Investigation of rotor downwash effects using CFD

Master Thesis in Automatic Control
by
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LITH-ISY-EX--09/4087--SE
Linköping 2009
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ABSTRACT

This paper is the result of a master thesis project on helicopter rotor downwash effects using computational fluid dynamics (CFD). The work was performed at the department of Aerodynamics and Flight Mechanics at Saab AB, Linköping in 2008. It completes the author’s studies for a M.Sc degree in Applied Physics and Electrical Engineering at the Department of Electrical Engineering at the Linköping institute of technology (LiTH), Linköping, Sweden.

The aim of the project was to study the rotor downwash effects and its influence on the helicopter fuselage. To fulfill this purpose, several CFD calculations were carried out and the aerodynamic forces and moments resulting from the calculations were implemented in an existing simulation model, developed in-house at Saab. The original (existing) model was compared to the updated model by studying step responses in MATLAB, Simulink. For some step commands, the comparisons indicated that the updated model was more damped in yaw compared to the original model for the hovering helicopter. When the helicopter was trimmed for a steady turn, the states in the updated model diverged much faster than the states in the original model for any given step command.

In order to investigate the differences between the original helicopter model and the updated model from a controlling perspective, a linear quadratic (LQ) state feedback controller was synthesized to stabilize the vehicle in a steady turn. The LQ method was chosen as it is a modern design technique with good robustness and sensitivity properties and since it is easily implemented in MATLAB. Before synthesising, a simplification of the helicopter model was made by reducing states and splitting them into lateral and longitudinal ones. Step responses from simulations with the original and the updated model were studied, showing an almost identical behavior.

It can be concluded that the aerodynamic coefficients obtained from the CFD calculations can be used for determining the aerodynamic characteristics of the helicopter. Some further validation is needed though, for example by comparing the results with flight test data. In order to build an aerodynamic data base that covers the whole flight envelop, additional CFD calculations are required.

Key words: Helicopter, Momentum Theory, Blade Element Theory, CFD, Rotor-Fuselage Interaction, Simulation, LQ controller
ACKNOWLEDGEMENTS

I would like to thank my examiner at LiTH, Professor Torkel Glad, for supporting my work, my supervisor at Saab, Aerodynamicist Erica Amundsson for guidance in the CFD jungle and feedback on my report, Mathematical Specialist, Per Weinelt for explaining the model and answering a 1000 questions, Aerodynamicist Roger Larsson for always finding a possible solution, Doctor Ola Härkegård for assistance in the control design, Propeller Expert Ingemar Persson for showing me the rotor split and finally, my co-worker Berenike Hanson, for good discussions and heavy weights at the gym.

Helena Johansson
Linköping, mars 2009
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NOTATION

\[ \dot{x} = \frac{dx}{dt} \]

Dot denotes differentiation with respect to time.

\( x, F \)

Bold style denotes vector or matrix.

ABBREVIATIONS

2-D Two dimensional
BET Blade Element theory
CFD Computational Fluid Dynamics
CFL Courant-Friedrichs-Lewy
DNS Direct Numerical Solution
c.g. Centre of Gravity
FEM Finite Element Method
FDM Finite Difference Method
FVM Finite Volume Method
LQR Linear Quadratic Regulator
MT Momentum Theory
N-S Navier-Stokes
RANS Reynolds-Averaged Navier-Stokes
RPM Revolutions Per minute
UAV Unmanned Aerial Vehicle

SUBSCRIPTS AND SUPERSCRIPTS

b Blade-fixed coordinate system
B Body-fixed coordinate system
E Earth-fixed coordinate system
h Hub-fixed coordinate system
mnr Main rotor
p Propeller
pad Hell-Hiller paddles
trl Tail rotor
T Transpose
### NOMENCLATURE

<table>
<thead>
<tr>
<th>Abbreviation</th>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>Angle of attack.</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta$</td>
<td>Flapping angle, angle of sideslip.</td>
<td>rad</td>
</tr>
<tr>
<td>$\beta_0, \beta_{\cos}, \beta_{\sin}$</td>
<td>Coning angle, cosine (longitudinal) flapping, sine (lateral) flapping.</td>
<td>rad</td>
</tr>
<tr>
<td>$\delta_{ij}$</td>
<td>Kronecker delta function.</td>
<td>-</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Blade pitch angle.</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_{\text{col}}, \theta_{\text{cos}}, \theta_{\text{sin}}$</td>
<td>Collective pitch angle, cosine (lateral) cyclic pitch, sine (longitudinal) cyclic pitch.</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_{\text{sw}}$</td>
<td>Swash plate angle.</td>
<td>rad</td>
</tr>
<tr>
<td>$\theta_{\text{sw col}}, \theta_{\text{sw cos}}, \theta_{\text{sw sin}}$</td>
<td>Collective pitch angle, cosine (longitudinal) cyclic pitch, sine (lateral) cyclic pitch.</td>
<td>rad</td>
</tr>
<tr>
<td>$\kappa, \kappa_t$</td>
<td>Laminar and turbulent conductivity.</td>
<td>W/(m K)</td>
</tr>
<tr>
<td>$\kappa_p$</td>
<td>Measured induced power at hover.</td>
<td>-</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Induced inflow ratio.</td>
<td>-</td>
</tr>
<tr>
<td>$\mu, \mu_t$</td>
<td>Laminar and turbulent coefficient of viscosity.</td>
<td>Pa s</td>
</tr>
<tr>
<td>$\mu_x, \mu_z$</td>
<td>Tip speed ratios.</td>
<td>-</td>
</tr>
<tr>
<td>$\rho, \overline{\rho}, \rho'$</td>
<td>Density: Instantaneous value, time averaged value and fluctuating part according to $\rho = \overline{\rho} + \rho'$.</td>
<td>kg/m³</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>Rotor solidity.</td>
<td>-</td>
</tr>
<tr>
<td>$\tau_{ij}$</td>
<td>Stress tensor in N-S equations.</td>
<td>Pa</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Induced angle of attack.</td>
<td>rad</td>
</tr>
<tr>
<td>$[\phi, \Theta, \Phi]^T$</td>
<td>Euler angles, i.e. scalar components of the orientation vector $\Omega_E$.</td>
<td>rad</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Rotor wake skew angle.</td>
<td>rad</td>
</tr>
<tr>
<td>$\psi$</td>
<td>Azimuth angle.</td>
<td>rad</td>
</tr>
<tr>
<td>$\omega_B$</td>
<td>Helicopter angular velocity vector.</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\Omega$</td>
<td>Rotor angular velocity.</td>
<td>rad/s</td>
</tr>
<tr>
<td>$\Omega_E$</td>
<td>Helicopter orientation vector.</td>
<td>rad</td>
</tr>
<tr>
<td>$A, B, C$</td>
<td>System matrices.</td>
<td>-</td>
</tr>
<tr>
<td>$A_0, A_{\psi}, A_7, A_{\beta}$</td>
<td>Rotation matrices used in the transformation matrix $A_{h2b}$.</td>
<td>-</td>
</tr>
<tr>
<td>$A_{E2B}, A_{B2E}$</td>
<td>Transformation matrices from earth- to body-fixed coordinate system and body- to earth-fixed coordinate system.</td>
<td>-</td>
</tr>
<tr>
<td>$A_{B2h}, A_{h2B}$</td>
<td>Transformation matrices from body- to hub-fixed coordinate system and hub- to body-fixed coordinate system.</td>
<td>-</td>
</tr>
<tr>
<td>$A_{h2b}, A_{b2h}$</td>
<td>Transformation matrices from hub- to blade-fixed coordinate system and blade- to hub-fixed coordinate system.</td>
<td>-</td>
</tr>
<tr>
<td>$c$</td>
<td>Local blade chord.</td>
<td>m</td>
</tr>
<tr>
<td>$C_C$</td>
<td>Side force coefficient.</td>
<td>-</td>
</tr>
<tr>
<td>$C_l$</td>
<td>Local lift coefficient or rolling moment coefficient.</td>
<td>-</td>
</tr>
<tr>
<td>$C_{\alpha}$</td>
<td>2-D lift curve slope.</td>
<td>1/rad</td>
</tr>
<tr>
<td>$C_m$</td>
<td>Pitching moment coefficient.</td>
<td>-</td>
</tr>
<tr>
<td>$C_n$</td>
<td>Yawing moment coefficient.</td>
<td>-</td>
</tr>
<tr>
<td>$C_N$</td>
<td>Normal force coefficient.</td>
<td>-</td>
</tr>
<tr>
<td>$C_T$</td>
<td>Rotor thrust coefficient or tangential force coefficient.</td>
<td>-</td>
</tr>
</tbody>
</table>
dF_z \quad \text{Incremental force in z-direction acting on blade element.} \\
dL \quad \text{Increment in lift.} \\
dT \quad \text{Incremental thrust, } dT=N_d dF_z. \\
D \quad \text{Drag force.} \\
E, \bar{E}, E^* \quad \text{Energy: Instantaneous value, density weighted averaged value and fluctuating part according to } E = \bar{E} + E^*. \\
F_I, F_V \quad \text{Inviscid and viscid flux matrices.} \\
I_b \quad \text{Inertia of the blade.} \\
I_{pad} \quad \text{Inertia of the Bell-Hiller paddle.} \\
J \quad \text{Cost function in LQR.} \\
k \quad \text{Turbulent kinetic energy.} \\
k_1, k_2, k_3, k_4 \quad \text{Factors for determining the induced velocity in the region where the momentum theory is invalid.} \\
k_s \quad \text{Weighting factors in linear inflow model according to Pitt & Peters} \\
L \quad \text{Lift force.} \\
mb \quad \text{Mass of rotor blade.} \\
N_b \quad \text{Number of rotor blades.} \\
[p \; q \; r]^T \quad \text{Roll, pitch and yaw rate, i.e. scalar components of the angular velocity vector } \omega_B. \\
p, \bar{p}, p' \quad \text{Pressure: Instantaneous value, time averaged value and fluctuating part according to } p = \bar{p} + p'. \\
Pr, Pr_t \quad \text{Laminar and turbulent Prandtl number.} \\
q \quad \text{Heat fluxes, dynamic pressure.} \\
Q \quad \text{Weighting matrix for control errors.} \\
Q_m \quad \text{Main rotor torque.} \\
r_E \quad \text{Position vector in Earth-fixed coordinate system.} \\
r \quad \text{Normalized chordwise location of the blade, } r = \frac{x}{R}. \\
R \quad \text{Weighting matrix for input signals.} \\
R \quad \text{Rotor radius.} \\
t \quad \text{Time.} \\
T \quad \text{Rotor thrust.} \\
T, \bar{T}, T^* \quad \text{Temperature: Instantaneous value, density weighted averaged value and fluctuating part according to } T = \bar{T} + T^*. \\
T_m, T_t \quad \text{Main and tail rotor thrust.} \\
T_{B2E} \quad \text{Transformation matrix for transformation of angular velocities from body- to earth-fixed coordinate system.} \\
[u \; v \; w]^T \quad \text{Scalar velocity components of helicopter velocity vector } \mathbf{V}_B. \\
U \quad \text{Vector of conserved variables in N-S equations.} \\
U \quad \text{Velocity at blade element.} \\
U_p, U_T \quad \text{Velocity components of } U, \text{ perpendicular and tangential to the rotor respectively.} \\
\mathbf{V}_B \quad \text{Helicopter velocity in body-fixed coordinate system.} \\
v_i \quad \text{Induced velocity.} \\
w_i, \bar{w}_i, w^*_i \quad \text{Velocity components: Instantaneous value, density weighted averaged value and fluctuating part according to } w_i = \bar{w}_i + w^*_i. \\
W \quad \text{Helicopter weight.}
1 INTRODUCTION

In contrast to ordinary fixed-wing aircraft, a helicopter has the ability to hover and to take off and land vertically. This high manoeuvring capability gives it a wide range of applications, such as reconnaissance and surveillance operations. Particularly interesting are unmanned aerial vehicles, also known as UAV’s, since they do not depend on a pilot for hazardous missions. These advantages, together with the recent progress of making sensors more accurate and reliable, have lead to increased research on UAV helicopters by numerous groups.

1.1 Background

Saab is one actor on the market following the trend of a growing interest in autonomous helicopters. A few years ago, the company got involved with the UAV helicopter project Skeldar. In order to increase the knowledge of helicopters, a physical/mathematical model has been developed in-house at Saab [1]. This model contains a standard method for determining the increment in velocity at the rotor disk, also known as the induced velocity. Since the induced velocity plays an important role for estimating the required rotor thrust of the helicopter, investigations of further more advanced inflow models are of interest. The current model also lacks contributions to fuselage aerodynamic forces and moments due to the downwash from the helicopter main rotor, which is another topic worth looking into. As a result, two master thesis projects have recently been carried out at the department of Aerodynamics and Flight Mechanics, one examining appropriate inflow models and the other using computational fluid dynamics (CFD) to calculate forces and moments acting on the helicopter. This report covers the latter.

1.2 Objectives and limitations

The purpose of this master thesis project is to investigate the possibility of using CFD for estimations of main rotor downwash effects on the helicopter fuselage. The aerodynamic forces and moments resulting from these calculations will be implemented in the existing simulation program simuskeldar.

The work done is split into four parts. The first part is the gathering of information on different subjects, such as helicopter theory, CFD calculations, control theory and how the existing model works. The second part involves building the computational model and calculating aerodynamic forces and moments. In order to be able to test the updated model, a linear quadratic regulator (LQR) for a steady turn is implemented in Simulink as a third step. The regulator is limited to control in the lateral direction. For longitudinal control, a regulator developed by Hanson [2] is used. Finally, the evaluation and discussion of simulation results is obtained with both the original and the updated model.
2  HELICOPTER THEORY

There are several different types of helicopters such as the ones with a single main rotor, coaxial rotors, side-by-side rotors, tandem rotors and multi-rotors [3]. Skeldar belongs to the category with a single main rotor and a tail rotor perpendicular to the main rotor, which is the most common configuration. This section gives an introduction to the control principles of the helicopter including the swash plate, the main and tail rotors and the Bell-Hiller paddles. It also gives a description of the helicopter blade motion.

2.1  Main rotor

The purpose of the main rotor is to produce the thrust necessary to fly forward, change altitude and make turns. The main rotor consists of two or more rotor blades, with angular velocity $\Omega$, that generate the thrust like rotating wings. Since the geometry of the blade is fixed, variations in thrust can be made by either changing the rotor RPM (revolutions per minute) or changing the pitch angle of the blades. Normally the thrust is varied by changing the blade pitch angle, except for some small radio controlled helicopters using the rotor RPM as control input. For a full scale helicopter, the change in RPM normally takes too long time due to the rotor’s large amount of inertia.

In climb or descent the pitch angles of all main rotor blades are changed simultaneously (or collectively) to the same degree, which is known as the collective pitch. In order to move forward, backward or sideways the thrust needs to be tilted in that direction. This is done by giving the rotor blades different pitch angles as they revolve, resulting in a main rotor tilt. Since the pitch angles vary cyclically it is called the cosine (lateral) and sine (longitudinal) cyclic pitch. Note that a change in pitch angle has to be applied approximately 90 degrees before action should take place due to phase lag [4]. This means that a lateral cyclic pitch (as for instance a left turn) is applied by increasing the pitch angle at the back of the rotor disk, while a longitudinal cyclic pitch (as in forward flight) is applied by increasing the pitch angle at the rotor disk’s left side, see figure 1.

![Figure 1a. Cosine (lateral) cyclic pitch.](image-url)
The thrust $T$ is always perpendicular to the rotor disk and in hover it is equal to the helicopter lift $L$. This equality does not hold when the rotor is tilted, which makes an increase in thrust necessary to sustain level flight. The tilt also has the effect that the thrust line no longer passes through the centre of gravity (c.g.) of the helicopter. In forward flight it acts behind the c.g. which results in a forward body tilt, as presented in figure 2. This nose-down attitude is not very comfortable for passengers and may increase the drag. One method to get rid of the problem is to design a helicopter with a tail-down attitude in hover. It can be made by either moving the c.g. backwards or inclining the hub and drive shaft slightly forward.

**Figure 1b.** Sine (longitudinal) cyclic pitch.

**Figure 2.** Main rotor tilted forward in forward flight.
2.2 Swash plate

The device used to change pitch angles of the main rotor is called a swash plate. It consists of two plates connected with each other by a bearing. Both plates are free to move vertically and tilt in any direction, but the upper plate also rotates with the shaft. If the swash plate is moved up or down, push rods connected to the upper plate and the blades will make a collective change in pitch. If the swash plate is tilted, the change in pitch is cyclic since the push rods are moving up and down periodically, see figure 3.

![Swash plate diagram]

Figure 3. Swash plate

2.3 Tail rotor

The lateral force generated by the tail rotor is used to balance the main rotor torque and for directional control, see figure 4a-c. To compensate for the force’s tendency to drift the helicopter sideways, the main rotor has to be slightly tilted giving a force in the opposite direction, see figure 4d. Since the tail rotor is directly connected to the main rotor gear box, the RPM of the tail rotor can not be changed separately. Neither is it possible to make cyclic pitch changes of the tail rotor blades.

