Dynamical Analysis and System Identification of the Gantry-Tau Parallel Manipulator

Examensarbete utfört i Reglerteknik vid Tekniska högskolan i Linköping
av
Johan Gunnar

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Linköping, 20 December, 2005
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This report presents work done in the field of linear and nonlinear system identification on robots. The subject of study has been a new parallel manipulator called Gantry-Tau. The work shall be seen as one of the first steps in the dynamical analysis of the robot. All practical work presented in the report was conducted on a prototype situated at University of Queensland.

The actuators have been analysed and modelled with the aim to gain knowledge of weaknesses and dynamical behaviour. The analysis resulted in a study of nonlinear grey-box identification of hysteresis in the drive train of the actuators. A very compact nonlinear hysteresis model was used together with a three-step identification procedure. The results show that a model of the nonlinear system can be successfully identified from measurement data.

Finally a method for estimation of parameters in the model for the inverse dynamics of the leg structure has been investigated. It turns out that the investigated method is not able to give accurate estimates. This is thought to be a result of unmodelled behaviour in the system and noisy data.

Keywords Robotics, identification, nonlinear, parallel, Tau
Abstract

This report presents work done in the field of linear and nonlinear system identification on robots. The subject of study has been a new parallel manipulator called Gantry-Tau. The work shall be seen as one of the first steps in the dynamical analysis of the robot. All practical work presented in the report was conducted on a prototype situated at University of Queensland.

The actuators have been analysed and modelled with the aim to gain knowledge of weaknesses and dynamical behaviour. The analysis resulted in a study of nonlinear grey-box identification of hysteresis in the drive train of the actuators. A very compact nonlinear hysteresis model was used together with a three-step identification procedure. The results show that a model of the nonlinear system can be successfully identified from measurement data.

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# Contents

1 Introduction 1  
1.1 The Thesis Project ............................................. 1  
1.2 Problem Statement ........................................... 2  
1.3 Outline of the Thesis ........................................ 2  

2 Robotics 3  
2.1 Historical Background ........................................ 4  
2.2 Modelling .................................................... 5  
2.2.1 Kinematics ................................................ 6  
2.2.2 Dynamics .................................................. 7  

3 Parallel Manipulators and the Gantry-Tau robot 9  
3.1 Comparison with Serial Structures ............................ 9  
3.2 The Tau-structure ............................................. 11  
3.3 Gantry-Tau .................................................... 11  
3.4 Kinematics and Dynamics for the Tau-structure .......... 13  

4 System Identification and Nonlinear Models 15  
4.1 Experimental design .......................................... 16  
4.2 Validation .................................................... 16  
4.3 Linear regression and least squares estimation .......... 17  
4.4 Nonlinear effects and models ................................ 17  

5 Analysis of the Actuators 19  
5.1 System description ............................................ 19  
5.2 Analysis ...................................................... 19  
5.3 Rigid body dynamics .......................................... 20  
5.3.1 Estimation methods ...................................... 20  
5.3.2 Motor parameters ........................................ 21  
5.3.3 Linear drive ............................................... 23  
5.3.4 Conclusions ............................................... 24  
5.4 Analysis of flexibilities .................................... 25  
5.4.1 Experimental design ...................................... 25  
5.4.2 Flexibility in the Belt Gear ............................... 25  
5.4.3 Total Flexibility and Flexibility in Lead screw ....... 26
Chapter 1

Introduction

In this chapter an introduction to the thesis is given. Some background and information about the project, the problem statement, and the outline of the report are given.

Industrial robots are today a mature product with plenty of applications in the manufacturing industry. Using basically the same concept with a single mechanical arm, technical improvement of especially the control system has made it possible to handle more and more tasks. However, this construction has some fundamental limitations that cannot be compensated for. This has meant that for some applications, where high stiffness, extreme accuracy or exceptional dynamical behaviour are demanded, industrial robots based on this concept can still not be used. With the ambition to overcome some of these limitations new manipulators have been designed, based on a concept with several arms linked together. The concept is generally referred to as parallel kinematic manipulators, PKMs. Today ABB has one manipulator on the market based on this technology, called the IRB 340 Flexpicker. The manipulator is mainly designed for pick-and-place tasks and can reach high accelerations and velocities. The manipulator studied in this thesis is designed by ABB for assembly of aeroplane components, a task today performed by large, inflexible and very expensive linear manipulators that can handle the demand of high stiffness and accuracy on a large workspace. The manipulator is based on a unique arm structure, called the Tau-structure, and is referred to as the Gantry-Tau manipulator. The Gantry-Tau is developed at University of Queensland, UQ, in Brisbane, Australia, where a prototype of the Gantry-Tau has been constructed. The research project is a cooperation between UQ and ABB Automation in Västerås, Sweden.

1.1 The Thesis Project

This report is a result of a thesis project conducted at the Department of Electrical Engineering at Linköpings University. The work has been done under Professor Svante Gunnarsson at the division of Automatic Control. All experiments and
a large part of the theoretical work has been conducted at School of Information Technology and Electrical Engineering at UQ, under supervision of Dr Geir Hovland, chief investigator at the research project. Dr Torgny Brogårdh, partner investigator of the research project, at ABB Automation has contributed with an industrial perspective and shared interesting ideas and problem formulations.

1.2 Problem Statement

The challenge in developing a robot that can replace the linear manipulators used today lies in assuring a more flexible and cost efficient solution with the same stiffness and accuracy. This puts high demand on joints, arms and the support structure, but the performance will also depend on the actuators used. Although additional sensors can be installed to reduce these effects a good knowledge of the dynamical behaviour of the actuators is essential. In this thesis the actuators used on the prototype at UQ will be analysed and modelled. The task is mainly to see where the flexibilities\(^1\) of the construction are located and if an accurate model of the system can be identified. The demands of high accuracy also mean that accurate models of the arm structure must be derived. At UQ, some work has been done on the development of estimation methods of parameters in the kinematic model of the structure. In the work reported here the dynamics of the structure will be considered. A method for estimation of parameters in an existing model of the structure will be investigated. The task is to apply a method used for serial manipulators to see if the parameters in the model can be estimated.

1.3 Outline of the Thesis

This thesis starts with an introduction and a historical background to robotics in Chapter 2. In Chapter 3 an overview of parallel kinematic manipulators in general and an introduction to the Gantry-Tau is given. Chapter 4 introduces the reader to system identification and nonlinear effects. In Chapter 5 a thorough analysis of the actuator is performed and in Chapter 6 the identification of a nonlinear grey-box model of the actuator is studied. Chapter 7 is focused on the dynamics of the Tau-structure and the estimation of parameters in the inverse dynamical model. Finally the conclusions from the different chapters are summarised in Chapter 8.

\(^1\)The word flexibility can have different meanings. Throughout this report, the word will denote elastic effects, which is a negative property.
Chapter 2

Robotics

In this chapter the reader is introduced to robotics in general. A brief summary of important moments in robotics are given as well as concepts used for kinematic and dynamical modelling.

In this work the term robot will be used for industrial manipulators as the one shown in Figure 2.1. Traditionally this type of robot is basically a mechanical arm, controlled by a computer. The Robot Institute of America [25] defines a robot as: A robot is a reprogrammable multifunctional manipulator designed to move material, parts, tools or specialized devices through variable programmed motions for the performance of a variety of tasks.

The words robot, industrial robot and manipulator will in this work refer to the same thing. Although the main part of the thesis will concern parallel kinematic manipulators, see Chapter 3, the introduction will use serial kinematic manipulators for a general discussion and for defining some fundamental modelling concepts. A serial kinematic manipulator is a robot with an open kinematic chain, as the one in Figure 2.1. A parallel kinematic manipulator is constructed of at least two kinematic chains connected at the end-effector [20]. The reason to introduce the reader to serial manipulators is that the concepts developed for these will be applied to the parallel manipulator in Chapter 3. Some of these concepts are however easier to understand by looking at a serial manipulator.

The serial manipulator is constructed of several mechanical links or axis connected by joints. The joints are either revolute or prismatic. A revolute joint allows relative rotation and a prismatic joint allows linear relative motion between the connected links. One can see the robot as an arm, a wrist and an end-effector, see Figure 2.1. The arm ensures mobility, the wrist, i.e. the joints closest to the end-effector, ensures dexterity and the end-effector performs the task. Robots constructed in this way are sometimes referred to as anthropomorphic robots, i.e. they imitate the anatomy of humans. The area that can be reached by the robot is referred to as its workspace. Normally a robot has six degrees-of-freedom (DOF) making it possible to place the end-effector in the workspace and at the same time
controlling its orientation. The term \textit{tool centre point}, TCP, is used frequently and is defined as the centre of the tool, \textit{i.e.} the centre of the end-effector. The term \textit{footprint} refers to how much surface area, or floor area, a robot occupies. [25]

2.1 Historical Background

Today robots are natural components in the manufacturing industry and is even expanding to other fields. Robots are however a pretty young product compared to other equipment used today. While it still is an evolving product it can be interesting to get a historical background to the robots used today. The background given here is based on [30] and has no ambition to give a complete picture, but to point out some important moments and to give a background to the European and Swedish robot production.

The robotic history began in the late 1950’s in USA where George Devol and Joseph Engelberger started what was to become the Unimation. Joseph Engelberger is sometimes called ”The Father of Robotics”. Unimation was the first company who delivered robots to the American industry and General Motors was the first customer in 1961. The word ”Robot” comes from the the Czech play
"Russums Universal Robots", performed in the 1920's.

The big breakthrough came 1964 when General Motors ordered 66 Unimate robots from Unimation, to be installed in their new top modern factory in Ohio. Even though the industry was hard to convince the public was now very interested in the robots, and Unimate robots appear in commercials and talk shows. Soon several other companies followed. IBM, AMF, Hughes Aircraft and Western Electric are just some of many companies who started their own production of robots.

In Europe the Scandinavian countries were early to adopt the new invention and several companies in Sweden, Norway and Finland started to develop robots or produce the Unimate robot on licence. Among the pioneers in Sweden one can mention Roland Kaufeldt, founder of Kaufeldt AB, the kitchen appliances company Electrolux, Esab, manufacturer of welding products and Asea, electronics manufacturer. In Norway Trallfa, manufacturer of wheelbarrows, constructed a robot for painting which became a success story.

In 1971 Asea started to develop a robot which would come to make ABB one of the main player on the market. The robot, which was to be called IRB 6, had a fully electronic control and power system, and was the first microprocessor controlled robot. It was also an anthropomorphic robot, i.e. it imitated the human anatomy. Using Harmonic Drives meant that it was much more compact than other robots. The production of IRB 6 started in 1973.

Other countries in Europe were not so eager to follow. Europe had a high unemployment rate and there was no need for robots since the pressure to raise productivity was relatively low. There was however some exceptions. The German company Kuka developed a robot used for welding, mainly sold to the European car industry. The European car industry also started to produce robots on their own.

While Europe had a high unemployment rate and no problem to get enough labour the situation was very much the opposite in Japan. The high economical growth in the 60's resulted in a lack of labour. This meant that the companies were very open for new ideas and the robots were embraced as a way to increase the production. The industry was quick to apply the robots in the production and soon there were many Japanese robot producers to compete on the growing market. In 1980 there were 150 Japanese robot producers and in 1988 nearly 70% of the 256 000 robots in use all over the world were installed in Japan. Some large Japanese robot manufacturers today are Fanuc, Yaskawa and Kawasaki.

In Scandinavia Asea became the main robot producer in the mid 80's when both Electrolux robot production and Trallfa was incorporated. After Asea and the Swiss company Brown Boveri merged in 1988 and formed ABB, the robot production of Cincinnati Milicroms, Graco Robotics and Esab were also incorporated.

2.2 Modelling

In this section some basic concepts of robotics will be defined and explained. To be able to work with control, identification or even to understand general papers
in robotics these concepts are fundamental. The concepts will only be introduced here. A more thorough discussion can be found in, e.g [24, 25].

The advantage of introducing these concepts for a serial kinematic manipulator is that the joint variables, $q$, are simple to identify and relate to. In this case the joint variables are the the angle of rotation for a revolute joint and the amount of linear displacement for a prismatic joint. All these variables can normally be measured as the angle of the motor shaft at the given joint.

### 2.2.1 Kinematics

The kinematic modelling is concerned with transforming positions and velocities in different base frames to each other. In position kinematics one is concerned with two problems, forward kinematics and inverse kinematics. The aim with forward kinematics can be stated as [25]: Given the joint variables, determine the position and orientation of the end-effector. This means that the forward kinematics is basically a transformation from the base frames in each joint to the Cartesian, room fixed, frame. The inverse kinematics is the inverse transformation, i.e to find the joint variables from a given position and orientation of the end-effector.

The forward kinematics can be seen as the transformation

$$\begin{pmatrix} x \\ r \end{pmatrix} = f(q)$$  \hspace{1cm} (2.1)

where $x \in \mathbb{R}^3$ is the position of the end-effector in the Cartesian frame and $r$ describes the orientation of the end-effector.

One realises that for a serial kinematic manipulator the forward kinematics can be derived relatively easy. Starting from the base of the robot one can determine the position and orientation of any joint in the kinematic chain from the joint variables, by applying the rotations and displacements in the same order as the joints. This is basically how the problem is solved. The inverse kinematics is generally a more difficult problem and often gives more than one solution, depending on the DOF of the robot considered. To make the calculations of the kinematic relationships more efficient one often use the Denavit-Hartenberg representation [24, 25].

