is indeterminate. But we cannot solve for $\hat{s}^*$ with our differential calculus method, since the function which relates $s^*$ to p is not differentiable in the limit where the inequality turns to an equality. (Strictly speaking, such a transition between a full employment and an unemployment regime would require a "large" change in p while we, by assumption, deal with infinitesimal changes.)

11. If preferences had incorporated labor-leisure, the effects of changed employment per se would have to be considered as well. This would change things quantitatively but not qualitatively. For example, instead of now being $V_y w_1 d\lambda$, the effect on x workers welfare of a change in first period employment would be $V_y (\bar{w}_1 - \bar{w}) d\lambda$, where $\bar{w}_1$ denotes the (shadow) supply price of labor. There would still be a gain from an increase in employment, though, since $\bar{w}_1$ must be lower than $\bar{w}$ as long as workers are rationed in the labor market.

12. That profits increase more than workers' incomes follows since 
$\hat{\pi}_1 = (-\hat{p})/[2 - (\theta_x + \theta_z)] > y^1_\lambda = y^1_\mu = 0$ and 
$\hat{\pi}_2 = (-\hat{p})[1 - (\theta_x + \theta_z)\eta^2_z]/[2 - (\theta_x + \theta_z)]$. But since $\gamma_z < \gamma_x$ and $\gamma_z < \gamma_x$ initially it follows from (14) that 
$2\eta^2_z < 1, [1 - (\theta_x + \theta_z)\eta^2_z]/[2 - (\theta_x + \theta_z)] > [2\eta^2_z - (\theta_x + \theta_z)\eta^2_z]/[2 - (\theta_x + \theta_z)] = \eta^2_\lambda$, and then $\hat{\pi}_2 > -\eta^2_\lambda p = \bar{w}$, hence, $\hat{\pi} > \hat{y}_\lambda = \hat{y}_\mu$. 

Appendix: Derivation of expression (23)

Since \( \eta_x + \eta_z = 1 \), we know from (21) that the initial situation is

\[
s^* = \frac{x}{\delta x^2 \eta_z^2}.
\]

Differentiating logarithmically, we have

\[
\hat{s}^* = \hat{x}^1 - \hat{x}^2 - \eta_z^2.
\] (A.1)

Define the new variable \( S = (1 + s) \) and differentiate the RHS of (A.1) with respect to \( s, p, \) and \( q, \) where, as in the text, we let \( \hat{q} = -\hat{p} \). For the two first terms, this yields

\[
\hat{x}^1 = \hat{\theta}_x^1 y_x^1 (S + \hat{p}) \quad \text{and}
\]

\[
\hat{x}^2 = \hat{\theta}_x^2 y_x^2 (S + \hat{p} - \hat{w})
\]

\[
= \hat{\theta}_x^2 y_x^2 (S + \hat{p} - \eta_x^2 (S + \hat{p}) - \eta_z^2 \hat{q})
\]

\[
= \hat{\theta}_x^2 y_x^2 \eta_z^2 (S + 2\hat{p})
\]

But by the Cobb-Douglas assumption, \( \hat{\theta}_x^1 \phi_x^1 = \hat{\theta}_x^2 \phi_x^2 = \hat{\theta}_x y_x \), and we get

\[
\hat{x}^1 - \hat{x}^2 = \hat{\theta}_x y_x [(1 - \eta_z^2)S + (1 - 2\eta_z^2)\hat{p}] .
\] (A.2)

Remains to find \( \hat{\eta}_z^2 \). The definition of \( \eta_z^2 \) is

\[
\eta_z^2 = \frac{\lambda z y_z}{\lambda x y_x + \lambda z y_z},
\]

which we differentiate to get

\[
\hat{\eta}_z^2 = (\lambda z \hat{y}_z) - (\lambda x \hat{y}_x + \lambda z \hat{y}_z).
\]

However, because of the Cobb-Douglas assumption \( \gamma_x = \sigma_x/(1 - \theta_x) \)

and \( \gamma_z = \sigma_z/(1 - \theta_z) \) are both constant. Using the definition of \( \hat{\eta}_x^2 \) and \( \hat{\eta}_z^2 \), we then get
\[ \eta_z^2 = \lambda_z - (\lambda_x (1 - \eta_z^2_x) + \lambda_z \eta_z^2) \]

and, since \( \lambda_x = - (\lambda_z / \lambda_x) \lambda_z \) because \( \lambda_x + \lambda_z = 1 \),

\[ \eta_z^2 = (1 - \eta_z^2_x) \lambda_z / \lambda_x. \]  \hspace{1cm} (A.3)