2.4 Pilot’s controls

The control lever at the pilots left side is the collective pitch control. It is mounted horizontally and moved up and down to make the helicopter change altitude. When the collective pitch is changed, an increase/decrease in engine power is required to keep the rotor RPM within its limits. A twist throttle, mounted at the end of the collective pitch control, is often used for this purpose. The vertical stick in front of the pilot is the cyclic pitch control, pushed in the direction the pilot wishes to fly. To control the collective pitch of the tail rotor and thereby its lateral force, the pilot uses two pedals on the floor. For a helicopter with an anti-clockwise rotating main rotor,
the tail pitch angle is increased when pushing the left pedal making the helicopter turn left. In order to perform a right turn, the tail pitch angle is decreased by a push on the right pedal.

Even though a fully autonomous helicopter such as Skeldar has no physical controls, the control principles described above are still the same.

![Diagram of helicopter control principles](image)

**Figure 4.** The lateral force $T_t$ generated by the tail is used to balance the rotor main torque $Q_m$ (a) and for directional control (b-c). Figure d) illustrates main rotor tilt to compensate for the tail force's tendency to drift the helicopter sideways.

### 2.5 Helicopter blade motion

A rotor blade is free to move around three axes, as shown in figure 5. The hinges at the flapping and lead-lag axes are implemented to allow the blade to move up and down or back and forth under the influence of varying airloads, while a pitch bearing is included to enable changes in the blade pitch angle around the feathering axis.

![Diagram of helicopter blade motions](image)

**Figure 5.** Rough draft showing the blade flapping, the lead-lag and the feathering motions.
2.5.1 Flapping

Hovering

Figure 6 shows a rotor blade flapping about a hinge with an offset of $e_R$ from the rotational axis. Since the gravitational force is much smaller than the other forces, it is neglected. The blade is thereby only affected by the lift and the centrifugal force, and will “cone” up to form a static balance between the forces.

If the length of a small element of the blade is $dy$ and the uniform mass per unit length is $m$, then the mass of the blade element is $m \cdot dy$. A constant rotational speed $\Omega$, about the axis gives the following contribution of this small element to the centrifugal force $dF_{CF}$, parallel to the plane of rotation:

$$dF_{CF} = m\Omega^2 y \, dy$$  \hspace{1cm} (2.1)

For a certain coning angle $\beta$, the component perpendicular to the blade is described by:

$$dF_{CF} \sin \beta = dF_{CF} \beta$$  \hspace{1cm} (2.2)

assuming small angles of $\beta$.

Since the centrifugal force is acting at a distance $y - e_R$ from the hinge, the centrifugal moment about the flap hinge is:

$$M_{CF} = \int_{e_R}^{R} m\Omega^2 y (y - e_R) \beta \, dy$$  \hspace{1cm} (2.3)

![Figure 6. Static balance between aerodynamic forces and centrifugal forces.](image)
The aerodynamic moment about the flap hinge is determined by the lift acting at the same distance as the centrifugal force, that is:

\[ M_{\text{aero}} = - \int_{eR}^{R} L(y - eR) \, dy \]  

(2.4)

If the centrifugal moment about the hinge, \( M_{\text{CF}} \), is equal and opposite to the aerodynamic moment, \( M_{\text{aero}} \), the blade has reached an equilibrium position and the coning angle, \( \beta_0 \), is given by:

\[ \beta_0 = \frac{\int_{eR}^{R} L(y - eR) \, dy}{\int_{eR}^{R} my\omega^2(y - eR) \, dy} \]  

(2.5)

Notice that the centrifugal force is much larger than the lift, resulting in quite small coning angles, typically between 3° and 6°.

**Dynamic condition:**

The blade flap angle \( \beta \), does not depend on the position of the blade in hover. It is constant as the blade revolves. In forward flight however, the asymmetric flow produces varying airloads which make the blade flap up and down. In addition to the airloads and the centrifugal force mentioned above, the blade element is also affected by an inertia force \( \int_{eR}^{R} m(y - eR)^2 \, dy \). Defining the moment positive in a direction that reduces \( \beta \), the momentum equilibrium about the flap hinge is given by:

\[ \int_{eR}^{R} m(y - eR)^2 \beta \, dy + \int_{eR}^{R} m\omega^2(y - eR) \beta \, dy - \int_{eR}^{R} L(y - eR) \, dy = 0 \]  

(2.6)

Since the mass moment of inertia about the flap hinge is

\[ I_b = \int_{eR}^{R} m(y - eR)^2 \, dy \]  

(2.7)

the flapping equation becomes

\[ I_b \ddot{\beta} + \left( I_b + eR \int_{eR}^{R} m(y - eR) \, dy \right) \omega^2 \beta = \int_{eR}^{R} L(y - eR) \, dy = M_b \]  

(2.8)
The flapping motion can be represented as an infinite Fourier series, but the higher harmonics of the blade motion are often neglected since their amplitudes are very small. The expression for the blade flapping motion is then given by:

$$\beta = \beta_0 + \beta_{\cos} \cos \psi + \beta_{\sin} \sin \psi$$  \hspace{1cm} (2.9)

with the coning angle \(\beta_0\), and the longitudinal and lateral flapping, \(\beta_{\cos}\) and \(\beta_{\sin}\) respectively. The position of the blade, in terms of an azimuth angle \(\psi\), is defined in figure 7.

![Figure 7. Definition of azimuth angle \(\psi\).](image)

Due to the 90° phase-lag when applying a cyclic pitch, a pure \(\theta_{\cos}\) displacement is equivalent to a \(\beta_{\sin}\) flapping motion. Similarly, the application of a \(\theta_{\sin}\) pitch displacement is equivalent to a \(-\beta_{\cos}\) flapping motion, since the disk is tilted back longitudinally. The lateral and longitudinal flapping, with and without coning, are shown in figure 8.

![Figure 8. The longitudinal flapping with and without coning is shown in (a)-(b) while the lateral flapping with and without coning is shown in (c)-(d).](image)
2.5.2 Lagging

Determining the equilibrium about the lead-lag hinge, as shown in figure 5, is also a balancing of forces. In contrast to the flapping motion the aerodynamic moment is generated by the drag force instead of the lift. As stated by Simons [5], the blade lag motion is though not only caused by variations in drag. When the blade flaps up, its tangential velocity decreases due to mass being moved slightly inwards (radially). The blade experiences a deceleration which gives rise to inertial forces proportional to lag moment of inertia of the blade. Especially at high rotor thrusts and large disk tilts, these moments can be large. Still, the lead-lag motion is often neglected when modelling helicopter rotors based on the assumption that the drag force is small compared to the lift.

2.5.3 Pitching (feathering)

The blade is also free to move around the feathering axis as shown in figure 5. This motion is known as the pitching motion discussed earlier in section 2.1-2.2. Using the azimuth angle $\psi$, together with the collective pitch $\theta_0$, and the first harmonics in a Fourier series, the blade pitch angle can be described as:

$$\theta = \theta_0 + \theta_{\cos} \cos \psi + \theta_{\sin} \sin \psi$$

(2.10)

where the lateral and longitudinal cyclic pitch are $\theta_{\cos}$ and $\theta_{\sin}$, respectively. Due to elastic deformations of the blade, the actual pitch differs from the commanded pitch.

2.6 Bell-Hiller paddle

Many helicopters have a mechanical system known as the Bell-Hiller paddle to improve the main rotor characteristics by stabilization. For instance, if a wind gust makes the rotor blade flap up, a required pitch correction is given by the paddles, since both the paddles and the blades are linked to the swash plate. The paddles may either be mounted below the rotor or, as for Skeldar, on top of it.

3 INFLOW THEORY

The increment in velocity at the rotor disk, also known as the induced velocity, $v_i$, plays an important role for determining the required rotor thrust of the helicopter. The velocity is often given in a non-dimensional form, normalized with the angular velocity $\Omega$ and the radius $R$ of the rotor, i.e. the rotor tip speed. The relation is given by equation:

$$\lambda_i = \frac{v_i}{\Omega R}$$

(3.1)

Since the induced inflow ratio $\lambda_i$ is a non-dimensional quantity, it is preferable to use when comparing different rotors.
The rotor thrust may also be given in a non-dimensional form, according to:

\[
C_T = \frac{T}{\rho A (\Omega R)^2}
\]  \hspace{1cm} (3.2)

The following sub-sections describe a uniform model for the induced inflow ratio derived from the momentum theory (MT), as well as the blade element theory (BET) used for estimating the rotor thrust. Since the induced velocity and the rotor thrust are coupled, the process of determining them is iterative.

### 3.1 Uniform inflow model based on the momentum theory (MT)

The momentum theory is a standard method for estimating the induced inflow ratio \( \lambda_i \). It is based on the following assumptions:

- The rotor is made up of an infinite number of blades, leading to a uniform thrust distribution with no tip losses.
- The disk is infinitely thin so that no discontinuities in velocity occur on the two sides of the disk.
- The velocity over the disk is uniform.
- The flow through the rotor is one-dimensional. This means that the fluid properties are constant across any plane parallel to the rotor plane and vary only in the vertical direction.
- The fluid is inviscid and incompressible.

Conservation of fluid mass, fluid momentum and energy together with the assumptions above give the following expressions for the induced inflow ratio \( \lambda_i \). 

**Hover and axial climb:**  
\[
\lambda_i = -\frac{\mu_x}{2} + \sqrt{\left(\frac{\mu_x}{2}\right)^2 + \frac{C_T}{2}}
\]  \hspace{1cm} (3.3)

**Axial descent:**  
\[
\lambda_i = -\frac{\mu_x}{2} - \sqrt{\left(\frac{\mu_x}{2}\right)^2 - \frac{C_T}{2}}
\]  \hspace{1cm} (3.4)

**Forward flight:**  
\[
\lambda_i = \frac{C_T}{2\sqrt{\mu_x^2 + (\mu_x + \lambda_i)^2}}
\]  \hspace{1cm} (3.5)

with the tip-speed ratios parallel and perpendicular to the rotor disk:  
\[
\mu_x = \frac{V_x \cos \alpha}{\Omega R}
\]  \hspace{1cm} and \hspace{1cm}  
\[
\mu_z = \frac{V_z \sin \alpha}{\Omega R}
\]  \hspace{1cm} (in hover \( \mu_z \) and \( \mu_x \) are equal to zero). The thrust coefficient \( C_T \) is defined in equation (3.2) and the velocity components \( (V_x \cos \alpha) \) and \( (V_z \sin \alpha) \) are defined in figure 9.
Figure 9. Momentum analysis in forward flight.

Note that the momentum theory is invalid in the region $-2\lambda \leq \lambda_c \leq 0$ due to the complicated recirculating flow pattern at the rotor disk. Since no definitive control volume exists, the conservation of fluid mass, momentum and energy cannot be applied. Instead a continuous approximation of the induced velocity, based on empirical data, is used for this region [6]:

$$\frac{\lambda_1}{\lambda_h} = \kappa_p + k_1 \left( \frac{\mu_z}{\lambda_h} \right) + k_2 \left( \frac{\mu_z}{\lambda_h} \right)^2 + k_3 \left( \frac{\mu_z}{\lambda_h} \right)^3 + k_4 \left( \frac{\mu_z}{\lambda_h} \right)^4$$  \hspace{1cm} \text{(3.6)}

where $\kappa_p = 1$ is the measured induced power factor at hover and $k_1 = -1.125$, $k_2 = -1.372$, $k_3 = -1.718$ and $k_4 = -0.655$.

3.2 Blade element theory (BET)

The blade element theory is used to calculate aerodynamic forces and moments acting on a blade. Each blade is divided into a number of independent thin strips, assuming that every strip acts as a two-dimensional airfoil. By finding the effective angle of attack for each strip, the forces and moments produced by this segment can be calculated. In order to obtain the total forces and moments for the blade, the increments have to be integrated from the root to the tip of the blade. Derived in this section is an expression for the rotor thrust $C_T$, which is the total force coefficient acting in the $z$-direction.

The velocity $U$ at the blade element may be divided into an in-plane velocity component $U_T$, tangential to the rotor and an out-of-plane velocity component $U_p$, perpendicular to the rotor:

$$U = \sqrt{U_T^2 + U_p^2}$$  \hspace{1cm} \text{(3.11)}

where

$$U_T = \Omega y + \mu_z \Omega R \sin \psi$$

$$U_p = (\mu_z + \lambda_i) \Omega R + y \beta(\psi) + \mu_x \Omega R \beta(\psi) \cos \psi$$  \hspace{1cm} \text{(3.12)}
The first term of the in-plane component $U_T$, is the blade rotation about the rotor shaft, while the second term is a free-stream (translational) part, which is zero in hover and axial flight. So are the last two terms in the out-of-plane component $U_P$, as they origin from perturbations in velocity at the blade element due to the flapping motion. The first term in $U_P$ represents the inflow velocity. For definition of velocities and aerodynamic forces, see figure 12.

The induced angle of attack for the blade element, also known as the inflow angle $\phi$, may be written as:

$$\phi = \tan^{-1}\left(\frac{U_P}{U_T}\right) = \frac{U_P}{U_T} \quad \text{for small angles of } \phi. \quad (3.13)$$

The effective angle of attack $\alpha$, is the difference between the pitch angle $\theta$, and the inflow angle $\phi$, that is:

$$\alpha = \theta - \phi = \theta - \frac{U_P}{U_T} \quad (3.14)$$

If $N_b$ is the number of rotor blades, the incremental thrust $dT$ for the rotor is given by:

$$dT = N_b dF_z \quad (3.15)$$

where

$$dF_z = dL \cos \phi - dD \sin \phi = dL \quad (3.16)$$

In equation (3.16) the approximation is made on the assumption that the inflow angle $\phi$ is small, leading to $\cos \phi = 1$ and $\sin \phi = 0$.

Figure 12. Velocities and aerodynamic forces acting on a blade element.
The increment in lift $dL$, may be written as a function of the local lift coefficient $C_l$, the local blade chord $c$, and the dynamic pressure $q = \frac{1}{2} \rho U^2$.

$$dL = \frac{1}{2} \rho U^2 c dy$$  \hspace{1cm} (3.17)

Using the expressions:

$$C_l = C_{l_u} \alpha = C_{l_u} (\theta - \phi) = C_{l_u} \left( \theta - \frac{U_p}{U_T} \right)$$  \hspace{1cm} (3.18)

$$r = \frac{y}{R}$$

together with equations (3.15)-(3.17) and the approximation $U = U_T$ (for small angles of $\phi$) gives the incremental thrust coefficient, $dC_T$:

$$dC_T = \frac{1}{2} \sigma C_{l_u} \left( \theta - \frac{U_p}{U_T} \right) \frac{U_T^2}{(\Omega R)^2} dr$$  \hspace{1cm} (3.19)

where the rotor solidity \( \sigma = \frac{N_c c}{\pi R} \).

The total thrust coefficient $C_T$, is obtained by integration of the incremental thrust coefficient $dC_T$, given in equation (3.19).

$$C_T = \frac{1}{2} \sigma C_{l_u} \int_0^1 \left( \theta - \frac{U_p}{U_T} \right) \frac{U_T^2}{(\Omega R)^2} dr $$  \hspace{1cm} (3.20)

Substituting $U_T$ and $U_p$ in equation (3.12) into equation (3.20) yields:

$$C_T = \frac{1}{2} \sigma C_{l_u} \int_0^1 \left( \theta (r + \mu_x \sin \psi)^2 - (\mu_z + \lambda_i) + \frac{r \beta}{\Omega} \right) \left( r + \mu_x \sin \psi \right) dr$$  \hspace{1cm} (3.21)

**Special case**: hover and axial flight:

$$C_T = \frac{1}{2} \sigma C_{l_u} \int_0^1 \left[ \theta r^2 - (\mu_z + \lambda_i) r \right] dr$$  \hspace{1cm} (3.22)

since $\mu_x \sin \psi = \frac{r \beta}{\Omega} = \mu_x \beta \cos \psi = 0$
4 CONTROL THEORY

As for most non-linear systems, the helicopter model may be written as a system of first order non-linear differential equations, that is:

\[
\begin{align*}
\dot{x} &= f(x, u) \\
y &= h(x)
\end{align*}
\]

(4.1)

where \(\dot{x}\) is the vector of state variables, \(u\) the vector of control inputs and \(y\) the vector of measured outputs. The process of synthesizing a linear state feedback controller for the model given in (4.1) is briefly described in the following sections.