The velocity kinematics is the problem of relating the linear and the angular velocities of the end-effector with the joint velocities. When deriving these relationships one of the most important quantities in the analysis of robot motion is defined, the manipulator Jacobian. Starting from the forward kinematics a set of equations that transforms the joint positions to end-effector position and orientation are given. The relationships between the velocities is then given by the Jacobian, hence the relationship is given by

$$\begin{pmatrix} \dot{x} \\ \dot{r} \end{pmatrix} = \begin{pmatrix} u \\ \omega \end{pmatrix} = J(q)\dot{q}$$  \hspace{1cm} (2.2)

The Jacobian is a matrix-valued function containing the partial derivatives of the position kinematic relationships. The Jacobian appears not only here but also
in problems as trajectory planning, determination of singular configurations and derivation of dynamical equations [25]. The inverse velocity kinematics is of course given by the inverse Jacobian, assuming that the inverse exists.

2.2.2 Dynamics

The dynamics refers to the problem of finding dynamical equations that describe the time evolution of a set of variables. The dynamic model is often divided into rigid body dynamics and flexible body dynamics. The dynamical modelling can also be separated in a direct (or forward) and an inverse problem [24].

The direct dynamics problem consists of finding \( \ddot{q}(t) \), \( \dot{q}(t) \) and \( q(t) \) given the joint torques \( \tau(t) \), possibly end-effector forces \( f(t) \), and initial position and velocity, \( q(t_0) \) and \( \dot{q}(t_0) \).

The inverse dynamics problem on the other hand consists of finding \( \tau(t) \) given \( \ddot{q}(t) \), \( \dot{q}(t) \) and \( q(t) \), assuming the end-effector forces \( f(t) \) to be known.

The direct dynamics is useful when simulating the robots motion, while the inverse dynamics is useful when implementing the control system.

The introduction of the terms direct and inverse dynamics is sufficient for understanding this report. For further details about dynamical modelling see [24, 25].
Chapter 3

Parallel Manipulators and the Gantry-Tau robot

In this chapter the reader is introduced to parallel manipulators and the Gantry-Tau robot in specific.

Parallel kinematic manipulators (PKMs) have recently attracted a lot of interest in the robot community. The main reason for this is some inherited properties of the structure, mainly high stiffness and dynamical advantages.

A parallel mechanism can be defined as [20]: Closed-loop mechanism in which the end-effector (mobile platform) is connected to the base by at least two independent kinematic chains. In other words a parallel kinematic manipulator consists of several kinematic chains, in contrast to the serial that only consist of one. This is a very general definition that opens up for many different constructions, with very different properties.

There are already some PKMs on the market. Figure 3.1 shows ABB’s IRB 340 Flexpicker, which is based on the Delta structure. Other examples of parallel structures are the Orthoglide [8], see Figure 3.2, the Hexaglide [9], the Triaglide [23] and the 14 [16, 22].

There is some common vocabulary that is used for parallel manipulators. The manipulator is said to consist of a mobile platform connected to a fixed base by several kinematic chains, called legs. If the number of legs is greater or equal to the degrees of freedom of the mobile platform and each arm having one actuated joint, the manipulator is called fully parallel. [13]

3.1 Comparison with Serial Structures

In [13] the different properties of serial and parallel manipulators are discussed. These are general properties, more or less true for different constructions, which give a background to raising interest in parallel robots and the problems inherited in the structure.
• **Workspace**  One of the main drawbacks with parallel robots is that they generally have a small workspace compared to the footprint of the robot.

• **Payload**  In a serial structure each actuator has to have the necessary power to move not only the manipulated object, but also the links and actuators located later in the kinematic chain. In a parallel structure the end-effector is directly supported by all actuators, and the actuators can be located close to the base, hence the payload can be much larger.

• **Accuracy**  In serial robots the errors from each link accumulate to a total error at the end-effector. An error in a joint closer to the base will also have a larger effect on the total error than an error in a joint closer to the end-effector. Parallel structures do not have these drawbacks at all, and are therefore remarkably rigid.

• **Dynamical behaviour**  The fact that the arm structure of a parallel robot can be made much lighter since the arms do not have to carry actuators, and the fact that errors do not accumulate, give them better dynamic performance than serial robots.

As seen in this comparison there are a lot of properties of a parallel structure that could make it interesting. The main drawback of the structure is the small workspace.
3.2 The Tau-structure

As mentioned above the definition of a parallel manipulator opens up for a wide range of constructions. We will here study a special group of parallel manipulators based on a mobile platform, six arms and three actuators. Depending on where the arms are connected to the platform, how they are grouped and what kind of actuators used the performance will be very different even within this group of manipulators. One example of constructions like this is the Orthoglide, see Figure 3.2.

In this group one can arrange the structures according to how the arms are grouped. The Orthoglide has a structure that would be named $2/2/2$, since the arms are grouped in pairs. If the arms are grouped as $3/2/1$, $3/1/2$, $2/3/1$, $2/1/3$, $1/3/2$ or $1/2/3$ the structure is referred to as a Tau-structure. One of the advantages of the Tau-structure is that the different configurations make it possible to get the highest stiffness in a desired direction. For a $2/2/2$ structure there is only one configuration. The Tau family of parallel kinematic manipulators was introduced by ABB Robotics, see [2, 3, 12].

3.3 Gantry-Tau

As stated before, the main drawback with parallel structures is the small workspace compared to the footprint of the robot. The Gantry-Tau is constructed to overcome this limitation while retaining most of the parallel structures advantages [4].
The Gantry-Tau has a total workspace larger than for a serial gantry robot with the same footprint [3]. The robot has been constructed for the assembly of airplane components. This is an application where very large and expensive machines are used today and a lighter and more cost efficient manipulator could compete. The Gantry-Tau prototype at UQ is shown in Figure 3.3. This is a prototype for a machine able to perform drilling, deburring and reveting of airplane components. In the figure the fixed, Cartesian frame is also defined. The prototype has a $1/2/3$ configuration, hence it has its highest stiffness in the $Z$-direction. The Gantry-Tau has 3 DOF and the platform only rotates around the $Y$-axis, defined by a frame located at the TCP, see Figure 3.4. The robot is mainly constructed to work in the X-Z-plane, and hence for tasks such as drilling the rotation around the $Y$-axis makes no difference.

The arms are connected by ball joints to the saddles and the platform. This makes some degree of rotational motion possible in all three directions. It also means that there are no torques acting on the arms, only forces in the same direction as the arms. The whole system from motor to saddle will throughout the report be referred to as the actuators. Actuator 1 has one arm connected at the center of the saddle. Actuator 2 has one arm connected $10\,\text{cm}$ out on each side of the saddle. The triangular link and the last arm are connected $10\,\text{cm}$ out.

**Figure 3.3.** The Gantry-Tau prototype at UQ.
3.4 Kinematics and Dynamics for the Tau-structure

In [4] some work on the kinematic and dynamic modelling of the Tau-structure is presented. While only considering the arm structure the kinematics will be a relation between actuator positions and platform position and orientation, while the dynamic will be a relationship between the forces acting on the actuators and the joint variables and their derivatives. There are some main differences when it comes to the difficulty of deriving the relationships compared to a serial robot. For a serial robot the forward kinematics was rather easy to derive, while the inverse kinematics was much harder. For the Tau-structure the situation is very much the opposite. The inverse kinematics can be derived easily from geometrical calculations but the forward kinematics is a much more difficult problem. As for the dynamics, a general approach for the inverse dynamics for a parallel structure is described in [14]. This approach has been applied to the Tau-structure.
Parallel Manipulators and the Gantry-Tau robot
Chapter 4

System Identification and Nonlinear Models

In this chapter the reader is introduced to system identification. A discussion on non-linear models are also given.

In this context the term system is used to denote an object or a group of objects, whose properties one wishes to study. External signals that affect the system are called input signals while system properties that can be observed are called output signals. Some input signals can be manipulated to control the system and are therefore called control signals. There are however signals that one can not manipulate but they still affect the system. These are called disturbance signals. System identification is a procedure to find a model of a system based on experimental data. The procedure should be seen in contrast to physical modelling, were physical laws and relationships are used to derive a model. The reason to model a system can be everything from an interest to understand the properties of the system better to designing a control system.

The system identification procedure can be described in six steps:

- Design experiments
- Collect and preprocess data
- Select a set of models to describe the data
- Choose a criterion to evaluate the models
- Determine the best model according to the criterion
- Validate the model

Each of these steps contains several options and considerations, and will not be described in detail here. There are many textbooks that deal with these issues in detail, and the interested reader is referred to, e.g [17, 27].
System identification in robotics is a vast research area and can be divided into, at least, three different levels or application areas. These levels involve the estimation of the kinematic description, the dynamic model (often divided into rigid body and flexible body dynamics), and the joint model (e.g., motor inertia, gear box elasticity and backlash, motor characteristics, and friction parameters). An overview of identification in robotics can also be found in [15].

4.1 Experimental design

In this work a lot of experiments will be conducted. One of the interesting things to look at is therefore experimental design. There are many considerations to be made when experiments are designed. An obvious problem is the choice of the excitation signal. The excitation signal is the input signal to the system under the experiment. There are several ways of choosing the excitation signal depending on the purpose and the properties of the model and the system. The basic guideline is to let the experiment resemble the situation under which the model is to be used. Another guideline is if a linear model is used to approximate a nonlinear system, the experiment should be carried out around some operating point [17]. The first guideline will have some impact on the experiment chosen in this work. In some cases one knows that the model is too simple to describe the system for some situations. This can be the use of a simple friction model that cannot describe the complex friction effects when the movement is started from zero velocity or to describe a system known to contain flexibilities with rigid body dynamics. In these cases a suitable excitation signal shall not excite the unmodelled properties in the system. In Chapter 6 a nonlinear model is identified. This causes some problems when choosing excitation signal. Identification of linear systems is a well studied problem. Here there are some general rules when choosing excitation signal. For nonlinear models there are very few guide lines to follow. What kind of excitation signal that shall be used will depend on the system of study and what kind of nonlinear effect one wishes to model, hence a more ad hoc procedure must be used, were different excitation signals are tested and the results compared.

4.2 Validation

When it comes to validation the term model fit or just fit will be used. Model fit is defined in [17] as

\[ \text{fit} = 100 \left( 1 - \frac{\sqrt{\sum_{t=1}^{N} (y(t) - \hat{y}(t))^2}}{\sqrt{\sum_{t=1}^{N} (y(t) - \bar{y})^2}} \right) \] (4.1)

where \( y(t) \) is the measured output, \( \hat{y}(t) \) is the predicted output and \( \bar{y} \) is the mean value of \( y(t) \). The model fit gives a number that tells how well the output of the system is described by the model. One important part of the validation is so called cross validation. Cross validation means that the model is tested on a different data set than the one used for the estimation [17].
4.3 Linear regression and least squares estimation

In the work reported here some of the models will be possible to write as a linear regression. The model will then be given by

$$F(q, \dot{q}, \ddot{q}) \theta = \Gamma$$

(4.2)

From a data set with N samples, \(\{q(t_i), \dot{q}(t_i), \ddot{q}(t_i), \Gamma(t_i)\}, i \in \{1, 2, 3, \ldots, N\}\) one gets the least squares problem

$$\begin{bmatrix}
F(q(t_1), \dot{q}(t_1), \ddot{q}(t_1)) \\
F(q(t_2), \dot{q}(t_2), \ddot{q}(t_2)) \\
\vdots \\
F(q(t_N), \dot{q}(t_N), \ddot{q}(t_N))
\end{bmatrix} \theta = 
\begin{bmatrix}
\Gamma(t_1) \\
\Gamma(t_2) \\
\vdots \\
\Gamma(t_N)
\end{bmatrix}$$

(4.3)

Under the condition that the noise level on \(q, \dot{q}, \ddot{q}\) is much less then the noise level on \(\Gamma\) the maximum-likelihood parameter estimation, MLE, is given by [26]

$$\hat{\theta}_{ml} = (\Phi^t \Sigma^{-1} \Phi)^{-1} \Phi^t \Sigma^{-1} Y$$

(4.4)

Here \(\Sigma\) is the diagonal covariance matrix of the measurements \(Y\). If the standard deviation for all components in \(Y\) are the same the maximum-likelihood reduces to the standard linear least squares estimation

$$\hat{\theta}_{ls} = (\Phi^t \Phi)^{-1} \Phi^t Y$$

(4.5)

These conditions will often be taken for granted in this thesis, but they will be discussed in some chapters.

4.4 Nonlinear effects and models

In the following chapters some nonlinear effects will be studied and modelled. As an introduction to this some discussion about the behaviour of a nonlinear system is suitable. In this thesis the main issue is flexibilities in the drive train. A common approximation is to apply a linear model. In the analysis of the actuator it is however interesting to see how the true system behaves and how large the approximation with a linear model is.

