I would like to thank my supervisor Henry Montgomery for his continues support. He developed the idea and mathematical functions that are the baseline of this work.
Reaching for the optimal -
The role of optimal alternatives in pre-decision making stages

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It was hypothesized that in order to find a promising alternative in a decision making situation, individuals will choose the alternative that is closest to their optimal alternative. Therefore, based on participants’ optimal alternative, a computer application generated five alternatives. Two of the alternatives were constructed to be most similar to the participant’s optimal alternative. Two other alternatives were associated with two non-compensatory rules, and one alternative did not link to any decision making rule. Content analysis of the think aloud protocols showed that participants did think of an optimal alternative. Furthermore, amongst given alternatives, participants chose the alternative that was most similar to their optimal alternative. In addition, this alternative also got higher preference ratings compared to other alternatives. These findings not only suggest the presence of optimal alternative, they also demonstrate the influence that such an optimal alternative have on the outcome in a decision-making situation.

Keywords: Decision making, promising alternative, optimal alternative, similarity, distance

Introduction

Imagine Jenny who has been looking for buying a house. She wants a house in the suburb, big yard, and preferably a bedroom with a view of the sunrise. She has not found this house yet or maybe she has but she cannot afford it. Still, when she looks for a promising house she looks for alternatives that are as close as possible to this optimal or ideal house of hers.

The above example demonstrates the hypothesis of this study, that individuals have optimal alternatives in mind and how these influence ones decision making. A promising alternative is an alternative that remains in the decision making process because it is found to be attractive to the decision maker (Dahlstrand & Montgomery, 1984; Montgomery & Svenson, 1989). An optimal alternative is the alternative that has attribute values that are most desirable and optimal for that individual (Zakay & Dil, 1984). Most of the decision-making theories from the different fields are either normative (Von Winterfeldt & Edwards, 1976) or descriptive (Beach, 1990; Camerer & Johnson, 1991; Kahneman & Tversky 1979; Montgomery, 1983,). However, Svenson (2003) divided the theories in structural and process theories. Several studies in the process tracing tradition have shown that decision makers (DM) often find a candidate for their decision – “a promising alternative” - at an early stage of the decision process. In the
ensuing process, the decision maker tries to strengthen the support for the promising alternative (Dahlstrand & Montgomery, 1984; Montgomery & Svenson, 1989). Other studies have shown that these promising alternatives are a part of a decision space structure where the distance between the promising alternative and an optimal alternative plays a role (Zakay & Dil, 1984).

To give an understanding of where the research stands in the areas of interest, a brief review of decision making rules, MAU, decision processes and decision alternatives, and similarity functions will be given.

**Decision making rules**

Individuals use different decision-making rules when making decisions. Decision-making rules are strategies to find an alternative that can be chosen from a given set of alternatives. The alternatives are characterized as being more or less attractive on a number of attributes (e.g. price, location, size, quality etc.). There are two types of decision-making rules: compensatory and non-compensatory rules. In compensatory rules, an unattractive attribute in a certain alternative can be compensated by other attributes (Montgomery & Svenson, 1976; Svenson, 1979). In contrast, in non-compensatory decision-making rules, attributes of an alternative cannot be compensated by other attributes (Elrod, Johnson & White, 2004). Non-compensatory rules do not require comparing across the different attributes and are therefore easier to use. Compensatory rules are applicable in more situations, but more difficult to use. They are complex and require much information processing, which can be problematic for a decision maker. Moreover, they might “force” the decision maker to give up some positive attributes in favor of other positive attributes (Montgomery, 1983). Non-compensatory rules are not considered optimal because they lead to missing important information. Dahlstrand and Montgomery found that individuals use non-compensatory rules when a promising alternative is involved (Dahlstrand & Montgomery, 1984). Lindeman and Markman (1996) found that individuals prefer comparable attributes more than non-comparable, which may make it difficult to use a compensatory rule. In their study, they gave students important non-comparable information about different colleges and some comparable information that was less important. They found that the comparable information was preferred.

There are many decision-making rules, some of interest for our study are: The **dominance rule**, often used when people look for justifying their choice, implies the choice of an alternative that is better than all other alternatives on at least one attribute and not worse on other attributes. Montgomery (1983) suggested that if the individual cannot find a dominating alternative, she will change the structuring of the alternatives so that one of the alternatives will eventually become dominant. Montgomery came to call this the dominance structuring theory (DST). Dahlstrand and Montgomery (1984) found that individuals, in a dominance search, tend to apply the **Lexicographic and Maximin** decision rule. When individuals apply the Lexicographic rule, they choose the alternative that has the best value on the most important attribute. The Maximin rule uses a similar concept but instead the alternative that has the best value amongst all attributes in all the alternatives is chosen. The above-mentioned rules are non-compensatory rules. The most commonly mentioned compensatory rule in the literature is the **Multiattribute utility (MAU) rule**. MAU theory states that each attribute has a utility or a value, which can be quantified. In MAU theory, you first start by identifying the alternatives. Second, you identify attributes, for each alternative that you find important. Third, you quantify the attributes. Lastly, you give importance weights to each attribute and then calculate the overall utility by summing the product of attribute values and weights. The goal according to MAU theory is to choose the alternative that has the highest utility.
Another theory that is along the same line as MAU theory is the Subjectively Expected Utility (SEU) theory which claims that decisions can be made by summing the utility of each alternative in a decision making situation, and weighting this by the probability of its occurrence (Wallsten, 1971). Many researchers have criticized MAU Theory. Typically, they argue that individuals are not perfectly rational entities but rather guided by bounded rationality. Therefore, they have a limited ability to base their decision on weighted utilities (Kahneman, 2003).