4.1 Linearization

In order to be able to use linear control theory, the model has to be linearized. A system with constant control signals \(\bar{u}_0\) and corresponding constant state variables \(\bar{x}_0\), such that \(f(\bar{x}_0, \bar{u}_0) = 0\), is in equilibrium. The linearization is an approximation of the system at an equilibrium point, given by the first order Taylor expansion around that point:

\[
\begin{align*}
\dot{x} &= f(\bar{x}, \bar{u}) + \frac{\partial f}{\partial x} (x - \bar{x}_0) + \frac{\partial f}{\partial u} (u - \bar{u}_0) \\
y &= h(\bar{x}) + \frac{\partial h}{\partial x} (x - \bar{x}_0)
\end{align*}
\]

(4.2)

If \(\frac{\partial f}{\partial x} = A\), \(\frac{\partial f}{\partial u} = B\), \(\frac{\partial h}{\partial x} = C\) and \(h(\bar{x}_0) = \bar{y}_0\) the system may be written as:

\[
\begin{align*}
\dot{x} &= A (x - \bar{x}_0) + B (u - \bar{u}_0) \\
(y - \bar{y}_0) &= C (x - \bar{x}_0)
\end{align*}
\]

(4.3)

or

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]

(4.4)

for \(x = x - \bar{x}_0\), \(y = y - \bar{y}_0\) and \(u = u - \bar{u}_0\).

4.2 Reduction of states

It is often necessary to make simplifications of complex models. One way of reducing the order of the model is a technique called singular perturbation. By making a separation in time scale, the model is divided into slow and fast states. Typically, the slow states are considered dominant, while the fast states are seen as deviations from the slow behavior [7]. By replacing the dynamics...
of the fast states with the corresponding static relations, it may be possible to eliminate them and still retain the system’s stationary characteristics [8].

The reduction is carried out by splitting the states of the model into two groups, \( x_1 \) and \( x_2 \), where \( x_1 \) are the slow states to be kept and \( x_2 \) are the fast states to be eliminated:

\[
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
\]
\[
y =
\begin{bmatrix}
C_1 & C_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
\]

(4.5)

The \( x_2 \) states converge much faster than those of \( x_1 \). If their dynamics are neglected, i.e. if \( \dot{x}_2 = 0 \), \( x_2 \) may be written as a function of \( x_1 \):

\[
x_2 = -A_{22}^{-1}(A_{21}x_1 + B_2u)
\]

(4.6)

Substituting \( x_2 \) in equation (4.5) by the expression for \( x_2 \) in equation (4.6) yields:

\[
\begin{align*}
x_1 &= A_{11}^{-1} - A_{12}A_{22}^{-1}A_{21}x_1 + B_1u \\
y &= C_1x_1
\end{align*}
\]

(4.7)

where the reduced system matrices are given by:

\[
A_r = A_{11} - A_{12}A_{22}^{-1}A_{21}
\]
\[
B_r = B_1 - A_{12}A_{22}^{-1}B_2
\]
\[
C_r = C_1 - C_2A_{22}^{-1}A_{21}
\]

(4.8)

The approximation may be verified by comparing step responses and Bode diagram for both the original and reduced system [5]. It may also be a good idea to check if the eigenvalues of the original matrix \( A \) in equation (4.4) coincide with the eigenvalues for \( A_{11} \) and \( A_{22} \) in equation (4.5).

### 4.3 Splitting the model

Designing a control system for a flying vehicle often involves splitting the model into a longitudinal and a lateral part. This is done to ease the design work and in order to get a better structure of the design. If \( x_{\text{long}} \) and \( x_{\text{lat}} \) denotes the longitudinal and lateral states, the model may be written as:

\[
\begin{bmatrix}
\dot{x}_{\text{long}} \\
\dot{x}_{\text{lat}}
\end{bmatrix} =
\begin{bmatrix}
A_{\text{long}} & A_{\text{long lat}} \\
A_{\text{lat long}} & A_{\text{lat}}
\end{bmatrix}
\begin{bmatrix}
x_{\text{long}} \\
x_{\text{lat}}
\end{bmatrix} +
\begin{bmatrix}
B_{\text{long}} \\
B_{\text{lat}}
\end{bmatrix} u
\]
\[
\begin{bmatrix}
\dot{y}_{\text{long}} \\
\dot{y}_{\text{lat}}
\end{bmatrix} =
\begin{bmatrix}
C_{\text{long}} & C_{\text{lat}}
\end{bmatrix}
\begin{bmatrix}
x_{\text{long}} \\
x_{\text{lat}}
\end{bmatrix}
\]

(4.9)
where subscript longlat and latlong indicates the lateral dependency of the longitudinal states and vice versa.

The split is only possible if the cross coupling effects between the longitudinal and lateral parts are small, i.e. if the $A_{\text{longlat}}$ and $A_{\text{latlong}}$ matrices in equation (4.9) are close to zero. If so, the new models may be written as:

**Longitudinal model:**

$$
\dot{x}_{\text{long}} = A_{\text{long}} x_{\text{long}} + B_{\text{long}} u
$$

(4.10)

$$
y_{\text{long}} = C_{\text{long}} x_{\text{long}}
$$

**Lateral model:**

$$
\dot{x}_{\text{lat}} = A_{\text{lat}} x_{\text{lat}} + B_{\text{lat}} u
$$

(4.11)

$$
y_{\text{lat}} = C_{\text{lat}} x_{\text{lat}}
$$

The second step of the model approximation is verified by checking that the eigenvalues of the reduced $A_r$-matrix in equation (4.7)-(4.8) coincide with the $A_{\text{long}}$ and $A_{\text{lat}}$ matrices in equation (4.9). The magnitudes of the elements in the $A_{\text{longlat}}$ and $A_{\text{latlong}}$ matrices should also be relatively small.

### 4.4 Linear Quadratic Regulator (LQR)

One control method for systems with several inputs and outputs is the linear quadratic control. It is a modern design technique with good robustness and sensitivity properties, especially if all states are available for feedback [8]. Since the equations involved are numerically easy to solve, the technique is implemented in many applications, such as for instance in MATLAB. Both the plant to be controlled and the optimal control law are linear. The later is given by minimizing the quadratic cost function:

$$
J = \int_0^\infty (z - r)^T Q (z - r) + u^T R u \, dt
$$

(4.12)

where $z$ are the regulated signals, $r$ the reference signals and $u$ the control signals. The matrices $Q = Q^T \geq 0$ and $R = R^T > 0$ are penalty or weighting matrices used as design variables.

The behavior of the closed system is changed by different settings of the weighting matrices. If the elements in $Q$ are chosen large compared to those in $R$, the state variables will get a fast response. On the other hand, if the the elements in $R$ are large compared to those in $Q$ the states will not converge as quickly since the control signals are penalized harder.
If all states are available, no observer is needed and the optimal control law is, according to figure 13, given by:

\[ u = L_r r - L_x \]  

(4.13)

with the feedback gain \( L \):

\[ L = R^{-1}B^T S \]  

(4.14)

\( S \) is the unique, positive semi definite solution to the algebraic Riccati equation:

\[ A^T S + SA + M^T Q M - S B R^{-1} B^T S = 0 \]  

(3.15)

\( L_r \) is chosen so that the closed-loop static gain equals identity.

Figure 13. The Linear Quadratic Regulator.

5 THE HELICOPTER MODEL

This section gives an overview of the in-house helicopter model, such as coordinate systems used, helicopter states and control inputs and equations of motion to solve.

5.1 Coordinate systems

Four types of reference systems are used in the helicopter model. One hub fixed \((h)\) and one blade fixed \((b)\) which deal with the description of the main/tail rotor and the Bell-Hiller paddles. The other two, the earth fixed \((E)\) and the body fixed \((B)\) coordinate systems, handle position, velocity and orientation of the helicopter. For transformations between coordinate systems, see appendix A.
5.1.1 Earth fixed coordinate system

There are different ways of defining an earth fixed coordinate system, such as in geocentric, geodetic or flat-earth coordinates. The flat-Earth coordinate system (E) is used in this work. It is a Cartesian system with its origin $O_E$ at the surface [9]. The $x$-axis points north, the $y$-axis points east and the $z$-axis points vertically down according to figure 14.

The position and orientation of the helicopter are given in the earth fixed coordinate system. The position vector $(r_E)$ has three coordinates $(x_E, y_E, z_E)$ and the orientation vector $(\Omega_E)$, also referred to as Euler angles, consists of three consecutive rotations $(\Phi, \Theta, \Psi)$:

\[
\begin{align*}
    r_E &= [x_E \ y_E \ z_E]^T \\
    \Omega_E &= [\Phi \ \Theta \ \Psi]^T
\end{align*}
\]

(5.1)

In order to obtain unique angles for all orientations, the Euler angles are limited to:

\[
\begin{align*}
    -\pi &\leq \Phi \leq \pi \\
    -\frac{\pi}{2} &< \Theta < \frac{\pi}{2} \\
    -\pi &\leq \Psi \leq \pi
\end{align*}
\]

(5.2)

5.1.2 Body fixed coordinate system

The body fixed coordinate system ($B$), which moves with the helicopter, is used in the development of the helicopters’ equations of motions. This frame has its origin $O_B$ at the centre of gravity (c.g.), where the $x$-axis points forward, the $y$-axis points to the right when looking from behind and the $z$-axis points downward according to figure 14.

![Diagram of earth fixed and body fixed coordinate systems.](image-url)

Figure 14. Body fixed and earth fixed coordinate systems.
The helicopter velocity vector \( \mathbf{V}_B \), and angular velocity vector \( \mathbf{\omega}_B \), both relative to the air, are given in the body fixed coordinate system:

\[
\begin{align*}
\mathbf{V}_B &= [u \ v \ w]^T \\
\mathbf{\omega}_B &= [p \ q \ r]^T
\end{align*}
\]

\( (5.3) \)

### 5.1.3 Hub-fixed and blade-fixed coordinate systems

The blade- and hub-fixed coordinate systems are used in calculations of forces and moments acting on the blade or hub, respectively. The hub-fixed coordinate system \((h)\) has the same orientation as the body-fixed frame, i.e. the \(x\)-axis points forward, the \(y\)-axis points to the right and the \(z\)-axis points downward, see figure 15. The frame has its origin at the centre of the hub shaft in level with the attached blade. The blade-fixed coordinate system \((b)\) is a rotational frame that follows the blade. It has its origin located at the flap hinge where the \(x\)-axis points backwards, the \(y\)-axis points out of the hub in spanwise direction and the \(z\)-axis points upward, see figure 15.

![Comparison between hub-fixed and blade-fixed coordinate systems](image)

**Figure 15.** Comparison between hub-fixed and blade-fixed coordinate systems.
5.2 Helicopter states and control inputs

According to equation (4.4), the helicopter model is after linearization given in terms of

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]  

(5.4)

and has twenty-four states \(x\), and four control inputs \(u\).

The states are the flapping angles for the main and tail rotors and the Bell-Hiller paddles \(\beta\), their time derivatives \(\dot{\beta}\), the helicopter velocity \(V_B\), position \(r_E\), angular velocity \(\omega_B\) and Euler angles \(\Omega_E\). As control input, the collective, cosine and sine cyclic pitch angle for the swash plate \(\theta_{sw \cos}\) and \(\theta_{sw \sin}\) as well as the collective pitch angle for the tail rotor \(\theta_{tlr}\) are used.

Helicopter states \(x\), and control inputs \(u\):  

\[
\begin{align*}
x &= [\beta \ \dot{\beta} \ V_B \ r_E \ \omega_B \ \Omega_E]^T \\
u &= [\theta_{sw \cos} \ \theta_{sw \sin} \ \theta_{sw \ col} \ \theta_{tlr}]^T
\end{align*}
\]  

(5.5)

where 

\[
\begin{align*}
\beta &= [\beta_{mnr \ col} \ \beta_{tlr \ col} \ \beta_{mnr \ sin} \ \beta_{pad \ cos} \ \beta_{pad \ sin}]^T \\
\dot{\beta} &= [\dot{\beta}_{mnr \ col} \ \dot{\beta}_{tlr \ col} \ \dot{\beta}_{mnr \ sin} \ \dot{\beta}_{pad \ cos} \ \dot{\beta}_{pad \ sin}]^T \\
V_B &= [u \ \nu \ \omega]^T \\
r_E &= [x_E \ \ y_E \ \ z_E]^T \\
\omega_B &= [p \ \ q \ \ r]^T \\
\Omega_E &= [\Phi \ \Theta \ \Psi]^T
\end{align*}
\]  

(5.6)

Subscript \(mnr\) stands for main rotor, \(tlr\) for tail rotor, \(sw\) for swash plate and \(pad\) for paddles. \(col\), \(cos\) and \(sin\) are short for collective, cosine and sine. The helicopter velocity, \(V_B\), and angular velocity, \(\omega_B\), are given relative to the body-fixed frame while position, \(r_E\), and Euler angles, \(\Omega_E\), are given relative to the earth-fixed frame.

Note that the pitch angle for the swash plate \(\theta_{sw}\), is equal to the pitch angle for the blade \(\theta\) only for collective pitch angles. A sine cyclic pitch is represented by \(\theta_{sw \sin}\) and a cosine cyclic pitch is represented by a negative \(\theta_{sw \cos}\). They are shifted 90° according to the relationship given by equation (5.7):

\[
\begin{align*}
\theta_{cos} &= k \cdot \theta_{sw \ sin} \\
\theta_{sin} &= k \cdot (-\theta_{sw \ cos})
\end{align*}
\]  

(5.7)

where \(k\) is a factor due to the linkage between the swash plate and the blade.
### 5.3 Equations of motion

The helicopter model in equation (5.4) has been implemented in Simuskeldar [1]. In the simulative tool two trim routines are used, one for forward flight, hover and axial flight and another for turning. During trim of the helicopter the rigid body dynamics as well as the flapping of the blade and of the stabilizing bar have to be solved. Pitch dynamics and lagging motions are neglected.

In order to be able to solve the rigid body dynamics, the blade flapping motion has to be solved first. This as the rotor blades contributes to the total forces and moments acting on the helicopter. The forces and moments from the paddles are not included in the model, but due to the coupling between the paddles and the rotor blades, via linkage to the swash plate, the flapping equation of the paddles is solved along with the flapping motion of the blades. They are given by equation (5.8)-(5.9):

**Flapping motion of the blades:**

\[
\frac{1}{2} \left( e+2 \right) \Omega^2 \beta + \left( e+2 \right) \left[ p \cos(\Omega t) \right] - q \sin(\Omega t) \Omega = \frac{M_{b,x}^{aero}}{I_{b,xx}} \\
(5.8)
\]

**Flapping motion of the paddles:**

\[
\ddot{\beta}_{pad} + \Omega^2 \beta_{pad} = \frac{M_{pad,x}^{aero}}{I_{pad,xx}} \\
(5.9)
\]

where \( I_{b,xx} \) and \( I_{pad,xx} \) are moments of inertia, \( M_{b,x}^{aero} \) and \( M_{pad,x}^{aero} \) are moments around the hinge, i.e. the x-axes of the blade and paddle respectively.

According to BET, \( M_{b,x}^{aero} \) is given by:

\[
M_{b,x}^{aero} = \int -y \left[ \sin \theta (D \cos \phi + L \sin \phi) + \cos \theta (L \cos \phi - D \sin \phi) \right] dy \\
(5.10)
\]

Solving equation (5.8) gives expressions for the blade angle \( \beta \), the blade velocity \( \dot{\beta} \), and the blade acceleration \( \ddot{\beta} \). By applying Newton’s second law of motion to the blade, the z-component of the blade force \( F_{b,z} \), may be written as:

\[
F_{b,z} = m \ddot{\beta} \\
(5.11)
\]

The blade force \( F_{b,z} \), is then split into an aerodynamic force and a hinge force, \( F_{b,z}^{aero} \) and \( F_{b,z}^{hinge} \), according to:

\[
F_{b,z} = F_{b,z}^{aero} + F_{b,z}^{hinge} = m \ddot{\beta} \\
(5.12)
\]
Since the hinge force $F_{b,z}^{hinge}$, is the only unknown in equation (5.12), it is easily solved using the blade acceleration $\ddot{\beta}$ from above, together with the aerodynamic force $F_{b,z}^{aero}$, recognized from equation (5.10). That is:

$$F_{b,z}^{aero} = \int \left[ \sin \theta (D \cos \phi + L \sin \phi) + \cos \theta (L \cos \phi - D \sin \phi) \right] dy$$

(5.13)

Transformation of the hinge force $F_{b,z}^{hinge}$ to the body-fixed coordinate system, gives the main (or tail) rotor blade contribution to the total force acting on the helicopter. In order to obtain the rotor contribution, terms from each blade have to be summed up. The transformation and summation is done according to:

$$F_{B, rotor} = \sum_{b}^{N_b} A_{b2b} A_{b2h} \begin{bmatrix} 0 & 0 & F_{b,z}^{hinge} \end{bmatrix}^T$$

(5.14)

where $N_b = 2$ is the number of blades. Similarly, the main (or tail) rotor contribution to the total moment on the helicopter is expressed in body-fixed coordinate system given by:

$$M_{B, rotor} = \sum_{b}^{N_b} A_{b2b} A_{b2h} \begin{bmatrix} r_{b,0} \times \begin{bmatrix} 0 & 0 & F_{b,z}^{hinge} \end{bmatrix}^T + \begin{bmatrix} M_{b,x}^{hinge} & 0 & 0 \end{bmatrix} \end{bmatrix}^T$$

(5.15)

where the hinge moment $M_{b,x}^{hinge}$, is related to the elasticity of the blade according to:

$$M_{b,x}^{hinge} = -k\beta$$

(5.16)

Beside forces and moments from the rotors, the helicopter is affected by the gravitational force and by fuselage aerodynamic forces and moments, see figure 16.