There are two terms which will be used to explain the behaviour of the system, backlash and hysteresis. Backlash is basically an effect of play in the gears. This means that, ideally, at first there is no or at least a very small stiffness in the system and the torsion will increase rapidly. Then, after a certain amount of torsion one has overcome the play and the system shows the same behaviour as a linear system. A more complex nonlinear effect is hysteresis. Hysteresis is, simply put, a nonlinear system with a memory of torsion. This memory can, at least partly, be explained by friction within the gears. Hysteresis is generally very hard to model. An approximation is to apply a nonlinear spring with a linear damper. This does however not model the memory correctly. In Chapter 6 a more complex hysteresis model, with a nonlinear damper, is used. In Figure 4.1 backlash, hysteresis and a nonlinear approximation is shown.
Figure 4.1. Nonlinear effects. All systems excited with a sinusoidal signal.
Chapter 5

Analysis of the Actuators

In this chapter the linear actuators of the Gantry-Tau robot will be analysed in detail. Parameters for moment of inertia and friction as well as a thorough analysis of the flexibilities, including some measurements of stiffness in the system are presented. The aim with the analysis is to give the reader a deeper understanding for the system and to identify its weaknesses.

5.1 System description

The main topic of this chapter is the actuators of the Gantry-Tau manipulator. For the prototype these are rodless actuators delivered by Tollo Linear AB. The actuators are powered by an AC-motor and the rotational movement of the motor axis is transformed into a linear movement of the saddle via a drive train. The drive train consists of two steps, first a belt gear that connects the motor to a lead screw and then a nut that connects the screw to the saddle, see Figure 5.1. More information about the actuator can be found in [6, 28].

5.2 Analysis

The analysis of the system is made in several steps. First the motor is dismounted from the drive train and studied separately. The reason to do this is both to get a good understanding of the different parts of the actuator but also to be able to compare the identified parameters with values given by the manufacturer. In the second step the procedure is repeated with the drive train connected. Using the parameters from step one it is now possible to derive the inertia and the friction in the drive train. The flexibilities of the drive train is studied by applying torque while fixing the drive train at different points. In this way both the flexibility in the belt gear and in the system in total can be studied. Finally some measurements of the stiffness of the system is done.
5.3 Rigid body dynamics

A basic understanding of the system can be found by looking at the location and size of the moment of inertia and the friction in the system. The analysis therefore starts by identifying the rigid body dynamics for the system. The identification is done in two steps to separate the motor from the drive train. The estimated parameters from this step will later be used as initial values in Chapter 6.

5.3.1 Estimation methods

To get a good estimate of the parameters, three different ways of finding them are compared. The first two methods use interpolation and integration of measured signals while the third method is a least square approach where all three parameters are estimated at once.

The rigid body dynamics is modelled by

\[ J\ddot{\theta}_m + F_v\dot{\theta}_m + F_c\text{sign}(\dot{\theta}_m) = \tau \]

(5.1)

where \( \theta_m \) is the angle of the motor axis, \( F_c \) is the Coulomb friction, \( F_v \) is the viscous friction and \( J \) is the moment of inertia and \( \tau \) is the torque.

The first method is based on measuring the torque required to achieve a constant speed. While the speed is kept constant the measured torque is a result only of the friction in the system. The torque is measured for several different velocities and the friction parameters \( F_c \) and \( F_v \) are adapted to the measured torque by solving a standard linear least square problem. Measuring the torque when the velocity is small, close to zero, one can use the approximation \( \tau \approx F_c \text{sign}(\dot{\theta}_m) \).

This shows that the parameter \( F_c \) can be estimated directly. However, this will
5.3 Rigid body dynamics

give a poor signal to noise ratio. This is especially the case when estimating the parameters in the motor, since the required torque is very small. The moment of inertia is estimated in the same way as in method two, i.e. (5.3) is used.

In the second method of estimating the friction parameters, \( F_c \) is estimated as in method one but \( F_v \) is estimated via some manipulation and integration of the rigid body dynamics, (5.1). By multiplying (5.1) with \( \dot{\theta}_m \) and observing that \( \int \dot{\theta}_m \ddot{\theta}_m \, dt = \frac{\dot{\theta}_m^2}{2} \), one realises that \( F_v \) can be estimated, without measuring \( \ddot{\theta}_m \), by choosing a time interval for the integration so that \( \dot{\theta}_m = 0 \) at both the start and the end point. The estimation of \( F_v \) is given by

\[
F_v = \frac{\int \dot{\theta}_m \tau_m \, dt - F_c \int \dot{\theta}_m \text{sign}(\dot{\theta}_m) \, dt}{\int \dot{\theta}_m^2 \, dt} \tag{5.2}
\]

This method has the potential of being less sensitive to measurement noise since the signals are integrated over some period of time. After estimating \( F_v \) via the equation above, \( J_m \) can be estimated the same way by integrating over time intervals where the angular velocity differs from zero at at least one of the endpoints, hence

\[
J_m = \frac{\int \dot{\theta}_m \tau_m \, dt - F_c \int \dot{\theta}_m \text{sign}(\dot{\theta}_m) \, dt - F_v \int \dot{\theta}_m^2 \, dt}{\int \dot{\theta}_m^2 \, dt} \tag{5.3}
\]

The third method tried here is to simply use the model described by (5.1), rewritten as a linear regression (5.4), and estimate the parameters as the solution to the least square problem (5.5), where \( t_i, \quad i \in \{1,2,\ldots,N\} \) are the sample points. The reason to try this is that this requires much less experiments than the methods described above and is therefore a more efficient method.

\[
\begin{pmatrix}
\dot{\theta}_m \\
\theta_m \\
\text{sign}(\dot{\theta}_m)
\end{pmatrix}
\begin{pmatrix}
J_m \\
F_v \\
F_c
\end{pmatrix} = \tau \tag{5.4}
\]

\[
\begin{pmatrix}
\dot{\theta}_m(t_1) \\
\theta_m(t_1) \\
\text{sign}(\dot{\theta}_m(t_1)) \\
\dot{\theta}_m(t_2) \\
\theta_m(t_2) \\
\text{sign}(\dot{\theta}_m(t_2)) \\
\cdots \\
\dot{\theta}_m(t_N) \\
\theta_m(t_N) \\
\text{sign}(\dot{\theta}_m(t_N))
\end{pmatrix}
\begin{pmatrix}
J_m \\
F_v \\
F_c
\end{pmatrix} =
\begin{pmatrix}
\tau(t_1) \\
\tau(t_2) \\
\cdots \\
\tau(t_N)
\end{pmatrix} \tag{5.5}
\]

5.3.2 Motor parameters

One of the advantages of parallel manipulators is that the structure very easily can be dismounted or fixed. In order to find the characteristics of the AC-motor it was dismounted from the belt gear, though the first gear wheel was kept on the motor axis. This is done of practical reasons and has the disadvantage that the estimated parameters cannot directly be compared with manufacturer’s values.

In method one, data were collected at four different velocities. Each data set included two intervals in which the motor was driven at the same speed in different direction. By doing this, bias effects in the measurements can be avoided. The
torque and velocity were then calculated as mean values over the whole data set. In Figure 5.2 the resulting torques for each velocity are shown. The figure shows that the simple friction model is valid at least while the angular velocity is large. In methods two and three, data where the angular velocity change sign are used. To reduce the problem that the model probably only is accurate for non-zero velocities, data sets with a relatively high frequency can be used. This is motivated by the discussion about stiction below. In method two, a sinusoidal signal was given as an input to the system. Using a periodical signal has the advantage that the integration to get $F_v$ and $J_m$ can be performed over many different intervals. The estimates are then taken as the mean value of all these estimates. In method three, different signals and frequencies have been evaluated. In the results presented here a sinusoidal signal was used as an input to the system.

The result of the methods is presented in Table 5.1 and a cross validation is given in Table 5.2. The methods give more or less equal estimates of the moment of inertia. The parameter is also specified in [6] as 1 kg cm$^2$, ($1 \cdot 10^{-4}$ kg m$^2$). The specified parameter is only for the motor. Since we have the first gear wheel connected to the motor axis we should get a slightly larger value. This shows that all methods are able to estimate the moment of inertia accurately. Method two tends to give $F_v$ a much higher value than the other methods. The drawback of method one, which is considered to give good parameters estimates since the model fit is high in the cross validation, see Table 5.2, is that it requires a lot of data from the system. As a comparison one can see that the parameters estimated with method three and a sinusoidal signal is not so far from these estimated with method one and the model fit is only slightly less. Method three only requires one data set.

The peak at 0 rad/s in Figure 5.2, represents a measurement of the torque
Table 5.1. Estimated motor parameters.

<table>
<thead>
<tr>
<th></th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_{cm}$</td>
<td>1.023</td>
<td>1.023</td>
<td>0.813</td>
</tr>
<tr>
<td>$F_{mv}$</td>
<td>1.203</td>
<td>2.886</td>
<td>1.204</td>
</tr>
<tr>
<td>$J_{m}$</td>
<td>1.041</td>
<td>1.041</td>
<td>1.048</td>
</tr>
</tbody>
</table>

Table 5.2. Cross validation. Model fit for each parameters from each method respectively. Data set is generated as close loop data with a sinusoidal wave, 5 Hz, as reference on the velocity.

<table>
<thead>
<tr>
<th></th>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model fit</td>
<td>96.20</td>
<td>93.82</td>
<td>95.47</td>
</tr>
</tbody>
</table>

required to get the motor to move at all. This shows that there is a so called static friction or stiction effect present in the system, i.e. the motor gets stuck when standing still. This parameter is given in the data sheet [6] as 0.030 Nm while the measurements gives 0.027 Nm, which is within the specified accuracy of 10%. As described in [1] this is a complicated effect which requires a dynamic friction model to be described accurately. In this work no effort is made to model it, hence the model used here is only accurate for non-zero angular velocities. According to the paper the stiction will increase with the time at zero velocity. This motivates the choice of a high frequency for method two and three. An interesting effect was discovered in the estimation with method three and a triangular wave. The input signal was a low frequency triangular wave given as a torque reference to the system. The main difference between this data set and the one where a sinusoidal wave was used is that the frequency is so low that the motor stands still under a significant time when the wave changes sign. Here the parameter $F_c$ was estimated to 0.029 Nm. The coulomb friction identified with this data set is thought to also include the static friction. As stated before this parameter is specified by the manufacturer as 0.030 Nm.

5.3.3 Linear drive

To identify the parameters in the linear drive the motor was re-attached and the data was once more collected, but now with the linear drive attached. Looking at the physical structure of the system one could see it as three connected masses, the motor, the screw and the saddle. If we rewrite (5.1) to include three different sets of parameters, one for each mass, we see that the parameters identified here are related as

$$F_c = F_c^m + r_1 F_c^l + r_1 r_2 F_c^s$$  \hspace{1cm} (5.6)

$$F_v = F_v^m + r_1^2 F_v^l + r_1^2 r_2^2 F_v^s$$  \hspace{1cm} (5.7)

$$J = J^m + r_1^2 J^l + r_1^2 r_2^2 m$$  \hspace{1cm} (5.8)
Here $J$ is the identified moment of inertia for the complete system, $J^l$ is the moment of inertia for the lead screw and $m^s$ is the mass of the saddle. According to [28], $m^s = 1.2$ kg. $F^l_c$, $F^l_v$, $F^s_c$ and $F^s_v$ are the parameters for the Coloumb and the viscous friction at the lead screw and the saddle respectively. $F_c$ and $F_v$ are the friction parameters for the complete system. $J_m$, $F^m_c$ and $F^m_v$ are the parameters for the motor model identified in Section 5.3.2. Using this and the fact that the gear ratios are known, $r_1 = 22/60$ and $r_2 = 32/(1000 \cdot 2\pi)$, we can determine the moment of inertia of the screw, using the motor parameters estimated with method 1 results in estimates presented in Table 5.3.

**Table 5.3.** Estimated parameters in the lead screw and the saddle.

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F^l_c + r_2 F^s_c$</td>
<td>20.626</td>
<td>20.413</td>
</tr>
<tr>
<td>$F^l_v + r_2^2 F^s_v$</td>
<td>14.633</td>
<td>5.226</td>
</tr>
<tr>
<td>$J^l$</td>
<td>7.388</td>
<td>7.249</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Method 1</th>
<th>Method 2</th>
<th>Method 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Model fit</td>
<td>83.71</td>
<td>89.07</td>
</tr>
</tbody>
</table>

As seen in Table 5.3 the moment of inertia in the lead screw is approximately seven times larger than the moment of inertia in the motor. One can also see that the friction in the remaining parts of the system is approximately ten times larger than the friction in the motor. The fact that most of the friction and the moment of inertia is located at the lead screw and the saddle is not so surprising. The motors are constructed to have very low friction and the screws are 2 m long and hence have much more rotating mass than the motor. Here one can also notice that the estimated parameter for the viscous friction with method 2 differs significantly from the other methods. One can also see that the least squares estimation tends to give a smaller value for the moment of inertia for the lead screw. Here method 2 gives a higher fit than method 1, see Table 5.4. The reason that method 2 works better then method 1 here and not for the motor parameters is somewhat unclear. One can also observe that the least squares estimation gives rather good fit even though this method only use one data set for the estimation.

**5.3.4 Conclusions**

Using simple estimation methods it was possible to estimate the rigid body model of the different parts of the actuators. The experiments showed that the friction is described rather well even by a simple friction model. The main effect that
remains unmodelled is the stiction. One can conclude that the main friction and inertia of the system are located in the lead screw and the saddle, which seems reasonable. The analysis showed that a least squares approach gives rather good accuracy and is preferable since only one data set is needed. The parameters identified here could, and will in Chapter 6, be used as initial values for a more advanced identification procedure.