So the next step is to find \( \lambda_z \). To do this we note that

\[
\lambda_z = \frac{n}{k + n} = \frac{\theta_{z q z}}{\theta_{z x} s x + \theta_{z q z}} = \frac{\theta_{z x}}{\theta_{z x} a_x + \theta_{z z}}
\]

However, because of our assumption \( \alpha_x = \alpha_z = 1/2 \) initially, we know that initially \( \lambda_z = \theta_{z x} \) and hence \( \lambda_x = \theta_{z z} \). Using this and the fact that \( \theta_{z x} \) and \( \theta_{z z} \) are constant, we differentiate the expression for \( \lambda_z \) to get

\[
\hat{\lambda}_z = \hat{\alpha}_z - \theta_{z x} \hat{\alpha}_x - \theta_{z z} \hat{\alpha}_z = \theta_{z x} (\hat{\alpha}_z - \hat{\alpha}_z)
\]

or, since \( \hat{\alpha}_x = - (\alpha_z / \alpha_x) \hat{\alpha}_z = - \hat{\alpha}_z \) (by assumption \( \alpha_x = \alpha_z \))

\[
\hat{\lambda}_z = -2 \theta_{z x} \hat{\alpha}_x.
\]  \hspace{1cm} (A.4)

From the definition of \( \alpha_x \)

\[
\hat{\alpha}_x = \hat{S} + \hat{p} + \hat{x} - \alpha_x (\hat{S} + \hat{p} + \hat{x}) - \alpha_z (\hat{q} + \hat{z})
\]

\[
= \hat{S} + \hat{p} + \hat{x} - \alpha_x (\hat{S} + \hat{x}) - \alpha_z \hat{z} - (\alpha_x - \alpha_z) \hat{p}
\]

\[
= \hat{p} + (1 - \alpha_x) (\hat{S} + \hat{x}) - \alpha_z \hat{z},
\]

where the last equality follows since \( \alpha_x = \alpha_z \) by assumption.

Developing this expression and simplifying, using that \( \alpha_x = 1/2 \),

we get

\[
\hat{\alpha}_x = (1 + \eta_z^2 \theta_{z x} \gamma_x + \eta_z^2 \theta_{z z} \gamma_z) (p + S/2).
\]
Finally, one shows easily that, because $\lambda_z = \theta_z$ and $\lambda_x = \theta_x$ initially,
\[\frac{n_x^2 \gamma_x}{n_x} = \frac{n_z^2 \gamma_z}{n_z},\] so we obtain
\[\hat{a}_x = (1 + 2n_x^2 \gamma_x)(\hat{p} + \hat{S}/2),\] (A.5)

It is now straightforward to substitute (A.5) into (A.4) and substitute the resulting expression into (A.3). Performing these operations and simplifying, we get
\[\hat{n}_z^2 = -n_x^2(1 + 2n_z^2 \gamma_x)(2\hat{p} + \hat{S}).\] (A.6)

Then we use (A.2) and (A.6) to rewrite (A.1). Doing this and simplifying, we have
\[\hat{s}^* = [n_x^2(1 + \theta_x \gamma_x)(2 + \gamma_x^2 - n_x^2)]\hat{s} + [2n_x^2 + \theta_x \gamma_x(4n_x^2 \gamma_x^2 + \gamma_x^2 - n_x^2)]\hat{p},\]

and since, from the definition of $S$, $\hat{S} = \hat{s}(s/(1+s))$
\[\hat{s}^*[1 - \frac{s}{1+s} \frac{n_x^2(1 + \theta_x \gamma_x)(2 + \gamma_x^2 - n_x^2)}{n_x^2(1 + \theta_x \gamma_x)(2 + \gamma_x^2 - n_x^2)}] = \]
\[= [2n_x^2 + \theta_x \gamma_x(4n_x^2 \gamma_x^2 + \gamma_x^2 - n_x^2)]\hat{p},\]

which is precisely our expression (23).
References

Corden, Max W., 1974, Trade policy and welfare (Oxford University Press, Oxford).


Horen, Jeff and Riezman, Raymond, 1981, Optimal dynamic tariff and subsidy policy with adjustment costs, The University of Iowa (mimeo).