![Figure 16. Forces and moments acting on the helicopter.](image-url)

The aerodynamic forces and moments are equal to zero in the original model, i.e. $F_{B, aero} = 0$ and $M_{B, aero} = 0$. 
Summation of the contributions gives the following expressions for the total force and the total moment around the centre of gravity acting on the helicopter.

\[
F_B = F_{B,mnr} + F_{B,aero} + F_{B,tlr} + mg_B
\]
\[
M_B = M_{B,mnr} + M_{B,aero} + M_{B,tlr} + (r_{B,cg} - r_{B,mnr}) \times F_{B,mnr} + (r_{B,cg} - r_{B,tlr}) \times F_{B,tlr}
\]

(5.18)

where \( r_{B,mnr} \) and \( r_{B,tlr} \) are positions of action points for the main and tail rotor forces.

The \( x \)-, \( y \)- and \( z \)-components of the total force and total moment in equation (5.18) represent the left hand side of the equations of motion for the helicopter, which according to Etkin [9] is given by:

\[
F_{B,x} = m(u + qw - rv)
\]
\[
F_{B,y} = m(v + ru - pw)
\]
\[
F_{B,z} = m(w + pv - qu)
\]
\[
M_{B,x} = I_{xx}(p - I_{yy}q^2 - r^2) - I_{xy}(r + pq) - I_{yx}(q - rp) - (I_{yy} - I_{zz})qr
\]
\[
M_{B,y} = I_{yy}q - I_{zx}(p^2 - r^2) - I_{xy}(p + qr) - I_{yx}(r - pq) - (I_{zz} - I_{xx})rp
\]
\[
M_{B,z} = I_{zz}r - I_{zx}(p^2 - q^2) - I_{yx}(q + rp) - I_{xy}(p - qr) - (I_{xx} - I_{yy})pq
\]

(5.19)

Solving equation (5.19), finally gives the helicopter velocity vector \( \mathbf{v}_B = [u \ v \ b]^T \), and the angular velocity vector \( \mathbf{\omega}_B = [p \ q \ r]^T \). The position of the helicopter \( \mathbf{r}_E = [x_E \ y_E \ z_E]^T \) is obtained by integration of the velocity vector \( \mathbf{v}_B \) transformed to earth-fixed coordinates, that is \( \mathbf{v}_E = [x_E \ y_E \ z_E]^T \). Similarly, the Euler angles \( \mathbf{\Omega} = [\phi \ \theta \ \psi]^T \) are given by integration of the Euler angle rates \( \dot{\mathbf{\Omega}} = \dot{\phi} \ \dot{\theta} \ \dot{\psi} \), which are transformations of the angular velocities in body-fixed coordinates \( \mathbf{\omega}_B \). Notice that a different transformation matrix is used for the angular velocity transformation, compared to the transformation of velocities. For further information on transformations between coordinate systems, see appendix A.

5.4 Induced velocity

The induced velocity used in the helicopter model is based on momentum theory and is given by equation (3.3)-(3.6). Since it is coupled to the thrust of the main rotor it is determined by iteration. An alternative inflow model according to Pitt and Peters has also been tested and evaluated by Hanson [2].

6 THEORY ON COMPUTATIONAL FLUID DYNAMICS (CFD)

The CFD calculation process can be separated into three different stages: preprocessing, solving and postprocessing. Given that a geometric configuration model is obtained, e.g. from a CAD program such as Catia, and that a corresponding mesh has been generated, preprocessing is the first step in the process. It involves discretization of the problem and setting up the flow boundary conditions and properties of the fluid material. Preprocessing is followed by the flow calculation, which includes applying a suitable algorithm for solving the equations of motion,
such as the Navier-Stokes equations for viscous flow and the Euler equations for inviscid flow. Finally, a postprocessor such as the visualization program EnSight is used for analysis of the results, as for instance integration of the pressure distribution to obtain the aerodynamic forces and moments.

6.1 Geometric configuration model

To begin with, the geometry of the object has to be generated. At Saab, the Ansys mesh generator ICEM [10] is used for that purpose, usually by re-arranging an imported CAD model. The geometry consists of surfaces, curves and points and in order to get a better structure of the model these have to be grouped into appropriate parts. For instance, a body, a tail, rotor axes and main/tail rotor disks may be appropriate parts for a helicopter, while a fuselage, left and right wing/canard and a fin may be appropriate parts for a fighter aircraft. It all depends on how complex the model is and what parts of the object that need separate solutions. The surfaces are given by the CAD model, while the curves and points have to be generated in ICEM by the user. A rule-of-thumb is that points are needed in intersections between curves or whenever there is a sharp corner. Curves are needed where the surface radius of a bend is small. The reason for having curves at large curvatures or at sharp edges is that they are used when setting up the mesh size, and it is always good to have a finer mesh where the pressure gradient may be large. Curves are also needed when splitting surfaces.

Before generating the computational mesh, a farfield and the material points LIVE and ORFN have to be defined. The farfield is usually a box or cylinder surrounding the model, marking the outer boundaries for the mesh. The LIVE represents the region within the farfield volume where the mesh is generated and the calculations subsequently are made, while the ORFN represents the dead volume inside the model, where no meshing (or calculations) is needed. The ORFN may be located arbitrary within the model and the LIVE may be located arbitrary outside the model but within the farfield. Figure 17 shows a rough draft of a helicopter model including a farfield and the material points LIVE and ORFN. A density box described in section 6.2 is also shown.

![Figure 17. A rough draft showing a helicopter model including a farfield, a density box and the material points LIVE and ORFN. Notice that the size of the helicopter is exaggerated for clarity.](image-url)
6.2 Mesh generation

In order to transfer the continuous flow into discrete counterparts, a mesh that enforces the model has to be generated. This mesh can either be structured or unstructured, as shown in figure 18. An unstructured mesh has the advantage of easier handling complex geometries, but in contrast to the structured mesh it needs a list of the connectivity, describing how the cells are connected. The structured mesh generally gives a more accurate solution of the flow field, but is often harder to generate for large and/or complex configurations. The actual points where the flow variables are calculated are called nodes. Their locations are usually cell-centered or vertex-centered, as shown in figure 18. For an unstructured mesh, calculations with vertex-centered nodes are much more time-efficient than calculations with cell-centered nodes, since the unstructured mesh contains more cells than vertices.

![Structured mesh](image1)

![Unstructured mesh](image2)

**Figure 18. Examples of structured and unstructured meshes.**

The size of the mesh has to be specified for all surfaces and curves, including the farfield. Note that not only may the size be given but also the width, i.e. the number of layers with maximum given size. In order to generate a finer mesh close to the object and thereby getting a better resolution of the flow field, density boxes surrounding the actual region is also created. Mesh size for these boxes are specified in the same way as for the surfaces and curves. An example of a density box is given in figure 17.

6.3 Preprocessing - Discretization

After building the model and generating the mesh, the problem has to be discretized. Techniques used in CFD are the finite difference method (FDM), the finite volume method (FVM) and, to some extent, the finite element method (FEM) originating from the field of structural mechanics. The FDM is the easiest method to implement, but it can only be used for simple geometries as it needs a structured mesh. Most widely spread is the FVM since it also handles unstructured meshes and thereby complex geometries. The FEM has the same geometric
flexibility as the FVM, but has the disadvantage of being mathematically rather complicated to implement. When it comes to accuracy, the FEM is better than the FVM.

The FVM is the discretization technique used at Saab. In this method, a control volume surrounding each node in the mesh has to be formed, see figure 19. This is done by the Edge preprocessor and used as an input to the solver. The control volume is in this case non-overlapping, but overlapping volumes exist for other codes. The fluxes are evaluated at the surfaces of each control volume. The philosophy of the method is that the flux is conserved, i.e. the flux leaving a given control volume is identical to that entering the same control volume from adjacent volumes. Both the preprocessor and the solver Edge are developed by FOI [11].

![Figure 19. Example of a control volume used in the finite volume method.](image)

### 6.4 Solving the equations of motion

Preprocessing is followed by the flow calculation, which includes applying a suitable algorithm for solving the equations of motion. Navier-Stokes equations are the fundamental equations for a fluid in equilibrium. They are based on expressions for conservation of mass, momentum and energy. For a three-dimensional flow, the Navier-Stokes equations can be written in a conservation differential form using a Cartesian coordinate system:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0 \quad \text{(mass conservation)}
\]

\[
\frac{\partial (\rho u_j)}{\partial t} + \frac{\partial (\rho u_i u_j)}{\partial x_j} + \frac{\partial p}{\partial x_j} \delta_{ij} - \frac{\partial \tau_{ij}}{\partial x_j} = f_i \quad \text{(momentum conservation)} \quad (6.1)
\]

\[
\frac{\partial (\rho E)}{\partial t} + \frac{\partial (\rho E + p) u_j}{\partial x_j} - \frac{\partial (u_i \tau_{ij})}{\partial x_j} + \frac{\partial q_j}{\partial x_j} = \mathbf{u} \cdot \mathbf{f} \quad \text{(energy conservation)}
\]

with density \( \rho \), velocity vector \( \mathbf{u} \) (components \( u_i \) or \( u_j \); \( i, j = 1, 2, 3 \)), total energy \( E \), pressure \( p \), Kronecker delta function \( \delta_{ij} \) (\( \delta_{ij} = 1 \) for \( i = j \) and \( \delta_{ij} = 0 \) otherwise), heat fluxes \( q_j \), stress tensor \( \tau_{ij} \) and body force \( \mathbf{f} \) (components \( f_i \)). Examples of body forces are gravitational, electric and magnetic forces. External forces generated by a propeller also fall into this category.
The stresses and the heat fluxes in equation (6.1) are given by:

\[
\tau_{ij} = \mu \left( \frac{\partial u_i}{\partial x_j} + \frac{\partial u_j}{\partial x_i} - \frac{2}{3} (\nabla u) \delta_{ij} \right)
\]

\[
q_j = -\kappa \frac{\partial T}{\partial x_j}
\]

(6.2)

where the thermal conductivity \( \kappa \) are found from the coefficient of viscosity \( \mu \) and a Prandtl number \( Pr \) according to:

\[
\kappa = \frac{\mu C_p}{Pr}
\]

(6.3)

Assuming a calorically perfect gas with the gas constant \( \gamma \), the equations (6.1) are closed by

\[
p = (\gamma - 1) \left[ \rho E - \frac{\rho u^2}{2} \right]
\]

(6.4)

If the Navier-Stokes equations are solved without any turbulence models, it is called a direct numerical simulation (DNS). These simulations have a very high computational cost, and exceed the capacity for today’s computers for most industrial applications. By time-averaging the Navier-Stokes system, approximate averaged solutions may be given with reasonable computer resources. These time-averaged equations are known as the Reynolds-average Navier-Stokes equations (RANS) and are described in Appendix B. Despite the simplification made, CFD calculations with RANS equations are still time-consuming, both when it comes to generating the mesh as well as performing the calculations.

Another possibility to reduce the complexity of the Navier-Stokes equations is to remove terms describing viscosity. If the viscosity is neglected, equations (6.1) are reduced to the inviscid Euler equations given by:

\[
\frac{\partial \rho}{\partial t} + \frac{\partial (\rho u_j)}{\partial x_j} = 0 \quad (\text{mass conservation})
\]

\[
\frac{\partial (\rho u_j)}{\partial t} + \frac{\partial (\rho u_j u_i)}{\partial x_i} + \frac{\partial p}{\partial x_j} \delta_{ij} = f_i \quad (\text{momentum conservation})
\]

(6.5)

\[
\frac{\partial (\rho E)}{\partial t} + \frac{\partial [\rho E + p] u_j}{\partial x_j} = u \cdot f \quad (\text{energy conservation})
\]

6.4.1 Modelling of the rotor

The flow field around a helicopter is complicated since it is strongly affected by the wake of the main rotor. This makes a numerical solution of the time-dependent Navier-stokes equations very costly. In order to reduce the computational complexity, three simplifications have been made when modelling the rotor. The first assumption is that the flow variables are well approximated
by the Euler equations given in (6.5). Secondly, the rotor is replaced by a thin rotor disk, also known as an actuator disk. The right hand side of the equations includes the rotor force $f$, which is zero everywhere in the flow field except in a region close to the rotor blades. By approximating the rotor by a thin disk, this body force is replaced by a surface force on the disk, which gives the following expression for the right hand side of the third equation in (6.5)

$$u \cdot f = \frac{1}{2} \left[ (u_{1u} + u_{1d}) f_1 + (u_{2u} + u_{2d}) f_2 + (u_{3u} + u_{3d}) f_3 \right]$$

(6.6)

where subscript $u$ and $d$ denote the values of the upstream and downstream sides of the rotor disk respectively. The last approximation made is time-averaging of the rotor force.

A propeller input file is created in order to handle the boundary condition for the rotor. This file contains rotor data such as the geometry and the advance ratio $J$ for the rotor as well as the blade pitch angle $\theta$. The sectional lift coefficient $C_l$ of the profile and the drag coefficient $C_d$ for the rotor blade are also included.

The advance ratio $J$ and the lift coefficient $C_l$ are calculated according to:

$$J = \frac{V_\infty}{N \frac{D}{60}}$$

(6.7)

$$C_l = C_{l_{\alpha}} (\theta - \phi)$$

(6.8)

where $V_\infty$ is the freestream velocity, $N$ is the number of revolutions per minute for a blade, $D$ is the rotor diameter, $C_{l_{\alpha}}$ is the 2-D lift curve slope of the airfoil sections and $\phi$ is the inflow angle shown earlier in figure 12. The propeller force $f = (f_1 f_2 f_3)$ in equation (6.5) is then given by the lift force coefficient, the drag force coefficient and the geometry of the blade according to a combined momentum and blade element theory.

The propeller/rotor model has been developed in-house at Saab [12] and is implemented in the solver Edge by Dr. Weinerfelt. The original intent was to use the model in the analysis of propeller-driven aircrafts.

### 6.5 Postprocessing

The last step in the CFD calculation process is the postprocessing. It involves analysis of the results such as visualization and integration of the pressure to obtain aerodynamic forces and moments. These forces and moments are often given as dimensionless coefficients as shown in section 6.5.1. Ensight [13] is the program used for postprocessing at Saab.
6.5.1 Integration of forces and moments

The total force acting on the helicopter is determined by a surface integration of the pressure coefficient and the unit normal vector, according to:

\[ F = \int \int_S C_p \cdot \hat{n} \, dS \]  \hspace{1cm} (6.9)

where \( S \) denotes the area of interest and \( dS \) denotes a segment area, see figure 20.

![Figure 20. Total helicopter area \( S \), with normal vector \( \hat{n} \) at segment \( dS \). \( r \) is the position vector for the segment and \( r_{ref} \) the reference point.](image)

Multiplying both sides of equation (6.9) with \( \times (r - r_{ref}) \), i.e. the difference between the position of the segment area and the reference point, gives the total moment for the helicopter.

\[ M = \int \int_S C_p \cdot [\hat{n} \times (r - r_{ref})] \, dS \]  \hspace{1cm} (6.10)

where the coordinates for the segment area and the reference point, both given in the body-fixed frame, are:

\[ r = \begin{bmatrix} x \\ y \\ z \end{bmatrix}, \quad r_{ref} = \begin{bmatrix} x_{ref} \\ y_{ref} \\ z_{ref} \end{bmatrix} \]  \hspace{1cm} (6.11)

Aerodynamic forces and moments are often given as dimensionless coefficients. The forces are normalized with the dynamic pressure \( q \), and a reference area \( S_{ref} \). In equation (6.12)-(6.13) the normal force coefficient \( C_N \), the tangential force coefficient \( C_T \) and the side force coefficient \( C_C \) are shown. In addition to \( q \) and \( S_{ref} \), the moments also need a reference length, \( c_{ref} \) or \( b_{ref} \), to be normalized. The longitudinal coefficient \( C_m \) is normalized with \( c_{ref} \), while the lateral coefficients, the yawing moment coefficient \( C_n \) and the rolling moment coefficient \( C_l \) are normalized with \( b_{ref} \). These coefficients are also given in equation (6.12)-(6.13).
7.1 Control design

In order to be able to work with both control design and CFD calculations at the same time, the original helicopter model, without CFD contributions to fuselage aerodynamic forces and moments, was used when synthesizing the lateral LQ state feedback controller. This section describes the process, starting with the linearization of the model.