5.4 Analysis of flexibilities

In order to analyse the flexibilities of the system three different experiments are conducted. The aim with the first two experiments is to determine the characteristics of the flexibilities in the system while the third experiment is conducted to investigate if and how the total flexibility is dependent on the position of the saddle.

5.4.1 Experimental design

In order to separate the different flexibilities in the system from each other, different parts of the structure are kept fixed in the experiments. A logical separation is to first look at the flexibility in the belt gear. The physical system makes it possible to fix the second gear wheel so that the measured motor angle is the torsion in the belt gear. This will be the first step in the analysis of the flexibilities. The second point that can be fixed is the saddle. Doing this means that the measured motor angle will be the total torsion in the system. This will be the second experiment. To establish the position dependence the second experiment is repeated at several points over the workspace of the actuator.

5.4.2 Flexibility in the Belt Gear

First the second gear wheel in the belt gear was fixed and the torsion was measured while applying torque. When conducting this experiment it is only possible to fix the second belt gear in one direction at the time. For that reason the data collected is a signal with either positive or negative torsion at the time. The experiment is conducted by applying a slowly varying triangular signal with an offset. From these measurements the characteristics of the flexibilities, as shown in Figure 5.3, were determined.

From these measurements one can determine that the flexibility in the belt gear is mainly of backlash character. In Figure 5.3 one can see that hardly any torque is needed in the interval where the torsion is between $-0.02$ and $0.02$ rad. Some hysteresis is also present in the belt gear, although this effect is much less significant than the backlash. The backlash of the gear is directly connected to how tightened the belt is. One can realise that a tighter belt would give less backlash but would instead increase the friction in the gear.
5.4.3 Total Flexibility and Flexibility in Lead screw

In the second experiment the saddle was fixed to the linear drive. The measured torsion is now a result of both the flexibility in the belt and the one in the screw. For this experiment there is no problem to cross the zero velocity. Using a simple controller to get the torsion to follow a slowly varying sinusoidal wave results in measurements shown in Figure 5.4.

Figure 5.4. The figure shows torque as a function of torsion. Here the total flexibility in the actuator is considered.

The total flexibility in the linear drive is clearly nonlinear and suffers from a strong hysteresis effect. The hysteresis in total is much more significant than
5.4 Analysis of flexibilities

the one in the belt gear. Translating the torsion in the flexibility into a linear inaccuracy at the saddle gives an upper limit of the inaccuracy as \( \max(\Delta x) \approx 0.5 \text{ mm} \).

It could be interesting to try to isolate the flexibility in the lead screw and the nut from the one in the belt gear. The main reason to do this is that while the flexibility of the belt gear hardly could be dependent of the position of the saddle, the other flexibilities in the system could very well vary with the position. It is also interesting to look at how the torsion in the belt gear affects the total flexibility. Using polynomials estimated from experiment one it is possible to subtract the effect of the flexibility in the belt gear from the total flexibility. Figure 5.5 shows measured torque, the approximated torsion in the belt gear and calculated torsion in the lead screw.

![Figure 5.5. Torque as a function of torsion. The figure shows the total flexibility in the actuator. The solid line is the measured torque, the dashed-dotted is the calculated torsion of the belt gear and the dashed line is the resulting torsion in the rest of the actuator.](image)

This shows that the belt gear mainly adds backlash and a linear flexibility of the total system.

5.4.4 Position dependence

In this final step of the analysis of the flexibilities dependence of the saddle’s position is investigated. An experiment where the torque is applied as a slowly varying sinusoidal wave with amplitude 1.17 Nm is repeated over the actuators workspace. To analyse the data the flexibility is approximated by a linear spring, \( \tau = k\Delta \theta \), where \( \tau \) is the torque, \( \Delta \theta \) the measured torsion and \( k \) the linear spring constant. The estimated spring constants are shown in Figure 5.6.

As seen in Figure 5.6 the stiffness of the actuator is clearly dependent on the saddles position. Assuming that the main cause of the weakness would come from torsion in the lead screw leads to an interesting result. If we see the lead screw as a
4.2 Linear spring constant as a function of saddle position

Figure 5.6. Estimated spring constant as a function of saddle position. The solid line shows the estimated spring constants and the dashed line estimated approximation of the spring constants.

If a straight symmetric rod, the stiffness of the system would be inversely proportional to the length of the rod, i.e. \( k \propto \frac{1}{X} \) where \( X \) is the position of the saddle [5]. Estimating the proportional constant gives the dotted line in the figure. As seen in the figure a large part of the dependence can be explained by this very simple model. This could be useful if one would build a very accurate model for the actuators. This could then relatively easy be made position dependent to gain better accuracy.

5.4.5 Stiffness analysis of the saddle

In the last step in the analysis the stiffness at the saddle is investigated. The stiffnesses considered here are the torque stiffnesses in \( x \) and \( z \) directions. This means that the torsion is measured while applied torque as \( M_z \) and \( M_x \) according to Figure 5.7. The force stiffness in the \( z \) direction is also measured through applying a force \( F_z \) and measuring the resulting displacement. The analysis is mainly done to compare the stiffnesses in different directions and of different kinds. The ambitions is not to estimate accurate stiffness constants, but to get values to compare through simple experiments.

Experiment design and data collection

The torques and forces are applied by placing weights at different points of the saddle. The torsion is measured as a linear displacement 10 cm from the centre point of the saddle. The displacement is measured with an accuracy of \( 2.54 \times 10^{-5} \) m but while the experiment design is rather primitive, the largest uncertainty here lies in the applied torque. The experiment design for measuring the stiffness for force in the \( z \) direction is here to place weights at the centre of the saddle. Since
5.4 Analysis of flexibilities

The measured displacements from this experiments turned out to be relatively small compared to the displacements when applying torque, the experiment for measuring the stiffness for torque in the \( x \) direction was simplified. Here weights are placed 10 cm out from the centre of the saddle on one side. The measured displacement is then really a combination of the force and the torque acting on the saddle, but since the displacement from the force is five to eight times smaller this is neglected. The design is shown by Figure 5.8.

The experiment design for measuring the stiffness for torque in the \( z \) direction is here to apply forces in opposite directions 10 cm out from the saddles centre so that a resulting torque \( M_z \) is created.
Results

Performing the experiments described above with four data sets for $M_x$, one for $M_z$ and two for $F_z$ resulted in the measurements in Figure 5.9.

![Figure 5.9](image)

**Figure 5.9.** Measured and approximated stiffness. The solid line is a linear approximation, crosses are measurements.

The estimated linear stiffness constants are shown in Table 5.5. It is clear that the saddle is much stiffer in the $z$-direction than in the $x$-direction when it comes to torques. An interesting comparison is to apply the same force at the center of the saddle and at the point where the arms are connected, which is 10 cm out from the center for actuator 2 and 3. The resulting torque will then be $\tau = 0.1 \cdot F$, where $\tau$ is the torque and $F$ is the applied force. For small forces the torsion can be approximated by a linear displacement $\Delta x \approx 0.1 \cdot \Delta \theta$. This shows that the structure is more than 5 times weaker for force in the $z$-direction applied 10 cm out, inducing a torque $M_x$, than for a force applied at the center. For the application considered here, the rather weak construction for torques could cause a problem. The tau structure will induce a torque in both the $x$ and the $z$ direction at actuator 2 and 3. If the position of the platform is calculated from the motor angle or the saddles position this will result in an inaccuracy in the position of the platform. There is also a play present in the saddles that is not considered here. This will also contribute to an inaccuracy in the manipulator. These effects will also cause unwanted dynamical behaviour of the system.
5.4 Analysis of flexibilities

Table 5.5. Estimation of stiffness parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$M_x$</th>
<th>$M_z$</th>
<th>$F_z$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$1.551 \times 10^3 , Nm/rad$</td>
<td>$4.121 \times 10^3 , Nm/rad$</td>
<td>$9.716 \times 10^5 , N/m$</td>
</tr>
</tbody>
</table>

5.4.6 Conclusions

The analysis has shown that the drive train contains a nonlinear flexibility, with both hysteresis and backlash present. The belt gear is shown to introduce backlash in the system, while the main hysteresis effect can be deduced to the lead screw. The flexibility is also shown to be dependent on the position of the saddle. The experiment showed that the dependence mainly comes from the lead screw and could be explained by a simple model as inversely proportional to the position of the actuator. Altogether this means that to be able to model the system accurately a rather complex model has to be used. The stiffness of the saddle is shown to be low for torques, which could cause a problem in this application since the tau-structure induces torques at actuator 2 and 3. This problem cannot be solved by extra sensors but should motivate a different construction where either the actuators are much stiffer or where the tau-structure doesn’t induce torques in the same way.
Analysis of the Actuators
Chapter 6

Nonlinear Grey-Box Identification

In this chapter a new identification procedure for a linear actuator is developed. The actuator dynamics contain both hysteresis and backlash resulting in a highly nonlinear system. The results in this chapter show that not only can a model of the nonlinear system be successfully identified from measurement data, but the model is also compact enough to be an ideal candidate for inclusion in a high-performance robot control system.

This chapter is to appear in the Proceedings of the 2006 IEEE International Conference on Robotics and Automation. The paper is reproduced unabridged, which means that some material from earlier chapters may be repeated.

6.1 Introduction

System identification in robotics is a vast research area and can be divided into, at least, three different levels or application areas. These levels involve the estimation of the kinematic description, the dynamic model (often divided into rigid body and flexible body dynamics), and the joint model (e.g., motor inertia, gear box elasticity and backlash, motor characteristics, and friction parameters). An overview of identification in robotics can also be found in [15].

In the work reported here the dynamics of the linear actuator of the Gantry-Tau Parallel Manipulator is investigated, including analysis and modelling of nonlinear hysteresis. The Tau family of parallel kinematic manipulators (PKMs) was introduced by ABB Robotics, see [2, 3, 12]. Other examples of PKMs utilising linear actuators are the Hexaglide [9], the Triaglide [23] and the I4 [16, 22]. One of the main benefits of PKMs is the high stiffness of the arm structures. However, the overall stiffness of the PKM depends not only on the arm structure, but also on the actuators and the support structure. The work presented in this chapter is a first step towards a PKM control system taking nonlinear actuator flexibilities
The hysteresis model considered in this paper was originally presented in [7]. The method for estimation of the parameters in [7] is here used in the initial steps of the identification procedure. The gain of applying nonlinear grey-box identification on the model parameters is investigated. A three-step approach inspired by the method developed in [29] is applied. The procedure proposed in [29] is used to identify parameters for rigid body dynamics, friction, and flexibilities in a two-mass model of an industrial serial-type robot. The first two steps find good initial parameter estimates for the iterative optimisation routine, while the main step is the last, where the parameters of a nonlinear physically parameterised model (a nonlinear grey-box model) are identified directly in the time domain. In the work presented here the procedure in [29] has been modified to suit the identification of a linear actuator containing hysteresis.

Backlash in drive trains have been studied before, e.g. in [11] where grey-box identification of a two-mass model with backlash was considered. Black-box modelling was used to find initial parameter values. In [10] identification of hysteresis with methods based on an augmented EKF-filter was considered.

### 6.2 Physical system

The physical system that is studied in this work is the linear actuator of a prototype of the Gantry-Tau robot. The actuator is made up of an AC-motor and a drive train, see Figure 6.1. The drive train consists of two steps, first a belt gear that

![Figure 6.1. The linear actuator on Gantry-Tau.](image)

connects the motor to a lead screw and then a nut that connects the screw to the saddle. The drive train transforms the rotation of the motor axis to a translatively movement of the saddle. The Tau-structure is connected to the three actuators via the saddles. More detailed information about the actuators can be found in [28].
Analysis of the system has showed that both the belt gear and the lead screw combined with the nut suffer from nonlinear characteristics and hysteresis. In Figure 6.2 the effect of the flexibilities between the motor and the saddle is illustrated. Some of the nonlinear effects in the belt gear can be understood by studying the construction of the actuator. The belt that connects the two gear wheels is not fully stretched when not exposed to torque. This introduces backlash in the system and hence trying to analyse the flexibilities between the belt gear and the saddle is not easy. Analysis has however showed that the main flexibility is located in the lead screw and the nut. This also means that the stiffness of the system will depend on the position of the saddle. The stiffness is on the other hand also the result of torsion in the structure that supports the saddle. The stiffness will therefore not only depend on the position of the saddle, but also on the distance to the nearest support structure element. In this work all experiments were performed with the saddle at the same position to avoid these effects.

6.3 Nonlinear Grey-box identification

In the work presented here a beta version of a nonlinear extension to the System Identification Toolbox (SITB), [18], for Matlab will be used for the identification process. The extension contains a nonlinear grey-box model structure nlgrey which has similar properties as the idgrey structure. In the current version, only output error (OE) models can be used, i.e. models with only additive white noise on the output. The model will be described by a continuous-time state space structure

\[
\dot{x} = f(t, x(t), \theta, u(t)) 
\]

\[
y = h(t, x(t), \theta, u(t)) + e(t) 
\]
where both \( f \) and \( h \) are nonlinear functions, \( x(t) \) are the states, \( u(t) \) are the input signals and \( y(t) \) are the output signals. \( e(t) \) is white measurement noise and \( \theta \) is the unknown parameter vector. The aim of the identification is to find the parameters that given data from the physical system will minimise the criterion

\[
V(\theta) = \frac{1}{N} \sum_{t=1}^{N} e^2(t, \theta)
\]  

(6.3)

where \( e(t, \theta) \) is the prediction error \( e(t, \theta) = y(t) - \hat{y}(t, \theta) \). The toolbox estimates the parameter vector \( \theta \) by applying a prediction error method, which performs a numerical optimisation of the criterion. The numerical optimisation is performed by an iterative numerical search algorithm.