7.1.1 Linearization

The linearization was made at a steady turn, with turn radius 60 m and forward helicopter velocity $\dot{x}_E = 20 \text{ m/s}$, given in the earth-fixed coordinate system. Under these flight conditions the helicopter trim bank angle was approximately 31 degrees. The linearization is considered to be accurate for bank angles between $31 \pm 3$ degrees. All trim states are given in Appendix C.

7.1.2 Reduction of states

The rotor states for a real helicopter, i.e. the flapping angles and their time derivatives, are non-measurable. One option is to make them available for feedback by estimations of an observer. Another option is to eliminate them from the model according to chapter 4.2. The elimination was chosen in this project, since the rotor states are faster than the other states and because it simplifies a split into longitudinal and lateral parts. The later due to cross coupling effects being avoided.

After the reduction of states a model validation was made, see appendix D. Eigenvalues of the original $A$-matrix in equation (4.4.) and the $A_{11}$ and $A_{22}$ matrices in equation (4.5) were compared, showing good agreement for fifteen of twenty-four eigenvalues (the fast modes having a better fit than the slow ones). Step responses in MATLAB, Simulink also indicated a good match between the original and reduced model for all input/output combinations, except for the roll rate $p$ with pitch command in $\theta_{\text{sw,cos}}$ and $\theta_{\text{sw,sin}}$. Over all, these results were considered sufficient for elimination of the fast states.
7.1.3 Splitting the model

Twelve states remain after reducing the model. Six of them are longitudinal and the other six are lateral, as specified below. The control signals for the longitudinal model are the collective and the cosine cyclic pitch angle for the swash plate, while the collective pitch angle for the tail rotor together with the sine cyclic pitch angle for the swash plate are the lateral control signals.

Helicopter states and control signals for the longitudinal model:

\[ x_{\text{long}} = [u \ w \ x \ z \ q \ \Theta]^T, \quad u_{\text{long}} = [0_{\text{sw,cos}} \ 0_{\text{sw,col}}]^T \] (7.1)

Helicopter states and control signals for the lateral model:

\[ x_{\text{lat}} = [v \ y \ p \ r \ \Phi \ \Psi]^T, \quad u_{\text{lat}} = [0_{\text{sw,sin}} \ 0_{\text{dr}}]^T \] (7.2)

As shown in appendix E, the magnitudes of the matrices \( A_{\text{longlat}} \) and \( A_{\text{latlong}} \) in equation (4.9) are fairly close to zero. Furthermore, the eigenvalues of the \( A_{r} \) matrix in equations (4.7)-(4.8) are close to those of matrices \( A_{\text{long}} \) and \( A_{\text{lat}} \) in equation (4.9). This means that the longitudinal and lateral cross coupling effects may be considered as small, and that the new models can be written as:

**Longitudinal model:**

\[
\begin{align*}
\dot{x}_{\text{long}} &= A_{\text{long}} x_{\text{long}} + B_{\text{long}} u \\
y_{\text{long}} &= C_{\text{long}} x_{\text{long}} + D_{\text{long}} u
\end{align*}
\] (7.3)

**Lateral model:**

\[
\begin{align*}
\dot{x}_{\text{lat}} &= A_{\text{lat}} x_{\text{lat}} + B_{\text{lat}} u \\
y_{\text{lat}} &= C_{\text{lat}} x_{\text{lat}} + D_{\text{lat}} u
\end{align*}
\] (7.4)

7.1.4 Linear-Quadratic Regulator

As the helicopter model is split into a longitudinal and a lateral part, a controller for each part is needed. The task for the longitudinal controller is to keep a constant forward velocity \( u \) at a constant altitude \( z \). Consequently, these are the reference signals of the longitudinal helicopter model. The position in \( x \)-direction varies as the helicopter flies forward and since the LQ-controller tries to keep states without a reference value close to zero, the \( x \)-position is removed from the longitudinal states. Further information on the longitudinal controller is given by Hanson [2].
The bank angle $\Phi$, and the velocity component $v$, are chosen as reference signals in the lateral helicopter model in order to perform a turn. The component $v$ is always kept zero, while the bank angle may vary resulting in a different turn radius. In the same way as for the longitudinal controller, the y-position and the Euler angle $\Psi$, are removed from the lateral states in order to ease the design since they vary in a turn. A rough draft of the regulator is shown in figure 21, while figure 47 in Appendix F shows the Simulink scheme.

![Figure 21. Linear quadratic regulator for the longitudinal and lateral helicopter model.](image)

Initially, the lateral model with four states was used for determining the lateral weighting matrices mentioned in equation (4.12). Since both the velocity in y-direction and the bank angle are reference signals they are the only states chosen to be penalized in the $Q$-matrix ($Q_{11}$ corresponds to $v$ and $Q_{44}$ to $\Phi$). The $Q$ and $R$ matrices were then given by:

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}, \quad R = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \quad (7.5)$$

A three degree step in bank angle was tested in MATLAB, Simulink on the lateral model with four states. It gave a fast step response with no overshoots. Due to the good result, the lateral controller together with the longitudinal controller given by Hanson [2] were tested on the original twenty-four state model. This time the system indicated a very oscillatory behavior for several of the states, which is not acceptable. Comparison of step responses for the lateral and the original twenty-four state model, with weighting matrices according to (7.5), is shown in figure 48, appendix F.
To improve the lateral controller, the components of the weighting matrices had to be changed. When the penalty on velocity component $v$ was increased, the oscillations declined for all states but gave a bank angle of 3.6 degrees instead of 3 degrees. When the bank angle was penalized harder the system became unstable, which may be the result if some of the states are driven to extreme values. For instance, by increasing the penalty on bank angle to 5, the flapping derivative $\beta_{mr,sin}$ reached 700 degrees/s after just 5 seconds. $Q_{11}$ was thereby increased to 100 000 while $Q_{44}$ remained 1.

Since the control signal $u_1 = \theta_{sw,sin}$ is coupled to $\Phi$, the offset in bank angle was adjusted by increasing the penalty on $\theta_{sw,sin}$ to 100, with the disadvantages of getting a slower system. The weighting matrices were finally given according to:

$$Q = \begin{pmatrix} 100000 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}, \quad R = \begin{pmatrix} 100 & 0 \\ 0 & 1 \end{pmatrix}$$

(7.6)

The comparison of step responses for the lateral and the original twenty-four state model, with weighting matrices according to (7.6), is shown in figure 49, appendix F. Steps, other than three degrees in bank angle, were also tested showing similar results.

7.2 Computational fluid dynamics

This section describes the CFD calculation process from building the model to solving the equations of motion and postprocessing the results. The implementation of the aerodynamic coefficients resulting from the simulations in simuskelda is also discussed.

7.2.1 Building the model and generating the computational mesh

Since no CAD model was available for Skeldar, the geometry was based on body cross section coordinates from a MATLAB file. When importing the file, curves for each cross section as well as one surface for the whole helicopter were automatically created. This in contrast to importing a CAD model, as it gives only the surfaces. The cross sections were delivered rather dense, so a lot of the cross section curves had to be removed. Some curves were added in the longitudinal direction, wherever the surface radius of a bend were small. Points were added in the intersection between the cross section curves and the new longitudinal curves. The surface was split into two parts; a body and a tail. This body/tail-configuration was used as a reference model in comparisons with earlier calculations made at Saab. One further model, with the main rotor included, was made for studying the rotor downwash effects. The main rotor was created using two cylinders, one as the rotor axis and the other as the rotor disk. The propeller disk was later split once again into an upper and a lower surface in order to be able to set up the boundary conditions defining the velocity of the flow passing through the disk as well as the direction of the flow. The tail rotor was not included in the model. The geometric configuration model with the main rotor is shown in figure 22-23.
7.2.2 Setting up boundary conditions for the helicopter

In order to handle the boundary condition for the rotor, a propeller input file was created for all cases presented in Table 2. Among others, this file contains the advance ratio $J$ and the sectional lift coefficient $C_l$ given in equation (6.7) and (6.8) respectively, as well as the drag coefficient $C_d$.

The advance ratio was calculated for the freestream velocity $V_{\infty} = 20$ m/s at a rotational speed of the rotor $N = 5500$ rpm. In order to calculate the sectional lift force coefficient, the 2-D lift curve slope $C_{l_{u}}$, the blade pitch angle $\theta$ and the inflow angle $\phi$ were needed. The 2-D lift curve slope was chosen as $C_{l_{u}} = 5.7$ according to wind tunnel measurements by Schaefer & Smith [14]. Since the pitch angle varies as the blade revolves, the collective pitch angle $\theta_{col}$ was chosen as a mean value for the blade pitch angle $\theta$. It was given from simulations with simuskeldar. The inflow angle $\phi$ was calculated according to equation (3.13), neglecting all terms including the azimuth angle $\psi$. That is, the inflow angle was, just like the blade pitch angle, approximated by its value for the hovering case. The lift force coefficient was determined for in total eleven

Figure 22. Curves and points in the geometric helicopter model.

Figure 23. Surfaces in the geometric helicopter model.
sections of the blade. Finally, the drag coefficient \( C_d \) was based on empirical data for a hovering helicopter, according to Carpenter [15].

### 7.2.3 Solving and postprocessing

In order to get a good overview of the flow field around the helicopter several cases with different angles of attack \( \alpha \) and angles of sideslip \( \beta \) were calculated. The angle of attack is defined positive when the wind is coming in from below, while the angle of sideslip is defined positive for wind coming in on the right hand side, see figure 24. Note that a -90° angle of attack with no sideslip represents the hovering case. The investigated cases are gathered in Table 1-2.

For every converged case, aerodynamic forces and moments acting on the helicopter were calculated. In order to do that, an appropriate reference point \( \mathbf{r}_{\text{ref}} = [x_{\text{ref}} \quad y_{\text{ref}} \quad z_{\text{ref}}] \), had to be determined. For this project, the intersection between the main and tail rotor axis was chosen, see figure 25. Furthermore, the helicopter cross section area was chosen as reference area \( S_{\text{ref}} \), while the body length and body width were chosen as reference lengths \( c_{\text{ref}} \) and \( b_{\text{ref}} \), see figure 25.
For comparison, aerodynamic forces and moments were also calculated for another reference point in order to match earlier calculations made at Saab. This reference point is located 10 cm below the chosen reference point.

In the following two sub-sections, the results from the calculations are presented, starting with the configuration without rotor and ending with the configuration with the main rotor added.

### 7.2.3.1 Helicopter configuration without the main rotor

Table 1 shows the calculated cases for the rotor-less configuration. These cases were chosen in order to verify data obtained from earlier calculations at Saab, with a structured mesh and a cylindrical farfield. The freestream velocity $V_\infty$ was set equal to the helicopter velocity at the given linearization point, i.e. 20 m/s.

<table>
<thead>
<tr>
<th>$\alpha$ [$^\circ$]</th>
<th>$\beta = -30^\circ$</th>
<th>$\beta = -25^\circ$</th>
<th>$\beta = -20^\circ$</th>
<th>$\beta = -15^\circ$</th>
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**Table 1. Calculated cases for the rotor-less configuration.**

All CFD calculations for the cases listed above converged, but it turned out that the unstructured grid had to be much finer than the structured grid in order to obtain the same resolution. Otherwise, pressure distribution plots from calculations with the structured/unstructured grids were similar. Comparisons of the aerodynamic coefficients on the other hand showed differences between the two grids. Especially the forces did not coincide so well. Since the total force acting on the helicopter body is determined by a surface integration of the pressure coefficient, the forces should at least have the same order of magnitude if the pressure distributions are alike. A control check of the pressure coefficients were made by plotting the values at different cross sections. Even these plots showed a good match between calculations with different grids. Furthermore, all parameters and settings in the indata files to the solver, as well as the scripts for generating the aerodynamic coefficients, where examined. No explanation, other than using different grids and farfields, were found for the differences between the aerodynamic coefficients. Figure 26-29 show a comparison of the pressure distributions from calculations with the structured and the unstructured grids, for a representative case ($\alpha = -30^\circ$, $\beta = 0^\circ$). Plots of the
aerodynamic coefficients (all $\alpha$, $\beta = 0^\circ$ and $\beta = 10^\circ$) along with pressure distribution plots for an additional case ($\alpha = 30^\circ$, $\beta = 0^\circ$) are presented in Appendix G.

Figure 26. Pressure distribution from a calculation with the structured grid at $\alpha = -30^\circ$, $\beta = 0^\circ$. View from above.

Figure 27. Pressure distribution from a calculation with the unstructured grid at $\alpha = -30^\circ$, $\beta = 0^\circ$. View from above.
Figure 28. Pressure distribution from a calculation with the structured grid at $\alpha = -30^\circ$, $\beta = 0^\circ$. View from below.

Figure 29. Pressure distribution from a calculation with the unstructured grid at $\alpha = -30^\circ$, $\beta = 0^\circ$. View from below.
7.2.3.2 Helicopter configuration with the main rotor

Table 2 presents the calculated cases for the main rotor configuration. These are fewer than those in Table 1, but they are still considered sufficient to accomplish an update of the helicopter model. The calculations at positive angles of sideslip $\beta$ are new. They were considered since the main rotor wake gives an asymmetric flow field around the helicopter, which make an extrapolation in $\beta$ difficult. As for the calculations with the rotor-less configuration, the freestream velocity was set to 20 m/s.

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Table 2. Calculated cases for the main rotor configuration (bold cross denotes convergence).

Since the boundary condition for the rotor originally was written for propeller-driven aircrafts, the orientation of the helicopter coordinate system turned out to cause problems. Initially, calculations at $\alpha = -90^\circ$, $\beta = 0^\circ$, i.e. a vertical climb, did not converge. This even though vertical climb for a helicopter is equivalent to forward flight for an aircraft, from a rotor/propeller point of view. The calculation failed because the rotor axis had to be oriented in the x-direction. In order to fulfil this criterion, the helicopter coordinate system was rotated.

Even with the rotated helicopter coordinate system, there were problems with the convergence. At zero angle of attack none of the solutions converged, due to the velocity component perpendicular to the rotor being zero. At other angles of attack, some cases converged after modification of the CFL-number. Reducing the CFL-number gives a smaller time step. A smaller time step results in a more stable calculation, but it increases the computational cost. The CFL-number was originally set to CFL = 0.8. For some of the restarted cases, it was set as low as CFL = 0.3. The convergent cases are denoted by a bold cross in table 2.

There were also some uncertainties about how to set up the boundary conditions for the rotor. The directions of the rotor axis and the rotation with respect to the rotor axis, as well as the definition of upstream and downstream side of the rotor disk, have to be given. For a climb, there are no doubt how to define these properties. They are set equally to a propeller-driven aircraft, as shown in figure 30. For a descent, on the other hand, they have to be switched according to
figure 31. The switch is carried out in order to obtain a correct pressure distribution on the rotor disk and a correct direction of the flow.

![Diagram](image)

**Figure 30.** Draft of the helicopter in a climb, showing positive directions of the rotor axis and the rotor rotation. The definition of upstream and downstream side of the rotor disk is also shown.

![Diagram](image)

**Figure 31.** Draft of the helicopter in a descent, showing positive directions of the rotor axis and the rotor rotation. The definition of upstream and downstream side of the rotor disk is also shown.

Figure 32-33 show the pressure distribution for a calculation at $\alpha = -30^\circ$, $\beta = 0^\circ$, i.e. the same case as presented for the rotor-less configuration in figure 26-29 above. Noticeable is the higher pressure on top of the helicopter nose and tail, due to the downwash from the main rotor. The appearance of the pressure distribution is confirmed by measurements in forward flight made by Leishman & Bi [16] in 1994, which show a significant increase in static pressure at these regions. The increase is caused by high dynamic pressure due to the induced velocity at the rotor disk. In Appendix H, the pressure distribution for an additional case ($\alpha = -30^\circ$, $\beta = 0^\circ$) is given. Even for this case, the pressure on the upper side of the helicopter body is higher due to the downwash, compared to the configuration without the main rotor.
Figure 32. Pressure distribution from a calculation with the main rotor configuration at $\alpha = -30^\circ$, $\beta = 0^\circ$. View from above.

Figure 33. Pressure distribution from a calculation with the main rotor configuration at $\alpha = -30^\circ$, $\beta = 0^\circ$. View from below.
Aerodynamic coefficients were obtained for all converged cases. As expected, these coefficients differ from the coefficients obtained from calculations with the rotor-less configuration. Comparisons at two different sideslip angles, $\beta = -10^\circ$ and $\beta = 0^\circ$, are presented in Appendix I. Appendix I also present the aerodynamic coefficients from calculations with the main rotor configuration at sideslip angle $\beta = 10^\circ$.

Especially interesting are the results at zero sideslip angles. Normally, the lateral coefficients, i.e. the side force $C_C$, the yawing moment $C_n$ and the rolling moment $C_l$, are zero at zero sideslip. Due to the asymmetry of the flow field caused by the rotor wake, this is not true for the main rotor configuration. The solutions at $\alpha = 0^\circ$, $\beta = 0^\circ$ did not converge, since there is no flow passing through the rotor disk, but the estimations indicate a positive side force, a positive rolling moment and a negative yawing moment for this case, see figure 34. According to measurements of the static pressure made by Leishman & Bi [16], large suction peaks are obtained on both sides of the helicopter fuselage since the flow accelerates over the sides. As the suction pressure on the retreating (port) side of the body is higher than the one on the advancing side, the result is a positive side force, which coincides with the sign of the side force given in figure 34. However, the magnitude can not be verified, since it is not given in the reference. The measurements also show that the peaks are located at the aft body, which should result in a positive yawing moment, i.e. the opposite sign of the yawing moment presented in this report. With increased tip speed ratio, the suction peaks are expected to move further aft along the body because the rotor wake is skewed back.

Figure 34. Lateral coefficients obtained from calculations with the main rotor configuration at $\beta = 0^\circ$. 
Furthermore, the estimates for the main rotor configuration at $\alpha = 0^\circ$, $\beta = 0^\circ$ indicate a positive normal force and a negative (nose-down) pitching moment, see figure 35. These results are harder to verify since the sign of the coefficients is highly affected by the disk loading and the tip speed ratio parallel to the rotor disk. Measurements by Leishman & Bi [16] show that the download on the fuselage decreases as the tip speed ratio goes up. When the tip speed ratio is high enough, a positive lift force is obtained. For the mentioned measurements, this occurs at tip speed ratios between $\mu_x = 0.1$ and $\mu_x = 0.125$, depending on the disk loading. The trend of increasing lift force with increased tip speed ratio has also been observed by Smith & Betzina in 1986 [17]. Similarly to the lift force, the pitching moment also change sign with increasing tip speed ratio from a positive moment in hover to a nose-down moment in forward flight. The influence of varying disk loading however is much higher for the pitching moment than for the lift force. At low disk loadings, the transition occurs at tip speed ratio $\mu_x = 0.075$ while $\mu_x = 0.15$ is the transition point for higher loads. As no comment was given on the effect of using different fuselage shapes and sizes, it is difficult to anticipate when the transition in lift force and pitching moment for the Skeldar helicopter should occur. The actual tip speed ratio for the calculation at $V_{\infty} = 20$ m/s and $\alpha = \beta = 0^\circ$ is $\mu_x = 0.1$.

Figure 35. Longitudinal coefficients obtained from calculations with the main rotor configuration at $\beta = 0^\circ$. 
7.2.4 Implementation of aerodynamic forces and moments

The purpose of the CFD calculations was to update the model with fuselage aerodynamic forces and moments as they are not included in the original model. In order to do that, curves were fitted to the calculated points in Table 2 for all aerodynamic coefficients. These curves where digitalized and later used as an input to simuskeldar, when determining the system matrix A for the updated model. They were also used for trimming the helicopter. For cases with missing solutions, the coefficients were determined by interpolation or, in absence of nearby \( \alpha \)-values, estimated by looking at the \( \alpha \)-trend at other sideslip angles. The curve fitting is shown in Appendix I.

The angle of attack and the angle of sideslip are not included in the helicopter model. By using the following definition:

\[
\begin{align*}
  u &= V_\infty \cdot \cos \alpha \cdot \cos \beta \\
  v &= V_\infty \cdot \sin \beta \\
  w &= -V_\infty \cdot \sin \alpha \cdot \cos \beta
\end{align*}
\]

(7.7)

the angles may be transformed to velocity components, which give a better representation of the coefficients since \( u, v \) and \( w \) are helicopter states. As the original helicopter model is analytic, the coefficients have to be delivered to simuskeldar as a perturbation model for a given equilibrium point, according to:

\[
\begin{bmatrix}
  C_x \\
  \frac{\partial C_x}{\partial u} \\
  \frac{\partial C_x}{\partial v} \\
  \frac{\partial C_x}{\partial w}
\end{bmatrix}
\]

(7.8)

where \( C_x \) is the aerodynamic coefficient at this point and \( \frac{\partial C_x}{\partial u}, \frac{\partial C_x}{\partial v} \) and \( \frac{\partial C_x}{\partial w} \) are the local coefficient derivatives with respect to the velocity components. These derivatives are calculated by using coefficient values at nearby points, both in the negative and in the positive \( u, v \)- and \( w \)-directions. That is:

\[
\begin{align*}
  \frac{\partial C_x}{\partial u} &= \frac{C_x (u + \Delta u) - C_x (u - \Delta u)}{2\Delta u} \\
  \frac{\partial C_x}{\partial v} &= \frac{C_x (v + \Delta v) - C_x (v - \Delta v)}{2\Delta v} \\
  \frac{\partial C_x}{\partial w} &= \frac{C_x (w + \Delta w) - C_x (w - \Delta w)}{2\Delta w}
\end{align*}
\]

(7.9)

where \( \Delta u, \Delta v \) and \( \Delta w \) are small steps from the given equilibrium point. Before the coefficients and the derivatives in (7.8) are delivered to simuskeldar, they are normalized with the rotor tip speed (\( \Omega R \)) instead of the freestream velocity \( V_\infty \). This normalization has been considered in order to obtain forces and moments even if the freestream velocity approaches zero.
7.3 Simulations

In order to investigate the differences between the original helicopter model and the updated model with CFD contributions to fuselage aerodynamic forces and moments, simulations in MATLAB, Simulink were made. Step responses were studied for both the open and the closed system, when the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity \( v_E = 20 \) m/s. This flight condition was chosen as it was used in the linearization of the model when designing the LQ controller. The velocity also corresponds to the freestream velocity given in the CFD calculations. For the open system, hover was chosen as an additional trim condition as the original helicopter model has been validated using flight tests at Saab with good results for the hovering case.

7.3.1 Open systems

Comparisons of step responses from simulations with the original and the updated model were made for all input/output combinations, excluding the rotor states. Before the pitch commands were applied, the helicopter was trimmed for two flight conditions; hover and a steady turn as mentioned above. The step size was set to two degrees while the simulations were carried out for one or three seconds, depending on the initial trim condition. Figure 61-64 in Appendix J show the comparison of step responses for the hovering case, while figure 65-68 show the comparison of step responses for a steady turn. Step responses for some states are also shown in section 7.3.2.1-7.3.1.2 below.

7.3.1.1 Hover

The behavior of the two models are similar for steps in \( \theta_{SW,cos} \), except for the yaw rate \( r \) and the yaw angle \( \psi \). When a positive cosine pitch command is applied, the rotor tilts forward and the velocity component \( u \) increases. As a side effect, the helicopter starts to yaw with an increasing velocity component \( v \). The increase in \( v \) gives a positive increasing angle of sideslip \( \beta \). In the updated model, this increase in \( \beta \) decreases the yawing moment. A negative increment in yawing moment counteracts the ongoing positive yawing motion with a smaller magnitude of \( r \) and \( \psi \) as a result, see figure 36. The updated model is thereby more damped in yaw than the original model, when a step in \( \theta_{SW,cos} \) is given.

![Figure 36. Step responses for a cosine pitch command of 2°. The states v, r and \( \psi \) are shown.](image-url)
The yaw motion mentioned above, occurs for any given pitch command of the main rotor swash plate. It is a result of a change in rotor moment due to the different pitch settings. As the moment generated by the tail rotor does not change, the helicopter start to rotate.

Similarly to a cosine pitch command, a step in $\theta_{sw,sin}$ gives discrepancies only in $r$ and $\psi$. When a positive sine pitch command is applied, the rotor tilts to the left and the velocity component $v$ increases. As $v$ is negative the helicopter experiences a negative angle of sideslip $\beta$, which corresponds to a positive yawing moment in the updated model. With a negative ongoing yaw motion, it is damped by the positive increment in yawing moment with a smaller magnitude of $r$ and $\psi$ as a result, see figure 37. It is thereby concluded that the updated model is more damped in yaw than the original model, even when a step in $\theta_{sw,sin}$ is given.

![Figure 37. Step responses for a sine pitch command of 2°. The states $v$, $r$ and $\psi$ are shown.](image)

For a step in $\theta_{sw,col}$, the differences between the two models are in velocity components $u$ and $v$, position $x$, pitch rate $q$ and pitch angle $\Theta$. When applying a positive collective pitch command, the helicopter climbs vertically and the velocity component $w$ increases. In the updated model, the velocity components $u$ and $v$ are close to zero, compared to the original model which moves slightly backward and to the right, see figure 38. The reason for $q$ being more negative in the updated model is due to a negative increment in pitching moment. As a result $\Theta$ becomes smaller in comparison to the original model. Since data for a vertical climb, i.e. $\alpha = -90^\circ$, is not included in the model, the coefficients are taken from data at the nearest angle of attack (in this case $\alpha = -30^\circ$). As the span between $\alpha = -90^\circ$ and $\alpha = -30^\circ$ is large, this approximation is probably not good enough. New calculations at higher angle of attack are needed to cover this region of the flight envelope.

![Figure 38. Step responses for a collective pitch command of 2°. The states $u$, $v$, $q$ and $\Theta$ are shown.](image)
Comparisons of step responses for the original and the updated model when a tail pitch command is applied show an almost exact match of all states. With a positive step in $\theta_{tlr}$, the force produced by the tail increases and the helicopter yaws to the left. As the velocity components $v$ and $w$ hardly changes, the angle of attack and the angle of sideslip obtained at the trimmed condition are maintained. No increments in forces and moments are thereby given in the updated model, which results in behaviors of the two models that are nearly the same. The initial conditions on the other hand differ between the models. It is not shown in the plots though, as they have been adjusted to the same level. The adjustment has been done for all pitch commands in order to facilitate the comparison of the simulations.

### 7.3.1.2 Steady turn

For a steady turn, the states in the updated model diverge much faster than the states in the original model when a pitch command is applied. As a result, the simulations were only carried out for one second for this flight condition. The analysis of step responses also becomes more complex as discrepancies are shown for additional states compared to the hovering case.

In contrast to applying a cosine pitch command in hover, the step in a steady turn is not only affecting $u$ and $v$ but also $w$, see figure 39. The increasing velocity components give a change in both angle of attack and sideslip angle, from being initially zero to approximately $\alpha = -11^\circ$ and $\beta = -2.5^\circ$ at $t = 1$ s. In the updated model a negative $\beta$ corresponds to a positive increment in yawing moment, which increases the magnitude of $r$ as the ongoing yaw motion is positive. The yaw angle $\psi$ on the other hand becomes negative at first, but if the simulations are carried out for a longer period of time it eventually changes sign and becomes larger compared to the original model. This phenomenon is not shown in the plots. When a cosine pitch command is applied, the helicopter initially experiences a nose-down moment due to the forward rotor tilt. As the pitch rate is positive from the beginning, this moment reduces the pitch rate, which finally becomes negative. Due to an additional negative pitching moment in the updated model, the pitch rate becomes even more negative. This trend is not shown in the hovering case as a step in $\theta_{sw,cos}$ does not change the angle of attack.

![Figure 39. Step responses for a cosine pitch command of 2°. The states $v$, $w$, $r$, $\psi$, $q$ and $\Theta$ are shown.](image-url)
When a positive sine pitch command is given, the helicopter rolls to the left with increasing (negative) velocity components $v$ and $w$ and a decreasing bank angle $\Phi$. As for the cosine pitch command, a positive increment in yawing moment is obtained, which increases the magnitude of $r$ and $\psi$, as shown in figure 40. Furthermore, due to a negative angle of attack the decline in pitch rate is reinforced by a negative pitching moment in the updated model. The reduction of $q$ is not as fast as in figure 39 though, due to a smaller (negative) angle of attack.

![Figure 40](image-url). Step responses for a sine pitch command of $2^\circ$. The states $v$, $w$, $r$, $\psi$, $q$ and $\Theta$ are shown.

When applying a positive collective pitch command, the helicopter usually climbs vertically and the velocity component $w$ increases. In a steady turn, the given pitch command also increases the velocity component $v$. The increased velocity components, give a negative angle of attack and a negative angle of sideslip. As mentioned before, these angles correspond to a negative increment in pitching moment and a positive increment in yawing moment in the updated model. The result is larger magnitudes of $r$, $\psi$ and $\Theta$ and a negative increment in $q$ as shown in figure 41.

![Figure 41](image-url). Step responses for a collective pitch command of $2^\circ$. The states $v$, $w$, $r$, $\psi$, $q$ and $\Theta$ are shown.
For a step in $\theta_{tlr}$, differences between the two models are shown in velocity component $v$, yaw rate $r$, yaw angle $\psi$ and pitch angle $\Theta$. When a positive pitch command is applied, the force produced by the tail increases and the helicopter yaws to the left. As the helicopter already is performing a right turn, the additional yaw motion decreases the yaw rate which eventually becomes negative. In the updated model, the yaw rate becomes even more negative as shown in figure 42, due to the negative increment in yawing moment at positive angles of sideslip.

![Figure 42. Step responses for a tail pitch command of 2°. The states $v$, $r$ and $\psi$ are shown.](image)

### 7.3.2 Closed systems

The original and the updated model were also compared from a controlling perspective by testing the lateral controller described in section 7.2 together with the longitudinal controller given by Hanson [2]. All twenty-four helicopter states were included in the models but only nine of them were available for feedback. The other states were only observed. A three degree step in bank angle was given in all simulations. As for the open system in hover, the simulation time was set to three seconds. Comparisons were carried out for one flight condition, a steady turn with turn radius 60 m and forward helicopter velocity $\dot{x}_E = 20 \text{ m/s}$. Figure 69-71 in Appendix K show the comparison of step responses from simulations with the original and the updated models. All states, including the rotor states are presented in the appendix as well as the input signals.

When applying a three degree step in bank angle, the step responses from simulations with the two models are nearly the same. The similar behaviors of the models origin from the fact that the velocity components $v$ and $w$ are close to zero during the whole simulation. This means that the angle of attack and the angle of sideslip do not change. As a result, no increments in forces and moments are given in the updated model during the simulation and consequently the behaviors of the two models become almost identical. Even if the step size is changed, the behaviors of the models are still the same. Some differences can be found though, with different settings in the weighting matrices. By choosing lateral weighting matrices according to (7.5), the amplitudes of the oscillations, particularly in the input signals (except $\theta_{sw,coil}$) and in the roll rate $p$ and the roll angle $\Theta$, become larger.
8 DISCUSSION, CONCLUSIONS AND FUTURE WORK

The aim of this master thesis project was to study the main rotor downwash effects and its influence on the fuselage using CFD. A lot of cases at freestream velocity $V_{\infty} = 20$ m/s with different angles of attack and different sideslip angles were calculated. The results from these calculations were gathered and the existing helicopter model at Saab was updated according to them. Finally, an LQ controller was designed and a comparison of the original and the updated helicopter model were carried out, for both the open and the closed systems. In the following sections some additional aspects and conclusions of the work done are given. Suggestions on further work are also presented.

8.1 Control design

In order to simplify the rather complex helicopter model and facilitate the control design, states were reduced and split into lateral and longitudinal ones. Both the reduction of states and the split were validated with satisfying result. After the simplifications were made, an LQ feedback controller was synthesized to stabilize the helicopter in a steady turn. At this flight condition the trimmed velocity components in y- and z-directions are approximately zero. This means that also the angle of attack and the angle of sideslip are close to zero. Even with a change in turn radius, obtained by making a step in bank angle, the angles hardly change. As a result, no increments in forces and moments are given in the updated model during the simulation and consequently the behaviors of the two models become nearly the same. It may thereby be appropriate to complete the analysis with an additional controller for other flight conditions such as climb, descent or sidewise flight. These flight conditions give a wider range of angles of attack and sideslip angles, which make them more suitable for the analysis and the comparison of the two models. They also make the analysis easier, since some of the cross-coupling effects that appear in a turn are eliminated.

8.2 CFD-calculations

The used version of Edge is not optimal for solving problems at low Mach numbers - the region where helicopters normally operate and, as a consequence, the region where the calculations were carried out. Another disadvantage is the fixed direction of the rotor/propeller axis in the boundary condition, as it leads to extra work rotating the helicopter coordinate system. After the calculations in this project were completed however, a new version of Edge has become available. In this version, the direction of the rotor axis can be set arbitrary. A preconditioner which handles the low speed difficulties has also been added. Possibly, this preconditioner will help with the convergence for some of the studied cases. Hence, all non-converged cases ought to be re-started with this new Edge version. Additional calculations are also suggested in order to properly investigate the effect of higher sideslip angles, higher angles of attack and other freestream velocities. The coefficients obtained from the existing calculations will be used in the built-up of the aerodynamic data base.
8.3 Simulations

Comparisons of step responses for simulations with the open systems show similar behavior for the two models if they are trimmed for hover. When applying a tail pitch command $\theta_{tlr}$, there are an almost exact match of all states. For steps in $\theta_{sw,sin}$ and $\theta_{sw,cos}$, there are discrepancies only in $r$ and $\psi$ and in such a way that the updated model becomes more damped in yaw, compared to the original model. For a step in $\theta_{sw,col}$ the discrepancies are in $u$, $v$, $x$, $q$ and $\Theta$, but however rather small. As the original helicopter model has been validated using flight tests at Saab with good results, the differences shown may be considered to be within reasonable limits.