### 6.4 Model

The dynamics of the system is approximated by a model consisting of two masses connected via a gear, a damper and a spring as shown in Figure 6.3. All flexibilities between the motor and the saddle, known to contain nonlinear effects, such as hysteresis and backlash, are lumped together and modelled as one flexibility.

The input of the model is the torque, \( \tau \), generated by the AC-motor and the output is the torsion in the flexibility, \( \theta(t) - z(t)/r \). The physical parameters of the model are defined as:

- \( J \) moment of inertia \([Nms^2]\]
- \( m \) mass of saddle \([kg]\]
- \( F_v \) viscous friction parameter \([Nm/s]\]
- \( F_c \) Coulomb friction parameter \([Nm]\]
- \( r \) gear ratio \([m/rad]\]

The first mass in the model represents the rotating part of the electrical motor but also all rotating parts between the motor and the saddle, i.e., gear wheels, the belt and the lead screw. This is motivated by looking at where the main flexibility
6.4 Model of the system is located. Analysis has showed that it is located in the lead screw and the nut which means that most of the moment of inertia between the motor and the saddle will be located at the motor side of the flexibility. This is however a significant approximation since some of the flexibility is located in the belt gear. The second mass in the model represents the saddle. Since the gear ratio is large, all friction is modelled at the motor side and the friction at the saddle is neglected.

Introducing the states $\mathbf{x}(t)^T = [\theta(t) - z(t)/r, \dot{\theta}(t), \dot{z}(t)]$, where $\theta(t)$ is the angle of the motor axis and $z(t)$ is the position of the saddle, and applying torque balances for the two masses gives the dynamic model

$$\dot{x}_1 = x_2 - x_3/r$$
(6.4)

$$\dot{x}_2 = \frac{1}{J}(-F_v x_2 - F_s \text{sign}(x_2) - g(x) + \tau)$$
(6.5)

$$\dot{x}_3 = \frac{g(x)}{r m}$$
(6.6)

The function $g(x)$ is the torque of the flexibility as a function of the states. A simple hysteresis model is added to approximate the nonlinear flexibility. The model was first used in [7], where hysteresis in a harmonic drive was studied. The hysteresis model has been chosen partly because of its simplicity. Since the model only contains four free parameters, $k_1$, $k_2$, $\alpha$ and $A$, it is an ideal candidate for use in identification. The introduction of the hysteresis model will add one extra state $x_4$ that corresponds to the nonlinear damping in the flexibility. The extra state equation is given by

$$\dot{x}_4 = -\alpha|d_2 - d_3/r|x_4 + A(x_2 - x_3/r)$$
(6.7)

and the torque function will be modelled as

$$g(x) = k_1 x_1 + k_2 x_1^3 + x_4$$
(6.8)

The hysteresis model is used in a similar way as in [7], although there are some differences in how the system is modelled. In [7] all nonlinear effects, both friction and flexibilities, in the system were modelled by the extra state and the nonlinear spring. In this report the Coulomb friction will be modelled separately and the hysteresis model will represent the friction and flexibility between the motor axis and the saddle.

While studying the ability to describe the nonlinear characteristics with different orders of odd polynomial functions of $x_1$ in $g(x)$, using step 2 in Section 6.5, no significant improvement was noticed between a function with two, and one with three terms. Therefore a lower order of polynomial function in $g(x)$ is chosen compared to [7]. The choice of a lower order function is also done to reduce the number or parameters in the model.

Since the hysteresis model only introduces one extra state, the model is, although describing some complicated nonlinear effects, very compact. This makes it an ideal candidate for inclusion in a control system.
6.5 Identification procedure

The aim of the identification is to estimate all the parameters in the model using nonlinear grey-box identification procedures. A problem of iterative search methods for solving nonlinear least squares problems is that only local convergence can be guaranteed. This means that good initial values of all parameters in the model are essential to achieve successful identification. The problem has been studied before, e.g. in [29] where a three-step method for the identification of a nonlinear model for industrial robots was proposed. Inspired by the results in [29] a three-step method to identify a model for the system is proposed here as well, although only the first step is common with the earlier proposed procedure. The main difference is that here the second mass is fixed in step 2 and 3. The reason for this is to enable an easier identification of the parameters in the flexibility. There are two advantages with constructing an experiment in this way and not try to identify the parameters through an experiment with the saddle able to move. First, it solves the problem of finding initial values for the hysteresis model since the procedure in [7] can be used for this. Secondly, it makes it easier to excite the flexibility enough to accurately identify the parameters. The mass in the saddle is so small, that without an extra load it would be very hard to excite the system enough to identify all parameters in the model through a single experiment. Since the load-carrying capability of the actuator is limited, adding a heavy weight was no alternative.

In the proposed procedure the first step initiates the parameters for the rigid body dynamics solving a least squares problem. In the second step the hysteresis model is initiated solving a nonlinear least squares problem. Finally, in the third step, the procedures for nonlinear grey-box identification are applied.

6.5.1 Step 1. Initial values for rigid body dynamics and friction

In the first step an experiment to identify the parameters of the rigid body dynamics is constructed with the saddle able to move. The rigid body dynamics of the system are described by

\[ J \ddot{\theta} + F_v \dot{\theta} + F_c \text{sign}(\dot{\theta}) = \tau \]  

(6.9)

where \( \theta \) is the angle of the motor axis, \( F_c \) is the Coulomb friction, \( F_v \) is the viscous friction. \( J_i \) is the total moment of inertia of the system. This means that \( J_i = J + r^2 m \). Rewriting equation (6.9) as a linear regression gives

\[
\begin{pmatrix}
\ddot{\theta} & \dot{\theta} & \text{sign}(\dot{\theta})
\end{pmatrix}
\begin{pmatrix}
J + r^2 m \\
F_v \\
F_c
\end{pmatrix} = \tau
\]  

(6.10)

The parameters can now be estimated by solving (6.10) as a standard least-squares problem, (4.5).
6.5 Identification procedure

6.5.2 Step 2. Initial values for the hysteresis model

While several methods for initiating parameters for springs and dampers in linear systems have been studied before, these are of little help when identifying a nonlinear model with a strong hysteresis. In [29] a method based on a linear approximation and the use of estimated Frequency Response Function (FRF) was used for initiating a nonlinear spring. As mentioned before the system studied here is very hard to excite when the mass of the saddle is small and the gear ratio large. This, in combination with the fact that the system suffers from hysteresis and a significant nonlinearity rules out the use of the method in [29]. Instead the procedure derived in [7] is used to initiate the parameters of the hysteresis model. The procedure is based on bringing the hysteresis into a steady-state loop while the second mass is fixed. If the second mass is fixed, then $x_3 = 0$ and the system is described by

$$\dot{x}_1 = x_2$$  (6.11)

$$\dot{x}_2 = \frac{1}{M} (-F_v x_2 - F_c \text{sign}(x_2) - g(x) + \tau)$$  (6.12)

$$\dot{x}_4 = -\alpha|x_2| x_4 + A(x_2)$$  (6.13)

$$g(x) = k_1 x_1 + k_2 x_3^2 + x_4$$  (6.14)

This will be the model used for step 2 and 3.

The steady-state hysteresis loop is described by the stiffness curve $\phi(x_1) = k_1 x_1 + k_2 x_3^2$, one ascending curve $C_{u}(x_1)$ for $\dot{x}_1 > 0$ and one descending curve $C_{d}(x_1)$ for $\dot{x}_1 < 0$. For an excitation signal $x_1 = Q_j \sin(\omega t)$ the curves are derived in [7] as

$$C_{u}(x_1) = -\frac{A}{\alpha} \left( -1 + 2 \frac{e^{\alpha(Q_j + x_1(t))}}{1 + e^{-2\alpha Q_j}} \right)$$  (6.15)

$$C_{d}(x_1) = \frac{A}{\alpha} \left( -1 + 2 \frac{e^{\alpha(Q_j - x_1(t))}}{1 + e^{-2\alpha Q_j}} \right)$$  (6.16)

Using the derived functions and measuring the torque while exciting the system so that $x_1 = Q_j \sin(\omega t)$ the parameters are estimated as the minimising arguments to the nonlinear least-squares problem

$$F = \sum_{j=1}^{m} \sum_{i=1}^{n} (\epsilon_{uj}(i)^2 + \epsilon_{dj}(i)^2)$$  (6.17)

where $\epsilon_{uj}(i) = \phi(x_{1j}(i)) + C_{uj}(x_{1j}(i)) - \tau_{nj}(i)$ are the model equation errors corresponding to the hysteresis loop $j$ and the data point $i$ for the ascending and descending curve, respectively. $m$ is the number of loops and $n$ is the number of data points. $\tau_n$ is the torque on the flexibilities, that is $\tau_n = \tau - F_v x_2 - F_c \text{sign}(x_2)$.

6.5.3 Step 3. Nonlinear grey-box identification

In the third step the nonlinear grey-box identification method is applied. The hysteresis parameters are estimated using data achieved with the saddle fixed and the model defined by equation (6.11) to (6.14).
6.6 Data collection

The equipment used for the experiments makes it possible to collect the data for the identification experiments only while the control system is in use. The data collected at the experiments will therefore be closed-loop data. Since the control system is not fully known to the user, the only way to use the collected data is for so-called direct identification, i.e., the data are used without any assumption about how it was generated. The estimates found in this way might get biased as an effect of the feedback in the system [17]. The equipment does however make it possible to give a reference either as speed or as torque to the control system. With a speed reference one can easily construct the experiments so that the saddle does not reach the endpoints of the linear drive and it is a suitable method to use when the saddle is free. The main limitation is that the control system is unable to follow very small variations of the motor axis, which is needed for identifying the flexibility. The system can more easily follow a torque reference and therefore most of the experiments will be performed in this way.

6.6.1 Experiment design

While applying the identification procedure described above there are needs for several different excitation signals. In step 1 where the parameters in the rigid body dynamics are identified one wishes to excite those parameters without exciting the flexibility. To be able to do this a signal with low frequency is chosen for the excitation. In step 2 the method is based on an excitation signal that brings the system to a state were \( x_1 = Q_j \sin(\omega t) \), for several different amplitudes \( Q_j \). In step 3 the whole frequency band where the model is supposed to be used should be excited as well as all the parameters in the model. In this case this means that frequencies up to at least 25 Hz should be excited.

Eight different data sets were used for the identification and validation. One was collected with the saddle free to move and with the signal as a reference speed for the controller.

- **Data set 1**: Sinusoidal signal. 10 s of data, with amplitude 210 rad/s and frequency of 1 Hz.

  Seven data sets were collected with the saddle fixed. All data sets were collected with the signal as torque reference for the controller. Data sets 2-5 were collected for the initiation of the hysteresis model. A PI-controller was used to make the motor angle follow the reference signal.

- **Data set 2-5**: Sinusoidal signals with different amplitude as reference angle. Angular velocity 0.5 rad/s, 12.3 s data. The amplitudes of the sinusoidal motion in these data sets have been chosen so that the maximum torque applied to the flexibility is in the interval between 0.9 and 1.6 Nm.

- **Data set 6**: Sinusoidal signals with an angular velocity of 2 rad/s and an amplitude of 1.4 Nm.
• **Data set 7-8:** Multi-sine signals (sum of sinusoids), 5s of data, frequencies in the interval $1 - 30$ Hz. Two different sets with different amplitude and phase were collected.

For details on the selection of excitation signals, see, *e.g.* [17, 21].

### 6.6.2 Preprocessing of data

In step 1 of the identification procedure the motor angular velocities and the angular acceleration are used. They are derived through numerical differentiation of the measured motor angle. All data are low-pass filtered to avoid high frequency noise. For this purpose a fourth order Butterworth filter with cut-off frequency $100$ Hz was used together with the Matlab command `filtfilt`. In some data sets the sample means have also been removed before filtering the signal.

### 6.7 Results

The three-step method described above has been applied to the data sets to identify the physical parameters in the actuator model. The gear ratio $r$ is known to be $r = 1.867 \cdot 10^{-3}$ m/rad and the mass of the saddle is known to be $m = 1.2$ kg.

#### 6.7.1 Step 1

Solving the least-squares problem formed by (6.10) and data set 1 gives the following parameter estimates including one standard deviation

\[
\begin{pmatrix}
    J + r^2m \\
    F_v \\
    F_c
\end{pmatrix} =
\begin{pmatrix}
    5.437(\pm 0.083) \cdot 10^{-4} \\
    1.330(\pm 0.140) \cdot 10^{-3} \\
    2.872(\pm 0.210) \cdot 10^{-4}
\end{pmatrix}
\]

The rather large standard deviation indicates that the model cannot fully describe the data set. This could mean that a more advanced friction model than the one used here must be applied to describe the data accurately. It can also mean that the flexibility, although not meant to, is excited in the data set so that the data cannot be modelled accurately with rigid body dynamics. Since the mass of the saddle is known the moment of inertia can be calculated. This gives $J = 5.396 \cdot 10^{-4}$ Nms$^2$.