When the helicopter is trimmed for a steady turn, the states in the updated model diverge much faster than the states in the original model for any given pitch command. In hover, the helicopter fuselage is only affected by the downwash from the rotor as it has no velocity of its own, while the downwash is added to the helicopter velocity in a steady turn. As the aerodynamic forces and moments are scaled by the total velocity, they are larger in a steady turn compared to hover, which gives larger differences between the original and the updated model for this flight condition. Furthermore, due to more distinguished lateral and longitudinal cross-coupling effects in a turn it is difficult to determine which behavior of the models that is the most realistic. Further investigations are needed to fully examine the behaviors.

From a controlling perspective, it does not matter which model is used. When applying a three degree step in bank angle, the step responses from simulations with the two models are nearly the same.

8.4 Summary

It can be concluded that the aerodynamic coefficients from the CFD calculations can be used for determining the aerodynamic characteristics of the helicopter. Some further validation is needed though, for example by comparing the results with flight test data. In order to build an aerodynamic data base that covers the whole flight envelop, additional CFD calculations are required. For this purpose, the use of a new version of Edge look promising as it should deal with the low speed difficulties with an adapted preconditioner.
REFERENCES


Appendix A: Transformation between coordinate systems.

If \( \mathbf{r}_P = [x_P \ y_P \ z_P]^T \) and \( \mathbf{r}_Q = [x_Q \ y_Q \ z_Q]^T \) are coordinates in the P and Q coordinate systems, \( \mathbf{r}_{Q,0} = [x_{Q,0} \ y_{Q,0} \ z_{Q,0}]^T \) is the origin of Q in the P system, and \( \mathbf{A}_{Q2P} \) is a rotation matrix, transformations from coordinate system Q to coordinate system P may in general be written as:

\[
\mathbf{r}_P = \mathbf{r}_{Q,0} + \mathbf{A}_{Q2P} \mathbf{r}_Q
\]

(A.1)

Applying the general description in equation (A.1) to \( E, B, h \) and \( b \) the positions yields:

\[
\begin{align*}
\mathbf{r}_E &= \mathbf{r}_{B,0} + \mathbf{A}_{B2E} \mathbf{r}_B \\
\mathbf{r}_B &= \mathbf{r}_{h,0} + \mathbf{A}_{h2B} \mathbf{r}_h \\
\mathbf{r}_h &= \mathbf{r}_{b,0} + \mathbf{A}_{b2h} \mathbf{r}_b
\end{align*}
\]

(A.2)

Assuming that \( \mathbf{r}_{b,0} \) and \( \mathbf{A}_{b2h} \) do not vary in time, differentiation with respect to time gives the corresponding velocities:

\[
\begin{align*}
\dot{\mathbf{r}}_E &= \dot{\mathbf{r}}_{B,0} + \dot{\mathbf{A}}_{B2E} \mathbf{r}_B + \mathbf{A}_{B2E} \dot{\mathbf{r}}_B \\
\dot{\mathbf{r}}_B &= \mathbf{A}_{h2B} \dot{\mathbf{r}}_h \\
\dot{\mathbf{r}}_h &= \dot{\mathbf{r}}_{b,0} + \dot{\mathbf{A}}_{b2h} \mathbf{r}_h + \mathbf{A}_{b2h} \dot{\mathbf{r}}_b
\end{align*}
\]

(A.3)

and after a second differentiation, the accelerations

\[
\begin{align*}
\ddot{\mathbf{r}}_E &= \ddot{\mathbf{r}}_{B,0} + \ddot{\mathbf{A}}_{B2E} \mathbf{r}_B + 2\dot{\mathbf{A}}_{B2E} \dot{\mathbf{r}}_B + \mathbf{A}_{B2E} \ddot{\mathbf{r}}_B \\
\ddot{\mathbf{r}}_B &= \mathbf{A}_{h2B} \ddot{\mathbf{r}}_h \\
\ddot{\mathbf{r}}_h &= \ddot{\mathbf{r}}_{b,0} + \ddot{\mathbf{A}}_{b2h} \mathbf{r}_h + 2\dot{\mathbf{A}}_{b2h} \dot{\mathbf{r}}_h + \mathbf{A}_{b2h} \ddot{\mathbf{r}}_b
\end{align*}
\]

(A.4)

Transformation: earth-fixed / body-fixed coordinate system.

The transformation from earth- to body-fixed coordinate system is determined by three consecutive rotations \((\Psi, \Phi, \Theta)\), also known as the Euler angles. The rotation matrices with respect to yaw angle \( \Psi \), pitch angle \( \Theta \), and roll angle \( \Phi \), are given by:

\[
\begin{align*}
\mathbf{A}_\Psi &= \begin{pmatrix} \cos \Psi & \sin \Psi & 0 \\ -\sin \Psi & \cos \Psi & 0 \\ 0 & 0 & 1 \end{pmatrix}, & \mathbf{A}_\Theta &= \begin{pmatrix} \cos \Theta & 0 & -\sin \Theta \\ 0 & 1 & 0 \\ \sin \Theta & 0 & \cos \Theta \end{pmatrix}, & \mathbf{A}_\Phi &= \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \Phi & \sin \Phi \\ 0 & -\sin \Phi & \cos \Phi \end{pmatrix}
\end{align*}
\]

(A.5)
Together they form the transformation matrix $A_{E2B}$ according to $A_{E2B} = A_\phi A_\theta A_\psi$, which is given in equation (A.6) below:

$$
A_{E2B} = \begin{pmatrix}
\cos \Theta \cos \Psi & \cos \Theta \sin \Psi & -\sin \Theta \\
\sin \Phi \sin \Theta \cos \Psi - \cos \Phi \sin \Psi & \sin \Phi \sin \Theta \sin \Psi + \cos \Phi \cos \Psi & \sin \Phi \cos \Theta \\
\cos \Phi \sin \Theta \cos \Psi + \sin \Phi \sin \Psi & \cos \Phi \sin \Theta \sin \Psi - \sin \Phi \cos \Psi & \cos \Phi \cos \Theta \\
\end{pmatrix}
$$

$A_{E2B}$ is needed in transformation of the gravitational force.

To decide the position of the helicopter relative to $E$, the velocity components in the body fixed coordinate system $[u, v, w]^T$ have to be transformed to earth fixed coordinates $[x_E, y_E, z_E]^T$ using the transformation matrix $A_{B2E}$, according to:

$$
V_E^G = \begin{bmatrix}
x_E \\
y_E \\
z_E \\
\end{bmatrix} = A_{B2E} \begin{bmatrix}
u \\
v \\
w \\
\end{bmatrix}
$$

where $A_{B2E}$ is the transpose of $A_{E2B}$ defined as:

$$
A_{B2E} = \begin{pmatrix}
\cos \Theta \cos \Psi & \sin \Phi \sin \Theta \cos \Psi - \cos \Phi \sin \Psi & \cos \Phi \sin \Theta \sin \Psi + \cos \Phi \cos \Psi \\
\cos \Theta \sin \Psi & \sin \Phi \sin \Theta \sin \Psi + \cos \Phi \cos \Psi & \cos \Phi \sin \Theta \sin \Psi - \sin \Phi \cos \Psi \\
-\sin \Theta & \sin \Phi \cos \Psi & \cos \Phi \cos \Theta \\
\end{pmatrix}
$$

The position of the helicopter $[x_E, y_E, z_E]^T$, is then given by integration of equation (A.7).

Notice that the time derivatives of the angular velocities in body-fixed frame $[p, q, r]^T$, do not correspond to the Euler angle rates $[\dot\Phi, \dot\Theta, \dot\Psi]^T$. The rates are instead given by the following expression:

$$
\begin{bmatrix}
\dot\Phi \\
\dot\Theta \\
\dot\Psi \\
\end{bmatrix} = T_{B2E} \begin{bmatrix}
p \\
q \\
r \\
\end{bmatrix}
$$

with the transformation matrix $T_{B2E}$ defined as:

$$
T_{B2E} = \begin{pmatrix}
1 & \sin\Phi\tan\Theta & \cos\Phi\tan\Theta \\
0 & \cos\Phi & -\sin\Phi \\
0 & \sin\Phi\sec\Theta & \sin\Phi\sec\Theta \\
\end{pmatrix}
$$

The Euler angles $[\Phi, \Theta, \Psi]^T$ are obtained by integration of equation (A.9).
Transformation: body-fixed / hub-fixed coordinate system.

Since the hub-fixed coordinate systems have the same orientation as the body-fixed coordinate system, no rotational matrix is needed in the transformation between them, that is:

\[
A_{h2h} = A_{h2B} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]  \hspace{1cm} (A.11)

The transformation is only a translation, due to different locations of origins.

Transformation: hub-fixed / blade-fixed coordinate system.

The transformation from hub- to blade-fixed coordinate system, is determined by the position of the blade, i.e. the azimuth angle \( \Psi \), the pitch angle \( \Theta \), and the flapping angle \( \beta \), see figure 15. The rotation matrices are given by:

\[
A_o = \begin{pmatrix} 0 & -1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & -1 \end{pmatrix}, \quad A_\psi = \begin{pmatrix} \cos \psi & \sin \psi & 0 \\ -\sin \psi & \cos \psi & 0 \\ 0 & 0 & 1 \end{pmatrix}
\]
\[
A_\theta = \begin{pmatrix} \cos \theta & 0 & -\sin \theta \\ 0 & 1 & 0 \\ \sin \theta & 0 & \cos \theta \end{pmatrix}, \quad A_\beta = \begin{pmatrix} 1 & 0 & 0 \\ 0 & \cos \beta & \sin \beta \\ 0 & -\sin \beta & \cos \beta \end{pmatrix}
\]  \hspace{1cm} (A.12)

Together they form the transformation matrix \( A_{h2b} \) according to \( A_{h2b} = A_\beta A_\theta A_\psi A_o \), which is given in equation (A.13) below:

\[
A_{h2b} = \begin{pmatrix} -\cos \Theta \sin \Psi \\ -\left(\sin \beta \sin \Theta \sin \Psi + \cos \beta \cos \Psi\right) \\ \sin \beta \cos \Psi - \cos \beta \sin \Theta \sin \Psi \end{pmatrix} \begin{pmatrix} -\cos \Theta \cos \Psi \\ \cos \beta \sin \Psi - \sin \beta \sin \Theta \cos \Psi \\ -\left(\cos \beta \sin \Theta \cos \Psi + \sin \beta \sin \Psi\right) \end{pmatrix} \begin{pmatrix} \sin \Theta \\ -\sin \beta \cos \Theta \\ -\cos \beta \cos \Theta \end{pmatrix}
\]  \hspace{1cm} (A.13)

For transformations from blade- to hub-fixed coordinate system, the transformation matrix \( A_{b2h} \) is used. It is the transpose of \( A_{h2b} \), defined as

\[
A_{b2h} = \begin{pmatrix} -\cos \Theta \sin \Psi \\ -\left(\sin \beta \sin \Theta \sin \Psi + \cos \beta \cos \Psi\right) \\ \sin \beta \cos \Psi - \cos \beta \sin \Theta \sin \Psi \end{pmatrix} \begin{pmatrix} -\cos \Theta \cos \Psi \\ \cos \beta \sin \Psi - \sin \beta \sin \Theta \cos \Psi \\ -\left(\cos \beta \sin \Theta \cos \Psi + \sin \beta \sin \Psi\right) \end{pmatrix} \begin{pmatrix} \sin \Theta \\ -\sin \beta \cos \Theta \\ -\cos \beta \cos \Theta \end{pmatrix}
\]  \hspace{1cm} (A.14)
Appendix B: Reynolds-Averaged Navier-Stokes equations.

The Reynolds-Averaged Navier-Stokes (RANS) equations can be obtained by using Boussinesq’s assumption for time-averaging the Navier-Stokes system, with first order closure. For a three-dimensional flow, the equations can be written in a Cartesian coordinate system as:

\[
\frac{\partial \mathbf{U}}{\partial t} + \nabla \cdot \mathbf{F}_1 + \nabla \cdot \mathbf{F}_v = 0
\]

(B.1)

where \( \mathbf{U} \) is the vector of conserved variables, \( \mathbf{F}_1 = f_{ij} \mathbf{e}_x^T + f_{ij} \mathbf{e}_y^T + f_{ij} \mathbf{e}_z^T \) and \( \mathbf{F}_v = f_{ij} \mathbf{e}_x^T + f_{ij} \mathbf{e}_y^T + f_{ij} \mathbf{e}_z^T \) are the inviscid and viscous flux matrices. \( \mathbf{U}, \mathbf{F}_1 \) and \( \mathbf{F}_v \) are given by:

\[
\mathbf{U} = \begin{pmatrix} \bar{p} \\ \bar{p} \bar{w}_1 \\ \bar{p} \bar{w}_2 \\ \bar{p} \bar{w}_3 \\ \bar{p} \bar{E} \end{pmatrix}, \quad \mathbf{F}_1 = \begin{pmatrix} \bar{p} \delta_i + \bar{p} \bar{w}_i \\ \bar{p} \delta_j + \bar{p} \bar{w}_j \\ \bar{p} \delta_k + \bar{p} \bar{w}_k \\ \bar{p} \delta_i + \bar{p} \bar{w}_i \\ \bar{p} \delta_j + \bar{p} \bar{w}_j \end{pmatrix}, \quad \mathbf{F}_v = \begin{pmatrix} 0 \\ \tau_{ij} \\ \tau_{ij} \\ \tau_{ij} \\ \tau_{ij} \end{pmatrix}
\]

(B.2)

with density \( \bar{p} \), velocity components \( \bar{w}_i \), energy \( \bar{E} \), pressure \( \bar{p} \), Kronecker delta function \( \delta_{ij} \) (\( \delta_{ii} = 1 \) for \( i = j \) and \( \delta_{ij} = 0 \) otherwise), heat fluxes \( q_i \), stress tensor \( \tau_{ij} \), coefficient of viscosity \( \mu \) (subscript \( t \) for turbulent) and turbulent kinetic energy \( k \).

The turbulent kinetic energy, the stresses and the heat fluxes are given by:

\[
\begin{align*}
    k &= \frac{1}{2} \bar{w}_i \bar{w}_i \\
    \tau_{ij} &= (\mu + \mu_t) \left[ \frac{\partial \bar{w}_i}{\partial x_j} + \frac{\partial \bar{w}_j}{\partial x_i} - \frac{2}{3} (\nabla \bar{w}) \delta_{ij} \right] \\
    q_i &= (\kappa + \kappa_t) \frac{\partial T}{\partial x_i}
\end{align*}
\]

(B.3)

where the laminar and turbulent conductivity, \( \kappa \) and \( \kappa_t \) respectively, which are found from the coefficient of viscosity (\( \mu \) and \( \mu_t \)) and a Prandtl number (\( \text{Pr} \) and \( \text{Pr}_t \)):

\[
\begin{align*}
    \kappa &= \frac{\mu C_p}{\text{Pr}} \\
    \kappa_t &= \frac{\mu_t C_p}{\text{Pr}_t}
\end{align*}
\]

(B.4)
In equation (B.2) the density $\overline{\rho}$ and the pressure $\overline{p}$ are time averaged values defined as:

$$\overline{\rho} = \frac{1}{T} \int_{0}^{T} \rho \, dt \quad \text{and} \quad \overline{p} = \frac{1}{T} \int_{0}^{T} p \, dt$$  \hspace{1cm} (B.5)

The instantaneous values, $\rho$ and $p$, are related to the time averaged values according to:

$$\rho = \overline{\rho} + \rho' \quad \text{and} \quad p = \overline{p} + p'$$  \hspace{1cm} (B.6)

where $\rho'$ and $p'$ are fluctuating parts with:

$$\overline{\rho}' = 0 \quad \text{and} \quad \overline{p}' = 0$$  \hspace{1cm} (B.7)

Furthermore, the energy $\tilde{E}$, temperature $\tilde{T}$ and the velocity components $\tilde{w}_i$, in equation (B.2) are density weighted averages defined as:

$$\tilde{E} = \frac{1}{T} \int_{0}^{T} \rho \omega \, dt \quad \tilde{T} = \frac{1}{T} \int_{0}^{T} \rho \omega \, dt \quad \text{and} \quad \tilde{w}_i = \frac{1}{T} \int_{0}^{T} \rho \omega \, dt$$  \hspace{1cm} (B.8)

The instantaneous values $E$, $T$ and $w_i$, are related to the time averaged values according to:

$$E = \tilde{E} + E' \quad T = \tilde{T} + T' \quad \text{and} \quad w_i = \tilde{w}_i + w_i'$$  \hspace{1cm} (B.9)

where $E'$, $T'$ and $w_i'$ are fluctuating parts.
Appendix C: The linearized helicopter model.