#### 6.7.2 Step 2

The methods for initiating the hysteresis model has been applied to data set 2 to 5. The amplitudes of the sinusoidal motion in these data sets have been chosen so that the maximum torque applied to the flexibility is in the interval between 3 and 6 Nm. The hysteresis loops can be seen in Figure 6.2. Each data set contains one period of data chosen at a time when the system has converged to a steady-state
loop. The nonlinear least-squares problem is solved with the \textit{lsqnonlin} command in Matlab. Fifty iterations give the following parameter estimates

\[
\begin{pmatrix}
  k_1 \\
  k_2 \\
  \alpha \\
  A
\end{pmatrix}
= \begin{pmatrix}
  1.104 \cdot 10^4 \\
  1.768 \cdot 10^2 \\
  7.192 \cdot 10^1 \\
  6.149 \cdot 10^1
\end{pmatrix}
\] (6.19)

Since the angular velocities in the data set are small the viscous friction was neglected when the parameters were estimated, using \( \tau_n \approx \tau - F_c \text{sign}(x_2) \).

### 6.7.3 Step 3

With the initial parameters derived in step 1 and 2, resulting in model \( m_0 \) defined by equation (6.11) to (6.14), the methods for nonlinear grey-box identification were now applied to the model. The optimisation was terminated after thirty iterations or when the expected improvement in the loss function was less then 0.01%. For estimating the parameters, data set 6 was used combined with data sets 7 or 8 as shown in Table 6.1. Data set 6 is included to stabilise the estimation of the parameters in the force function. Two models where the rigid body dynamics are kept fixed during the identification is also presented. The quality of the models is assessed using the model fit, defined in [17] as

\[
\text{fit} = 100 \left( 1 - \sqrt{\frac{\sum_{t=1}^{N} (y(t) - \hat{y}(t))^2}{\sum_{t=1}^{N} (y(t) - \bar{y})^2}} \right)
\] (6.20)

where \( y(t) \) is the measured output, \( \hat{y}(t) \) is the predicted output and \( \bar{y} \) is the mean value of \( y(t) \).

<table>
<thead>
<tr>
<th></th>
<th>( m_0 )</th>
<th>( m_{67f} )</th>
<th>( m_{68f} )</th>
<th>( m_{67} )</th>
<th>( m_{68} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J )</td>
<td>5.396</td>
<td>5.396*</td>
<td>5.396*</td>
<td>4.991</td>
<td>5.659</td>
</tr>
<tr>
<td>( F_v )</td>
<td>1.330</td>
<td>1.330*</td>
<td>1.330*</td>
<td>1.413</td>
<td>1.360</td>
</tr>
<tr>
<td>( F_c )</td>
<td>2.872*</td>
<td>2.872*</td>
<td>2.872*</td>
<td>2.065</td>
<td>2.614</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>1.104</td>
<td>1.080</td>
<td>1.009</td>
<td>1.131</td>
<td>1.010</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>1.768</td>
<td>1.751</td>
<td>1.872</td>
<td>1.443</td>
<td>1.859</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>7.192</td>
<td>10.113</td>
<td>9.493</td>
<td>11.250</td>
<td>10.207</td>
</tr>
<tr>
<td>( A )</td>
<td>6.149</td>
<td>4.406</td>
<td>4.672</td>
<td>4.249</td>
<td>4.427</td>
</tr>
</tbody>
</table>

Table 6.1. Estimation of parameters, \( m_0 \) from step 1 & 2, \( m_{67} \) from step 3 using data sets 6 and 7 and \( m_{68} \) from step 3 using data sets 6 and 8. Parameter values marked with * in \( m_{67f} \) and \( m_{68f} \) are kept fixed during identification.

The model fit, see Table 6.2, is improved significantly through the nonlinear grey-box identification. The fit is increased from 83 to 88-89 for data set 6, from
6.7 Results

Table 6.2. Model fit when validating the models with estimated parameters on data set 6, 7 and 8

<table>
<thead>
<tr>
<th></th>
<th>m0</th>
<th>m67f</th>
<th>m68f</th>
<th>m67</th>
<th>m68</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data set 6</td>
<td>82.52</td>
<td>89.37</td>
<td>88.46</td>
<td>88.27</td>
<td>89.31</td>
</tr>
<tr>
<td>Data set 7</td>
<td>66.96</td>
<td>75.64</td>
<td>74.63</td>
<td>78.02</td>
<td>75.11</td>
</tr>
<tr>
<td>Data set 8</td>
<td>51.98</td>
<td>66.78</td>
<td>65.73</td>
<td>70.44</td>
<td>68.47</td>
</tr>
</tbody>
</table>

66 to 75-78 for data set 7 and from 52 to 66-70 for data set 8. One can notice that data set 8 was generated with a similar signal as data set 7, but with a smaller amplitude. It is probably the backlash in the system that makes it harder to predict data generated with a small amplitude. As seen by the increase in fit, this is where most is gained by applying nonlinear grey-box identification. In Table 6.1 one can notice that the significant differences, compared to the estimates given by step 1 and 2, are in the parameters for the non-linear damping, \( \alpha \) and \( A \).

Clearly the non-linear grey-box identification improves the parameter estimations compared to the initial ones. While the initialisation method only optimises the parameters with data from low frequency waves and only in a steady-state loop, the optimisation in step 3 optimises with data from a more complex signal that excite a broader band of frequencies.

To analyse the relative importance on the model fit for each parameter, the estimated model \( m_67 \) is used and each parameter is perturbed \( \pm 20\% \), one at a time. In Table 6.3 the difference in model fit can be seen for the three data sets. It is clear that the parameters for the rigid body dynamics have small influence on the model fit. For data set 6 and 8 the moment of inertia and the viscous friction hardly influence the fit at all, while they, probably due to higher velocities, have more effect on the fit for data set 7. For this reason two models where these parameters are kept fixed are also presented. As seen in Table 6.1 this does however not affect the estimates of the hysteresis parameters in a significant way. The poor

Table 6.3. Difference in model fit for \( m_67 \), compared to Table 6.2, when each parameter is perturbed \( \pm 20\% \) (one at a time)

<table>
<thead>
<tr>
<th></th>
<th>Data set 6</th>
<th>Data set 7</th>
<th>Data set 8</th>
</tr>
</thead>
<tbody>
<tr>
<td>( J )</td>
<td>0.001/-0.001</td>
<td>-2.463/0.118</td>
<td>-0.023/-0.003</td>
</tr>
<tr>
<td>( F_v )</td>
<td>-0.009/-0.002</td>
<td>0.116/0.120</td>
<td>-0.040/0.002</td>
</tr>
<tr>
<td>( F_c )</td>
<td>-0.124/0.013</td>
<td>-0.101/0.150</td>
<td>-1.087/0.201</td>
</tr>
<tr>
<td>( k_1 )</td>
<td>2.758/-1.192</td>
<td>-4.091/0.539</td>
<td>-3.400/0.504</td>
</tr>
<tr>
<td>( k_2 )</td>
<td>2.139/-2.000</td>
<td>-0.333/0.631</td>
<td>0.900/0.357</td>
</tr>
<tr>
<td>( \alpha )</td>
<td>-2.357/-1.943</td>
<td>0.499/0.609</td>
<td>1.844/-0.047</td>
</tr>
<tr>
<td>( A )</td>
<td>-1.898/-1.983</td>
<td>-0.085/0.664</td>
<td>-2.399/0.370</td>
</tr>
</tbody>
</table>
influence of the parameters for the rigid body dynamics shows that, to get an accurate model, one has to find a way to combine the method described here with experiments where these parameters are more excited.

![Validation of nonlinear model](image)

**Figure 6.4.** Torque as a function of torsion. Dash-dotted curve is measured data, solid is simulated from model m67.

In Figure 6.4 the measured and simulated torque is plotted against applied torque. This gives another validation of the fact that the estimated parameters in the hysteresis model actually models the flexibility satisfactorily. One can notice that even though the nonlinear grey-box identification has decreased the damping of the system compared to the initial values, the model has some problems to adapt to the data around zero torque.

### 6.8 Conclusions

A three-step procedure to identify the rigid body dynamics, friction and hysteresis using only measurements on the motor side has been proposed and exemplified with real data. The work shows that the parameter estimates for the hysteresis model derived according to the common initialisation steps can be improved using nonlinear grey-box identification. Especially the estimation of the nonlinear damping is significantly improved. The work also shows that although it is a simple model, the hysteresis model is able to model a great part of the behaviour of the flexibility in the linear drive.

The relatively simple hysteresis model introduces one new dynamic state and hence is an ideal candidate for inclusion in a model-based PKM control system. Such a model-based controller will have its highest benefit for fast dynamic operations, such as pick-and-place.

There are several aspects of the work that are subjects for future studies. The poor excitation of the parameters in the rigid body dynamics is the most obvious one. An interesting topic for future work is to examine the possibilities
to identify the system with the saddle free. This would open the possibility for better excitation of those parameters. Many problems are to be solved to make this possible. One is the problem with initiating the parameters in the hysteresis model. Another is to excite the system enough to be able to separate the masses and to make the hysteresis parameters identifiable.
Chapter 7

Identification of Parameters in the Inverse Dynamics

In this chapter a method for identification of the mass and inertia of the arms as well as the mass and centre of gravity of the platform is derived. The model for the inverse dynamics of the structure is rewritten in a suitable form and used for the identification. Optimal trajectories for the identification are calculated and the method is validated with experimental data.

7.1 Theory

The methods used in this chapter are based on the paper [26]. This paper deals with the problem of finding optimal trajectories for identification of the rigid body dynamics of a serial robot. The theory is based on a dynamic model of the robot as a linear regression. We will here use the notation

\[ \Gamma = F(q, \dot{q}, \ddot{q})\theta \] (7.1)

for the linear regression. \( \Gamma \) is a vector consisting of the forces from the tau structure on each linear actuator respectively, \( F \) is the regression matrix and \( \theta \) is a vector with the parameters to be identified. Under the assumption that the noise level on all measured actuator forces are the same and that the noise level on the forces are much greater than the noise level on the angle measurements, the maximum-likelihood parameter estimation is given by

\[ \hat{\theta}_{ls} = (F^t F)^{-1} F^t \Gamma \] (7.2)

In [26] a method to generate optimal excitation trajectories for the identification is suggested. In the method the trajectories for each joint is a finite Fourier series
Identification of Parameters in the Inverse Dynamics

\[ q_i(t) = \sum_{l=1}^{N} \frac{a_i^l}{\omega_f l} \sin(\omega_f l t) - \frac{b_i^l}{\omega_f l} \cos(\omega_f l t) + q_{i0} \]  \hfill (7.3)

\[ \dot{q}_i(t) = \sum_{l=1}^{N} a_i^l \cos(\omega_f l t) + b_i^l \sin(\omega_f l t) \]  \hfill (7.4)

\[ \ddot{q}_i(t) = \sum_{l=1}^{N} -a_i^l \omega_f l \sin(\omega_f l t) + b_i^l \omega_f l \cos(\omega_f l t) \]  \hfill (7.5)

where \( \omega_f \) is the fundamental angular frequency of the Fourier series. The fundamental angular frequency is chosen common for all joints to preserve the periodicity of \( T_f = 2\pi/\omega_f \) of the overall excitation. This parametrisation will give \( 2 \times N + 1 \) parameters for each joint. The parameters \( q_{i0} \) is the offset of the trajectory, i.e. the operating point at which the excitation will be carried out. This choice of excitation signal has several advantages. While it guarantees a periodic band-limited trajectory one of these advantages is that time-domain averaging can be used to increase the signal to noise ratio. In [26] it is shown that minimising the criterion \(-\log \det(F^TF)\) where \( F \) is the regression matrix gives a small uncertainty in the estimated parameters. An alternative would be to minimise the condition number of \( F \) directly, but this tends to give poorer estimates.

7.2 Model

The inverse dynamics of the Tau-structure have been derived in [4] using methods from [14]. In the work reported here a method for identifying the parameters in the model will be derived. The base frames used here are introduced in Chapter 3.

The inverse dynamic model of the parallel structure is given by

\[ \Gamma = J_r^T \left[ F_p + \sum_{i=1}^{m} \frac{\delta X_i}{\delta X_p} J_i^{-T} H_i \right] \]  \hfill (7.6)

For the Gantry-Tau \( \Gamma \) is a \( 3 \times 1 \) vector of the linear actuator forces, \( J_r \) is the direct Jacobian, \( F_p \) is a \( 3 \times 1 \) vector of the platform forces, \( X_i \) is the Cartesian coordinates of the terminal point of leg \( i \), \( X_p \) is the Cartesian coordinates of the tool point, \( J_i \) is the Jacobian matrix of leg \( i \) and \( H_i \) is the inverse dynamic model of leg \( i \). The platform forces can be expressed as

\[ F_p = m_p (\ddot{X}_p - \begin{pmatrix} 0 \\ 0 \\ g \end{pmatrix}) + \omega_p \times (\omega_p \times MS_p) \]  \hfill (7.7)

where \( m_p \) is the mass of the platform, \( g \) is the acceleration of gravity, \( \omega_p = [0, \dot{\alpha}, 0]^T \) is the angular velocity of the platform. The angle \( \alpha \) describes the orientation of the platform and is calculated in the inverse kinematics. The vector \( MS_p \) contain the first moments (moments of mass) of the platform with respect to the TCP, i.e. \( MS_p = m_p \cdot [c_x, c_y, c_z]^T \) where \( c_x, c_y, c_z \) is the centre of gravity for the platform.
7.2.1 Approximations

The model for the inverse dynamics is quite complicated and here some assumptions and approximations are made to simplify the identification. The complete model with all arms modelled separately is used, but under the assumption that the mass and the constant inertia matrix are common for all six arms. Apart from this approximation, the inertia matrix is assumed to be diagonal and the inertias with respect to the x and y axis are assumed to be equal. The last assumption is easy to motivate since the arms are symmetric with respect to the z axis. Using these approximations means that the constant inertia for all six arms can be described by only two parameters, which will reduce the number of parameters to identify drastically. This does however mean that all differences between the arms with respect to mass and inertia are neglected. If the differences between the arms are big this approximation will naturally lead to a less accurate model.

7.2.2 The model as a linear regression

In order to identify the parameters one has to rewrite the model as a linear regression. This is done in several steps, starting by rewriting $F_p$. Evaluation of the cross products and some rewriting gives

$$F_p = m_p \begin{pmatrix} \ddot{X}_p \\ \ddot{Y}_p \\ \ddot{Z}_p - g \end{pmatrix} - \dot{\alpha}^2 \begin{pmatrix} c_x \\ 0 \\ c_z \end{pmatrix}$$

(7.8)

This can be rewritten into a linear regression in the parameters $m_p, c_x, c_z$ according to

$$F_p = \begin{pmatrix} \ddot{X}_p \\ \ddot{Y}_p \\ \ddot{Z}_p - g \end{pmatrix} - \dot{\alpha}^2 \begin{pmatrix} m_p \\ m_p c_x \\ m_p c_z \end{pmatrix}$$

(7.9)

Since the parameters $m_p, c_x, c_y, c_z$ do not appear in the rest of the model, i.e. all other terms in the expression for $\boldsymbol{\Gamma}$ are independent of these parameters, the continued rewriting will concern the masses and inertias of the arms. Analysing the remaining terms in the expression also reveals that $(\frac{\partial \gamma}{\partial X_p})^T H_i$ is linear in $m_i, I_{11}^i, I_{22}^i, I_{33}^i$, where $m_i$ is the mass of leg $i$ and $I_{jj}^i$ are the diagonal elements of the constant inertia matrix associated with leg $i$. The only dependence of these parameters is in $H_i$. This means that we can rewrite the whole expression for the actuator forces as a linear regression. The expression is given by

$$\boldsymbol{\Gamma} = \boldsymbol{F} \boldsymbol{\Theta}$$

(7.10)

where $\boldsymbol{\Theta}$ is a row vector consisting of $m_p, m_p c_x, m_p c_z$ and $m_i, I_{11}^i, I_{22}^i, I_{33}^i$, $i \in \{1, 2, 3, 4, 5, 6\}$ and $\boldsymbol{F} = J^T \begin{pmatrix} F_{F_p}, F_{1}, F_{2}, F_{3}, F_{4}, F_{5}, F_{6} \end{pmatrix}$ with

$$F_{F_p} = \begin{pmatrix} \ddot{X}_p \\ \ddot{Y}_p \\ \ddot{Z}_p - g \end{pmatrix}$$

(7.11)
and

\[ F_i = \left( \frac{\delta X_i}{\delta X_p} \right) J_i^{-T} \tilde{H}_i , \quad i \in \{1, 2, 3, 4, 5, 6\} \]  \hspace{1cm} (7.12)

where \( \tilde{H}_i \) is given by \( H_i = \tilde{H}_i \cdot (m_i, I_i^{11}, I_i^{22}, I_i^{33})^T \).

Under the assumptions described above the model can be reduced significantly. Using that the mass and the inertia matrix are the same for all six legs and the notation \( m_a = m_i, I^{1122} = I_i^{11} = I_i^{22}, I^{33} = I_i^{33}, \quad i \in \{1, 2, 3, 4, 5, 6\} \), gives the model

\[ \Gamma = F\Theta \]  \hspace{1cm} (7.13)

where

\[ \Theta = [m_p, m_p c_x, m_p c_z, m_a, I_i^{1122}, I_i^{33}]^T \]  \hspace{1cm} (7.14)

and \( F = J_r^T (F_{Fp}, F_i^1 + F_i^2, F_i^3) \) with

\[ F_{Fp} = \begin{pmatrix} \ddot{X}_p & -\dot{a}^2 & 0 \\ \ddot{Y}_p & 0 & 0 \\ \ddot{Z}_p - g & 0 & -\dot{a}^2 \end{pmatrix} \]  \hspace{1cm} (7.15)

and \( F_i^1 \) as the \( i \):th column of

\[ F_i = \sum_{i=1}^{6} \left( \frac{\delta X_i}{\delta X_p} \right) J_i^{-T} \tilde{H}_i \]  \hspace{1cm} (7.16)

where \( \tilde{H}_i \) is given by \( H_i = \tilde{H}_i \cdot [m_i, I_i^{1122}, I_i^{33}] \).

### 7.3 Optimal trajectories

The trajectories will be optimised according to [26]. Since the structure of the Gantry-Tau is completely different from the robots used for the experiment in that study, one has to come up with a way to apply the method to this structure. There are mainly two alternatives. Either to optimise the trajectory of each actuator directly or to optimise the trajectory of the platform. In this study it is the trajectory of the platform that has been optimised. This is mainly because of computational advantages. The inverse kinematics of the structure have been derived analytically in [4] while, at the time of this study, one has not been able to derive an analytical form of the forward kinematics. Optimising the trajectory for the platform means that we can use the derived analytical forms in the computation of \( F \). The motion of the platform will, in each direction, be described by a finite Fourier series. The amplitudes will be the parameters to adjust in the optimisation of the trajectory. The positions and velocities of the actuators can easily be calculated with the inverse kinematics from the optimised trajectories and used as reference signal to the control system. These will also be used to constrain the optimisation while one has to consider the limited speed of the actuators.
The optimisation problem is solved in Matlab using the command `fmincon` [19]. This command performs an optimisation under nonlinear constraints. `fmincon` uses sequential quadratic programming to solve the optimisation problem. Several different constraints have been tested in the optimisation. Here the results of two different constraints will be presented. In both cases the position of the platform is constrained to a central part of the manipulator workspace. The maximum velocity of the saddles are $0.58 \text{ m/s}$, which with some margin gives the constraints below. The optimisations were terminated after 1000 iterations. The first trajectory was optimised with constraints on the velocity of the platform. This gives the optimisation problem stated as

$$
\min \ - \ \log \ \det (F^t F)
$$

$$
\max (\dot{X}^2 + \dot{Y}^2 + \dot{Z}^2) - 0.5^2 < 0
$$

$$
\max ((X - 0.20)^2 + Y^2 + (Z - 0.75)^2) - 0.25^2 < 0
$$

The result is a trajectory clearly dominated by two terms, $\ddot{Y} = 0.49 \cos(\omega_f 5t)$ and $\ddot{Z} = 0.49 \sin(\omega_f 5t)$. Since these terms were so dominating two different excitations signals will be derived, one where only the dominating terms are included and one where all terms are included. The constraints on the velocity were rather conservative and the optimised trajectories do not result in maximum velocity of the actuators. Scaling the parameters so that the maximum velocity is achieved will give a lot smaller value on the optimisation criterion as shown in Table 7.1.

The second optimisation was done with constrains directly on the velocity of the actuators. In order to do this the actuator velocities were calculated via the inverse kinematics from the platform velocity and position. This gives the optimisation problem stated as

$$
\min \ - \ \log \ \det (F^t F)
$$

$$
\max (|\dot{X}_1|) - 0.55 < 0
$$

$$
\max (|\dot{X}_2|) - 0.55 < 0
$$

$$
\max (|\dot{X}_3|) - 0.55 < 0
$$

$$
\max ((X - 0.20)^2 + Y^2 + (Z - 0.75)^2) - 0.25^2 < 0
$$

A problem during the optimisation is that the calculations may return a complex valued matrix if the trajectories go outside of the workspace. To reduce this problem the absolute value of all parameters were also constrained to be less than 1. Even with this constraint the optimisation will go outside the workspace and here this problem were dealt with by only considering the real part of $\det (F^t F)$.
Table 7.1. Minimised criterion for the excitation signals.

<table>
<thead>
<tr>
<th>Trajectory</th>
<th>(-\log \det(F^t F))</th>
<th>Condition number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Trajectory 1</td>
<td>-85.36</td>
<td>16.10</td>
</tr>
<tr>
<td>Trajectory 1 (Scaled)</td>
<td>-88.54</td>
<td>4.65</td>
</tr>
<tr>
<td>Trajectory 1 (Dominating terms)</td>
<td>-85.27</td>
<td>16.06</td>
</tr>
<tr>
<td>Trajectory 1 (Scaled dominating terms)</td>
<td>-90.09</td>
<td>4.62</td>
</tr>
<tr>
<td>Trajectory 2</td>
<td>-87.43</td>
<td>3.79</td>
</tr>
</tbody>
</table>

7.4 Data collection

To generate the data for the identification experiment a simple controller with a reference for actuator torque and a feed-forward for the actuator velocities is used. This is sufficient to follow the reference for the actuators with good accuracy. In the experiment the applied torque and the motor position are measured. To measure the acceleration of the platform an accelerometer has been mounted at the TCP.

7.4.1 Experimental design

The reference signals are generated from the optimised trajectories using the inverse kinematics and the robot Jacobian \( J_r \). The matrix \( J_r \) is derived by differentiating the equations for the inverse kinematics and describes the relationship between the actuator’s and the platform’s velocities. The experiments are initialised by driving the platform to the correct operating point. The data are collected over 13 periods, where 2 periods in the beginning are used for ramping the excitation signal to the correct amplitude and 2 periods in the end are used for ramping the amplitude down so that the robot starts and stops smoothly to avoid transient effects. This means that 11 periods with the correct amplitude are collected.

- **Data set 1**: Reference signals derived from the scaled version of Trajectory 1.
- **Data set 2**: Reference signals derived from the scaled version of Trajectory 1 with the dominating terms only.
- **Data set 3**: Reference signals derived from Trajectory 2.

7.4.2 Preprocessing of data

The accelerations of the platform in the Cartesian base frame are calculated using the redundant angle \( \alpha \), which describes the orientation of the platform. The angle \( \alpha \) is calculated in the inverse kinematics. The accelerations are then band-pass
7.5 Results

Filtered to remove drift and high frequency noise. The velocity of the platform is estimated by integrating the acceleration. For the actuators the velocities and accelerations are estimated by differentiation of the measured positions. The position of the platform is calculated by using a numerical solution of the forward kinematics. After this all signals are low-pass filtered and time-averaged.

In order to calculate the forces from the structure on the actuators, two different methods can be used. The first idea is to redo the experiments with the structure dismounted. The forces can then be calculated as the difference between the measured forces in the experiments with the structure connected and the experiments without the structure connected. The other idea is to make use of the models derived in Chapter 5. Both ways will be used here to compare the results.

7.5 Results

With the preprocessed data from the three experiments the matrix $F$ is calculated. The optimisation criterion $-\log \det (F^t F)$ and the condition number for the matrix are calculated and presented in Table 7.2. This shows that the optimisation criterion decrease compared to the values presented in Table 7.1. But one also sees that the condition number increase significantly compared to the expected values.

<table>
<thead>
<tr>
<th>Measurement</th>
<th>$-\log \det (F^t F)$</th>
<th>condition number</th>
</tr>
</thead>
<tbody>
<tr>
<td>Measurement 1</td>
<td>-93.59</td>
<td>18.92</td>
</tr>
<tr>
<td>Measurement 2</td>
<td>-93.70</td>
<td>27.00</td>
</tr>
<tr>
<td>Measurement 3</td>
<td>-93.69</td>
<td>32.47</td>
</tr>
</tbody>
</table>

Estimating the parameters using Equation (7.2) and the forces derived by either subtracting the modelled dynamics of the actuators or the measured dynamics without the structure mounted gives the estimates presented in Tables 7.3 and 7.4.

To evaluate the estimates one can compare these with the values in Table 7.5. These values have been achieved by accurately weighing the masses of the platform and the arms. The centre of gravity has been determined by calculations on the platform structure. The moments of inertia for the arms are calculated under the approximation that the arms can be seen as a straight rod. The most accurate of these values are therefore the masses. Compared to these values the estimates are very poor for all three cases. An interesting thing to look at is the estimation of the total mass in the structure, i.e. $m = m_p + 6m_a$. The correct value for this, assuming that the parameters in Table 7.5, called parameter set 0, are correct would be 10.080kg. With parameter set 1 the total mass is estimated to $9.434 \pm 0.275$kg and $10.472 \pm 0.320$kg respectively. With parameter set 2 the total mass is estimated to $9.682 \pm 0.365$kg and $10.788 \pm 0.404$kg respectively and with parameter set 3 the total mass is estimated to $5.359 \pm 0.529$kg and $5.205 \pm 0.580$kg.
Table 7.3. Estimated parameters with excitation signal 1 and 2. The a columns refer to forces derived by subtracting measurements and the b columns from subtracting the actuator model. For each estimate the standard deviation is shown.