Linearization at a steady turn, with turn radius 60 m and forward helicopter velocity \( v_E = 20 \text{ m/s} \), gives the following trim states:

**Trim states:**

\[
\beta = \begin{bmatrix}
\hat{\beta}_{\text{nr}} \\
\hat{\beta}_{\text{dr}} \\
\hat{\beta}_{\text{mnr}} \\
\hat{\beta}_{\text{mnr}} \\
\hat{\beta}_{\text{pad}} \\
\hat{\beta}_{\text{pad}} 
\end{bmatrix} = 0
\]  
(C.1)

\[
\beta = \begin{bmatrix}
\beta_{\text{nr}} \\
\beta_{\text{dr}} \\
\beta_{\text{mnr}} \\
\beta_{\text{mnr}} \\
\beta_{\text{pad}} \\
\beta_{\text{pad}} 
\end{bmatrix} = \begin{bmatrix}
3.92 \\
0.83 \\
0.02 \\
-0.20 \\
0.33 \\
-3.15 
\end{bmatrix}
\]  
(C.2)

\[
V_B = \begin{bmatrix}
u \\
v \\
w 
\end{bmatrix} = \begin{bmatrix}
20.00 \\
-0.03 \\
-0.06 
\end{bmatrix}
\]  
(C.3)

\[
X_E = \begin{bmatrix}
x_E \\
y_E \\
z_E 
\end{bmatrix} = \begin{bmatrix}
0.00 \\
0.00 \\
0.00 
\end{bmatrix}
\]  
(C.4)

\[
\omega_B = \begin{bmatrix}
p \\
q \\
r 
\end{bmatrix} = \begin{bmatrix}
0.06 \\
9.88 \\
16.35 
\end{bmatrix}
\]  
(C.5)

\[
\Omega_B = \begin{bmatrix}
\Phi \\
\Theta \\
\Psi 
\end{bmatrix} = \begin{bmatrix}
31.15 \\
-0.19 \\
0.00 
\end{bmatrix}
\]  
(C.6)

All angles and angular velocities above, are given in [deg] and [deg/s] respectively, while velocities and positions are given in [m/s] and [m].
Appendix D: Validation of model reduction.

If the states of the linearized model are split into two groups, $x_1$ and $x_2$, where $x_1$ are the slow states to be kept and $x_2$ are the fast states to be eliminated, the model may be written as:

$$
\begin{bmatrix}
\dot{x}_1 \\
\dot{x}_2
\end{bmatrix} =
\begin{bmatrix}
A_{11} & A_{12} \\
A_{21} & A_{22}
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix} +
\begin{bmatrix}
B_1 \\
B_2
\end{bmatrix} u
$$

$$
y =
\begin{bmatrix}
C_1 \\
C_2
\end{bmatrix}
\begin{bmatrix}
x_1 \\
x_2
\end{bmatrix}
$$

Eigenvalues of the original $A$-matrix $A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}$ and eigenvalues of matrices $A_{11}$ and $A_{22}$ are given in (D.1) below.

$$
\lambda_A =
\begin{bmatrix}
0 \\
0 \\
0 \\
0 \\
-33.70 \pm 543.50i \\
-35.93 \pm 243.47i \\
-6.52 \pm 242.04i \\
-36.00 \pm 122.53i \\
-38.09 \\
-37.15 \\
-2.05 \pm 9.16i \\
-3.11 \pm 3.57i \\
-0.12 \pm 1.35i \\
0.10 \pm 0.81i \\
-0.36 \\
0.15
\end{bmatrix}, \quad 
\lambda_{A_{11}} =
\begin{bmatrix}
-35.96 \pm 243.70i \\
-35.99 \pm 122.68i \\
-6.48 \pm 241.84i \\
-36.02 \pm 1.72i \\
-6.50 \pm 0.17i \\
-33.70 \pm 543.50i
\end{bmatrix}, \quad 
\lambda_{A_{22}} =
\begin{bmatrix}
0 \\
0 \\
0 \\
-0.01 \pm 1.34i \\
0.39 \pm 0.57i \\
-0.30 \pm 0.5i \\
-0.62 \\
-0.30
\end{bmatrix}
$$ (D.1)
Figure 43 presents a comparison of step responses for a cosine pitch command of 2°, before and after the reduction of the rotor states. Solid lines represent the original model with twenty-four states and the dashed lines represent the reduced model, with the remaining twelve states. The simulation was done with the open system.

Figure 43. Step response for a cosine pitch command of $\theta_{sw,\cos} = 2^\circ$. 
Figure 44 presents a comparison of step responses for a sine pitch command of 2°, before and after the reduction of the rotor states. Solid lines represent the original model with twenty-four states and the dashed lines represent the reduced model, with the remaining twelve states. The simulation was done with the open system.

Figure 44. Step response for a sine pitch command of $\theta_{sw,\sin} = 2^\circ$. 
Figure 45 presents a comparison of step responses for a collective pitch command of 2°, before and after the reduction of the rotor states. Solid lines represent the original model with twenty-four states and the dashed lines represent the reduced model, with the remaining twelve states. The simulation was done with the open system.

Figure 45. Step responses for a collective pitch command of \( \theta_{\text{sw, coll}} = 2^\circ \).
Figure 46 presents a comparison of step responses for a tail pitch command of $2^\circ$, before and after the reduction of the rotor states. Solid lines represent the original model with twenty-four states and the dashed lines represent the reduced model, with the remaining twelve states. The simulation was done with the open system.

Figure 46. Step response for a tail pitch command of $\theta_{tlr} = 2^\circ$. 
Appendix E: Validation of model split.

After elimination of the fast helicopter states the reduced \( A \)-matrix \( A_r \), may be written as:

\[
A_r = \begin{bmatrix}
    A_{\text{long}} & A_{\text{longlat}} \\
    A_{\text{latlong}} & A_{\text{lat}}
\end{bmatrix}
\]  

(E.1)

where

\[
A_{\text{long}} = \begin{bmatrix}
    -0.03 & -0.19 & 0 & 0 & 1.14 & -9.81 \\
    0.03 & -0.59 & 0 & 0 & 20.96 & 0 \\
    1 & 0 & 0 & 0 & 0 & 0 \\
    0 & 0.86 & 0 & 0 & 0 & -20 \\
    0.09 & 0.05 & 0 & 0 & -3.31 & 0 \\
    0 & 0 & 0 & 0.86 & 0 & 0 
\end{bmatrix},
A_{\text{lat}} = \begin{bmatrix}
    -0.02 & 0 & -1.12 & -20 & 9.81 & 0 \\
    0.86 & 0 & 0 & 0 & 0 & 0.07 \\
    0 & 0 & -14.45 & 0.04 & 0 & 0 \\
    0.09 & 0 & -0.10 & 0 & 0 & 0 \\
    0 & 0 & 1 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0.86 & 0 & 0 
\end{bmatrix}
\]  

(E.2)

\[
A_{\text{longlat}} = \begin{bmatrix}
    0.28 & 0 & 0.06 & -0.03 & 0 & 0 \\
    0 & 0 & -0.07 & 0 & 0 & 0 \\
    0 & 0 & 0 & 0 & 0 & 0 \\
    0.52 & 0 & 0 & 0 & 0 & 0 \\
    0.02 & 0 & 0.08 & 0 & 0 & 0 \\
    0 & 0 & 0 & -0.52 & -0.33 & 0 
\end{bmatrix},
A_{\text{latlong}} = \begin{bmatrix}
    -0.28 & -0.01 & 0 & 0 & 0.07 & 0 \\
    0 & -0.52 & 0 & 0 & 0 & 0 \\
    0 & -0.01 & 0 & 0 & 0.68 & 0 \\
    -0.08 & 0.13 & 0 & 0 & 0.19 & 0 \\
    0 & 0 & 0 & 0 & 0.33 & 0 \\
    0 & 0 & 0 & 0 & 0.52 & 0 
\end{bmatrix}
\]  

(E.3)

Eigenvalues of \( A_r \)

\[
\lambda_{A_r} = \{ 0 \, 0 \, 0 \, -14.45 \, -3.75 \, -0.10 \pm 1.36i \, 0.10 \pm 0.79i \, -0.35 \, 0.15 \} 
\]  

(E.4)

Eigenvalues of \( A_{\text{long}} \) and \( A_{\text{lat}} \)

\[
\lambda_{A_{\text{long}}} = \{ 0 \, 0 \, -3.73 \, 0.09 \pm 0.56i \, -0.38 \} \\
\lambda_{A_{\text{lat}}} = \{ 0 \, 0 \, 0.01 \pm 1.37i \, -14.45 \} 
\]  

(E.5)
Appendix F: The linear quadratic regulator.

Figure 47 shows the Simulink scheme for the LQ controller, including both the longitudinal and the lateral control loops. For longitudinal control, $u$ and $z$ are chosen as reference signals, while $v$ and $\Phi$ are chosen as reference signals for lateral control.

\[ x' = Ax + Bu \]
\[ y = Cx + Du \]

State–Space

Figure 47. Simulink scheme of the LQ controller.
Figure 48 shows a comparison of step responses for the four state lateral model and the original twenty-four state model, with weighting matrices according to (7.5). A 3° step in bank angle $\Phi$ has been given. The solid lines represent the original model and the dashed lines represent the lateral model. In order to obtain a better damped system, the weighting matrices were later changed according to (7.6).

Figure 48. Step response for the lateral and the original twenty-four state model. A step of 3° in bank angle $\Phi$ has been given.
Figure 49 shows a comparison of step responses for the four state lateral model and the original twenty-four state model, with the final weighting matrices according to (7.6). A 3° step in bank angle $\Phi$ has been given. The solid lines represent the original model and the dashed lines represent the lateral model. Compared to the originally tested weighting matrices (7.5), this system is better damped but has the disadvantage of being slower.

Figure 49. Step response for the lateral and the original twenty-four state model. A step of 3° in bank angle $\Phi$ has been given.
Appendix G: Comparison of CFD calculations with structured/unstructured grids.

Figure 50 presents a comparison of the aerodynamic coefficients from CFD calculations with the structured/unstructured grids at sideslip angle $\beta = -10^\circ$. The coefficients are plotted as a function of angle of attack $\alpha$. The dashed lines represent the structured grid and the solid lines represent the unstructured grid.

Figure 50. Aerodynamic coefficients from CFD calculations with the structured/unstructured grids, at sideslip angle $\beta = -10$. 
Figure 51 presents a comparison of the aerodynamic coefficients from CFD calculations with the structured/unstructured grids at sideslip angle $\beta = 0^\circ$. The coefficients are plotted as a function of angle of attack $\alpha$. The dashed lines represent the structured grid and the solid lines represent the unstructured grid.

![Graphs showing aerodynamic coefficients](image)

Figure 51. Aerodynamic coefficients from CFD calculations with the structured/unstructured grids, at sideslip angle $\beta = 0$. 
Figure 52-55 present a comparison of the pressure distribution from CFD calculations with structured and unstructured grids at $\alpha = 30^\circ$, $\beta = 0^\circ$. Both a view from above and a view from below are shown.

![Figure 52. Pressure distribution from a CFD calculation with the structured grid at $\alpha = 30^\circ$, $\beta = 0^\circ$. View from above.](image)

![Figure 53. Pressure distribution from a CFD calculation with the unstructured grid at $\alpha = 30^\circ$, $\beta = 0^\circ$. View from above.](image)
Figure 54. Pressure distribution from a CFD calculation with the structured grid at $\alpha = 30^\circ$, $\beta = 0^\circ$. View from below.

Figure 55. Pressure distribution from a CFD calculation with the unstructured grid at $\alpha = 30^\circ$, $\beta = 0^\circ$. View from below.
Appendix H: Pressure distributions from CFD calculations with the main rotor configuration.

Figure 56-57 present the pressure distribution from a CFD calculation with the main rotor configuration at $\alpha = 30^\circ$, $\beta = 0^\circ$. Both a view from above and a view from below are shown.

**Figure 56.** Pressure distribution from a CFD calculation with the main rotor configuration at $\alpha = 30^\circ$, $\beta = 0^\circ$. View from above.

**Figure 57.** Pressure distribution from a CFD calculation with the main rotor configuration at $\alpha = 30^\circ$, $\beta = 0^\circ$. View from below.
Appendix I: Comparison of CFD calculations with rotor-less/main rotor configurations.

Figure 58 presents a comparison of the aerodynamic coefficients from CFD calculations with the rotor-less and the main rotor configurations at sideslip angle $\beta = -10^\circ$. The coefficients are plotted as a function of angle of attack $\alpha$. The dashed lines represent the rotor-less configuration and the circles represent the main rotor configuration. The solid lines show the curve fitting to calculations with the main rotor configuration.

Figure 58. Aerodynamic coefficients from CFD calculations with rotor-less/main rotor configurations at sideslip angle $\beta = -10$. 
Figure 59 presents a comparison of the aerodynamic coefficients from CFD calculations with the rotor-less and the main rotor configurations at $\beta = 0^\circ$. The coefficients are plotted as a function of angle of attack $\alpha$. The dashed lines represent the rotor-less configuration and the circles represent the main rotor configuration. The solid lines show the curve fitting to calculations with the main rotor configuration.

Figure 59. Aerodynamic coefficients from CFD calculations with rotor-less/main rotor configurations at sideslip angle $\beta = 0$. 
Figure 60 presents the aerodynamic coefficients from CFD calculations with the main rotor configurations at sideslip angle $\beta = 10^\circ$. The coefficients are plotted as a function of angle of attack $\alpha$. The solid lines show the curve fitting to the calculations.

Figure 60. Aerodynamic coefficients from CFD calculations with main rotor configurations at sideslip angle $\beta = 10^\circ$. 
**Appendix J: Step responses from simulations with the open systems.**

Figure 61 presents a comparison of step responses for the original and the updated model. Before a cosine pitch command of 2° was given, the helicopter was trimmed for hover. The solid lines represent the original model and the dashed lines represent the updated model.

![Step response graphs](image)

**Figure 61.** Step response for a cosine pitch command of $\theta_{sw,cos} = 2^\circ$. 
Figure 62 presents a comparison of step responses for the original and the updated model. Before a sine pitch command of $2^\circ$ was given, the helicopter was trimmed for hover. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 62. Step response for a sine pitch command of $\theta_{sw, sin} = 2^\circ$. 
Figure 63 presents a comparison of step responses for the original and the updated model. Before a collective pitch command of $2^\circ$ was given, the helicopter was trimmed for hover. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 63. Step response for a collective pitch command of $\theta_{sw, col} = 2^\circ$. 
Figure 64 presents a comparison of step responses for the original and the updated model. Before a tail pitch command of 2° was given, the helicopter was trimmed for hover. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 64. Step response for a tail pitch command of $\theta_{\text{tlr}} = 2^\circ$. 
Figure 65 presents a comparison of step responses for the original and the updated model. Before a cosine pitch command of $2^\circ$ was given, the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity $\dot{x}_E = 20$ m/s. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 65. Step response for a cosine pitch command of $\theta_{sw,cos} = 2^\circ$. 
Figure 66 presents a comparison of step responses for the original and the updated model. Before a sine pitch command of $2^\circ$ was given, the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity $\dot{x}_E = 20$ m/s. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 66. Step response for a sine pitch command of $\theta_{\text{sw, sin}} = 2^\circ$. 
Figure 67 presents a comparison of step responses for the original and the updated model. Before a collective pitch command of $2^\circ$ was given, the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity $\dot{x}_E = 20$ m/s. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 67. Step response for a collective pitch command of $\theta_{sw, col} = 2^\circ$. 
Figure 68 presents a comparison of step responses for the original and the updated model. Before a tail pitch command of \( \theta_{\text{tlr}} = 2^\circ \) was given, the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity \( \dot{x}_E = 20 \) m/s. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 68. Step response for a tail pitch command of \( \theta_{\text{tlr}} = 2^\circ \).
Appendix K: Step responses from simulations with the closed systems.

Figure 69 presents a comparison of input signals for the original and the updated model. Before a 3° step in bank angle was given, the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity $\dot{x}_E = 20$ m/s. The solid lines represent the original model and the dashed lines represent the updated model.

Figure 69. Input signals for a 3° step in bank angle.
Figure 70 presents a comparison of step responses for the original and the updated model. Before a 3° step in bank angle was given, the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity $x_E = 20$ m/s. The solid lines represent the original model and the dashed lines represent the updated model. All states, except the rotor states, are presented.

Figure 70. Step responses for a 3° step in bank angle.
Figure 71 presents a comparison of step responses for the original and the updated model. Before a 3° step in bank angle was given, the helicopter was trimmed for a steady turn with turn radius 60 m and forward helicopter velocity \( x_E = 20 \) m/s. The solid lines represent the original model and the dashed lines represent the updated model. Only the rotor states are presented.

![Figure 71: Step responses for a 3° step in bank angle.](image)