<table>
<thead>
<tr>
<th></th>
<th>1a</th>
<th>1b</th>
<th>2a</th>
<th>2b</th>
</tr>
</thead>
<tbody>
<tr>
<td>mp</td>
<td>2.901 (± 0.082)</td>
<td>3.492 (± 0.096)</td>
<td>2.715 (± 0.1124)</td>
<td>3.305 (± 0.124)</td>
</tr>
<tr>
<td>mp,cx</td>
<td>-0.036 (± 0.023)</td>
<td>-0.024 (± 0.027)</td>
<td>-0.128 (± 0.026)</td>
<td>-0.107 (± 0.028)</td>
</tr>
<tr>
<td>mp,cz</td>
<td>0.054 (± 0.017)</td>
<td>-0.002 (± 0.020)</td>
<td>0.159 (± 0.021)</td>
<td>0.115 (± 0.023)</td>
</tr>
<tr>
<td>ma</td>
<td>1.087 (± 0.032)</td>
<td>1.163 (± 0.037)</td>
<td>1.161 (± 0.042)</td>
<td>1.247 (± 0.047)</td>
</tr>
<tr>
<td>I_{1122}</td>
<td>-0.122 (± 0.011)</td>
<td>-0.013 (± 0.013)</td>
<td>-0.101 (± 0.022)</td>
<td>0.049 (± 0.024)</td>
</tr>
<tr>
<td>I_{33}</td>
<td>0.033 (± 0.016)</td>
<td>-0.106 (± 0.018)</td>
<td>0.0577 (± 0.015)</td>
<td>-0.1145 (± 0.016)</td>
</tr>
</tbody>
</table>

Table 7.4. Estimated parameters with excitation signal 3. The a column refer to forces derived by subtracting measurements and the b column from subtracting the actuator model. For each estimate the standard deviation is shown.

<table>
<thead>
<tr>
<th></th>
<th>3a</th>
<th>3b</th>
</tr>
</thead>
<tbody>
<tr>
<td>mp</td>
<td>1.488 (± 0.170)</td>
<td>1.534 (± 0.186)</td>
</tr>
<tr>
<td>mp,cx</td>
<td>-0.203 (± 0.048)</td>
<td>-0.190 (± 0.052)</td>
</tr>
<tr>
<td>mp,cz</td>
<td>0.099 (± 0.032)</td>
<td>0.031 (± 0.035)</td>
</tr>
<tr>
<td>ma</td>
<td>0.645 (± 0.060)</td>
<td>0.612 (± 0.066)</td>
</tr>
<tr>
<td>I_{1122}</td>
<td>0.229 (± 0.033)</td>
<td>0.412 (± 0.036)</td>
</tr>
<tr>
<td>I_{33}</td>
<td>0.415 (± 0.024)</td>
<td>0.340 (± 0.027)</td>
</tr>
</tbody>
</table>

respectively. This shows that the first two results at least give a decent estimate of the total mass of the structure, while the third estimate clearly is too low.

Another problem is the fact that the estimates give unrealistic values for the inertia parameters. These parameters cannot have a negative value, since this does not have any physical interpretation. Both estimate 1 and 2 have unrealistic values for the inertia parameters. The fact that the centre of gravity is given a small negative value in some directions is not totally unrealistic.

Looking at the model fit for each set of estimated parameters, see Table 7.6, one can make some observations. To start with one sees that parameter set 0 gives a rather poor fit for all three data sets. This shows that the data sets are not very well described by the model, since these parameter values are accurate. This points out that all three datasets probably contain unmodelled behaviour in the system. From Chapter 5 it is clear that the actuators and especially the saddles will introduce flexibilities in the system. This could contribute to the poor result.

The second observation from the fit is made by looking at the cross validation, i.e. the fit for a parameter set estimated with data set i for data set j. As seen in the table all parameter estimates give a larger fit than set 0 on the data set they were estimated from. One can also see that the sets estimated from data set
### 7.5 Results

Table 7.5. Measured and calculated parameters. By accurate measurements on the dismounted structure and calculations, these values have been derived for the parameters.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m_p$</td>
<td>1.860</td>
</tr>
<tr>
<td>$m_p c_x$</td>
<td>0.000</td>
</tr>
<tr>
<td>$m_p c_z$</td>
<td>0.074</td>
</tr>
<tr>
<td>$a$</td>
<td>1.370</td>
</tr>
<tr>
<td>$I_{11}$</td>
<td>0.257</td>
</tr>
<tr>
<td>$I_{33}$</td>
<td>0.000</td>
</tr>
</tbody>
</table>

Table 7.5 gives rather high fit for data set 2 and the other way around. This is hardly surprising since the data sets are very similar. There are however three parameter sets that stand out. Parameter set 1b, 3a and 3b give higher fit than parameter set 0 for all data sets. Of these three data set 1b has clearly the highest fit and gives even higher fit for data set 3 than the parameters estimated from this data set. Unfortunately this parameter set has unrealistic values for both inertia parameters.

By considering the standard deviation and the condition numbers for the matrices $F$, one can make some interesting observations. Looking at the optimised trajectories in Table 7.1 one would presume to get the best estimation with Trajectory 3. The smallest condition number of $F$ would give the smallest standard deviation for the estimates [26]. Here one sees that some practical aspects of the experiment have a large input of which trajectories that will give the best estimates. First, the only things controllable in the experimental equipment are the position and velocity of the actuators. It will therefore be much harder to get the platform to follow the reference signal in Trajectory 3 than in Trajectory 1 and 2, which basically consist of only one frequency component. Secondly, while the model does not model any flexibilities in the structure, some data sets will be less accurately described by the model than others. Trajectory 3 is a more aggressive excitation, which has much more zero crossings, where unmodelled properties like play in the saddles will have a larger impact than for Trajectory 1 and 2. These two observations can also be seen in the condition numbers in Table 7.2, where the condition number for Trajectory 3 is larger than the condition number for the

Table 7.6. Model fit. The a columns refer to forces derived by subtracting measurements and the columns b from subtracting the actuator model. The parameters in table 7.5 is referred to as model 0.

<table>
<thead>
<tr>
<th>Dataset</th>
<th>0a</th>
<th>0b</th>
<th>1a</th>
<th>1b</th>
<th>2a</th>
<th>2b</th>
<th>3a</th>
<th>3b</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dataset1</td>
<td>35.76</td>
<td>32.53</td>
<td>45.57</td>
<td>50.88</td>
<td>45.24</td>
<td>46.32</td>
<td>37.13</td>
<td>36.17</td>
</tr>
<tr>
<td>Dataset2</td>
<td>31.47</td>
<td>25.48</td>
<td>42.98</td>
<td>50.32</td>
<td>45.39</td>
<td>47.51</td>
<td>33.43</td>
<td>28.97</td>
</tr>
<tr>
<td>Dataset3</td>
<td>35.81</td>
<td>36.43</td>
<td>33.42</td>
<td>39.37</td>
<td>32.43</td>
<td>34.23</td>
<td>37.73</td>
<td>39.25</td>
</tr>
</tbody>
</table>
other trajectories. This gives an idea that one should choose a trajectory as simple as possible with a fairly low condition number among the optimised trajectories, but maybe not the trajectory with the smallest condition number if this trajectory will be hard to generate in the real system.

The estimation in this chapter was performed by solving the standard linear least squares problem (7.2). This is the maximum-likelihood estimation of the parameters under two assumptions. At least one of these, that the noise level on the measured forces is much higher than the noise level on the joint variables, may cause some trouble in this case. To be fair this is rather hard to motivate here. In fact, the measurements of the accelerations are noisier than the torque measurements used to calculate the forces. This means that the regression matrix contains noise that maybe cannot be neglected. Taking this into account would mean that the estimation cannot be solved as a linear least squares problem and becomes much harder, generally a nonlinear least squares problem.

7.6 Conclusions

A method for identification of parameters in the model for the inverse dynamics has been proposed and evaluated with data from the Gantry-Tau prototype. The work shows that the inverse dynamics can be rewritten as a linear regression in six parameters. Optimal trajectories have been calculated with the methods from [26]. Experimental data from three excitation signals have been used to evaluate the method.

The evaluation showed that it is not possible to get accurate estimates from the method used here. Several parameters got unrealistic values and the estimates were far from the previously known parameter values which have been measured accurately.

Several reasons for this are conceivable. One observes that the experimental data contain unmodelled behaviour in the system. This is believed to contribute to the problem with unrealistic, negative, parameters. The conclusion is that either the model for the inverse dynamics fails to model some behaviour of the system, or that the way the actuator forces are calculated here is not accurate enough. Probably it is a combination of both these factors. It is also hard to motivate that the conditions for using standard linear least squares for the estimation are fulfilled. One reason for the poor estimation could therefore also be that the regression matrix contains too much noise.

One could conclude that the optimised trajectory with the smallest condition number is not always the best. Here it was shown that a simple trajectory with one dominating frequency gave better accuracy in practice.

An interesting idea for further study is to extend the model for the inverse dynamic with the actuator dynamics. This would give a model from joint variables to applied motor torque. The problem of trying to calculate the actuator forces from measurements or from separate actuator models will then be replaced by trying to estimate the actuator dynamics together with the dynamics of the structure. In such a work one could also look at more complex friction models and
also try to model the play, mainly believed to be located in the saddles.
Identification of Parameters in the Inverse Dynamics
Chapter 8

Conclusions and Future Work

This chapter summarise the report by restating the results from Chapters 5, 6 and 7. Some ideas of future work are also summarised.

8.1 Actuator dynamics

Using simple estimation methods it was possible to estimate the rigid body model of the different parts of the actuators. The experiments showed that the friction is described rather well even by a simple friction model. The main effect that remains unmodelled is the stiction. One can conclude that the main friction and inertia of the system are located in the lead screw and the saddle, which seems reasonable. The analysis showed that a least squares approach gives rather good accuracy and is preferable since only one data set is needed. The analysis further showed that the drive train contains a nonlinear flexibility, with both hysteresis and backlash present. The belt gear is shown to introduce backlash in the system, while the main hysteresis effect can be deduced to the lead screw. The flexibility is also shown to be dependent on the position of the saddle. The experiment showed that the dependence mainly comes from the lead screw and could be explained by a simple model as inversely proportional to the position of the actuator. Altogether this means that to be able to model the system accurately a rather complex model has to be used. The stiffness of the saddle is shown to be low for torques, which could cause a problem in this application since the tau-structure induces torques at actuator 2 and 3. This problem cannot be solved by extra sensors but should motivate a different construction where either the actuators are much stiffer or where the tau-structure doesn’t induce torques in the same way.
8.2 Nonlinear Grey-Box Identification

A three-step procedure to identify the rigid body dynamics, friction and hysteresis using only measurements on the motor side has been proposed and exemplified with real data. The work shows that the parameter estimates for the hysteresis model derived according to the common initialisation steps can be improved using nonlinear grey-box identification. Especially the estimation of the nonlinear damping is significantly improved. The work also shows that although it is a simple model, the hysteresis model is able to model a great part of the behaviour of the flexibility in the linear drive.

The relatively simple hysteresis model introduces one new dynamic state and hence is an ideal candidate for inclusion in a model-based PKM control system. Such a model-based controller will have its highest benefit for fast dynamic operations, such as pick-and-place.

8.3 Identification of Parameters in the Inverse Dynamics

A method for identification of parameters in the model for the inverse dynamics has been proposed and evaluated with data from the Gantry-Tau prototype. The work shows that the inverse dynamics can be rewritten as a linear regression in six parameters. Optimal trajectories have been calculated with the methods from [26]. Experimental data from three excitation signals have been used to evaluate the method.

The evaluation showed that it is not possible to get accurate estimates from the method used here. Several parameters got unrealistic values and the estimates were far from the previously known parameter values which have been measured accurately.

Several reasons for this are conceivable. One observes that the experimental data contain unmodelled behaviour in the system. This is believed to contribute to the problem with unrealistic, negative, parameters. The conclusion is that either the model for the inverse dynamics fails to model some behaviour of the system, or that the way the actuator forces are calculated here is not accurate enough. Probably it is a combination of both these factors. It is also hard to motivate that the conditions for using standard linear least squares for the estimation are fulfilled. One reason for the poor estimation could therefore also be that the regression matrix contains too much noise.

One could conclude that the optimised trajectory with the smallest condition number is not always the best. Here it was shown that a simple trajectory with one dominating frequency gave better accuracy in practice.

8.4 Proposal for Future Work

There are several aspects of the work that could be studied further. An interesting continuation of the analysis of the actuator is to try to eliminate weakness and the
8.4 Proposal for Future Work

play in the saddles. This could also mean that the model for the inverse dynamic
would describe the system better since the system becomes more rigid.

The poor excitation of the parameters in the rigid body dynamics in the non-
linear grey-box identification is an interesting system identification problem. An
interesting topic for future work is to examine the possibilities to identify the sys-
tem with the saddle free. This would open the possibility for better excitation of
those parameters. Many problems are to be solved to make this possible. One is
the problem with initiating the parameters in the hysteresis model. Another is
to excite the system enough to be able to separate the masses and to make the
hysteresis parameters identifiable.

An interesting idea is to extend the model for the inverse dynamic with the
actuator dynamics. This would give a model from joint variables to applied motor
torque. The problem of trying to calculate the actuator forces from measurements
or from separate actuator models will then be replaced by trying to estimate the
actuator dynamics together with the dynamics of the structure. In such a work
one could also look at more complex friction models and also try to model the
play, mainly believed to be located in the saddles.
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