Decision processes and decision alternatives

In the late 50s, Festinger (1957) suggested in his dissonance theory that decision-making processes differ depending on being pre- or post-decision processes. He concluded that pre-decision processes are mostly objective and information-based, while post-decision processes have more emotions involved and are characterized by finding arguments that satisfy the decision made. This implies that the individual’s evaluation of chosen alternatives will be more positive, whereas the evaluation of non-chosen alternatives will become more negative (Festinger, 1964; Gerard & White, 1983; Frey, 1986). These findings are somewhat coherent with DST. Dahlstrand and Montgomery (1984) as well as Montgomery and Svenson (1989) reported that participants indeed did pay more attention to the alternative they finally chose and as well enhanced their positive evaluation of the alternative. Decades earlier, Jecker (1963) also found indications that individuals early in the pre-decision processes choose an alternative that they found more attractive than other alternatives. Another theory, the Differentiation and Consolidation theory (DiffCon) argues that the pre and post processes are the same. Instead, an alternative gradually differentiates from the rest of the alternatives (Svenson, 1992). Mills (1968) found that in the pre-decision stage individuals try to increase the likening of their choice. In another study, Tyszka (1985) found that participants look for more information about the alternatives they found more promising, when incomplete information was given to participants. In a study conducted by Brownstein, Read and Simon (2004) participants, in a simulated horse race, rated each horse’s chance of winning before placing a bet and after placing a bet. They found that the chosen horse ratings increased in the pre-decision as well as in the post-decision process. The authors concluded that promising alternatives became more favorable.

Comparable to DST (for a review of research concerning the validity of DST see Montgomery, 1989) the Action Control Theory states that individuals intentionally process or act more on information that strengthens what they really want to choose. As a result, the decision-making process will be ended if a threatening alternative appears (Kuhl, 1984). Englander and Tyszka (1980) found another indication of the existence of a promising alternative. In their study, participants could ask any questions they wanted about a set of alternatives. They found that participants asked more questions about the alternative they eventually chose. This shows that an alternative has become promising but the participants look for more information about it. This finding was confirmed by another study conducted by Tyszka (1986) where participants in a think-aloud experiment asked more questions about the alternative they eventually choose. Russo and Medvec (1996) reported that existing preferences could lead to distortion of new information in support of the chosen alternative. They referred their findings to confirmation bias and explained it by saying that individuals desire to maintain a consistency of their beliefs and therefore reject information that goes against it. They also explained it by mentioning that individuals want to reduce the effort by confirming already set-up preferences. This is contradictory to what Gerard (1983) had found, which was that participants spend more time on the alternatives they do not choose.
It can be concluded that there is evidence for the idea that decision makers find a promising alternative early in the decision process. Many researchers have confirmed this. There, however, are nearly no findings on where this promising alternative comes from. Zakay and Dil (1984) found that alternatives that had the shortest distance to an optimal alternative in terms of weighted sum of the differences between each attribute value would cause more post-decisional confidence. They also found indications that the preference of the given alternatives differs depending on its distance from the optimal alternative. Even though Zakay and Dil (1984) did not exclusively study the optimal alternative they found evidence of such an alternative and that there is a relation between a promising alternative and the optimal alternative.

**Similarity functions**

As already mentioned, individuals, already at an early stage find an alternative as the promising alternative. It may be asked on what grounds the promising alternative is found. One possibility for finding the promising alternative may be to select the alternative that is most similar to an optimal alternative (Gati & Winer, 1987; Zakay & Barak, 1984, Zakay & Dil, 1984). A suitable definition of similarity in the present context was made by Tenenbaum (1999), if the similarity between A and B increases, then if B has property X we know that the probability of A also having property X increases. The idea behind similarity is that we have mental representations of objects and similarity of different objects is derived from the comparisons between these representations (Yushi, 2006).

One common similarity approach is multidimensional scaling (MDS) models. In MDS, stimuli, in our case a set of choice alternatives, are represented as points in a multidimensional space, where each point has a certain location on one or more dimensions. Similarity is assumed to be inversely related to the distances between the points (Shepard, 1962). Estimates of the distance are typically based on similarity judgments or correlations between judgments of each stimulus. The most applied distance or similarity functions (or metrics as they are called in mathematics) have been the city-block and Euclidean distance functions. In mathematics, Euclidean distance is the length of the straight line between two points and it is calculated by repeatedly applying the Pythagorean Theorem. City-block is smallest distance between two points in a grid or a mesh, which can only traverse along each dimension. The city-block distance is calculated as the sum of the absolute difference of the alternative’s coordinates. In psychology, Euclidean metrics are usually best suitable for integrated data such as brightness and saturation, while city block metrics are suitable for separated data such as brightness and size (Attneave, 1950; Goldstone & Son, 2005; Tversky, 1977, Fishburn, 1970).

Even though decision making is a domain with researchers from behavioural science, business science, and mathematics, not many of them have studied the influence that ones ideal or optimal image have in decision making processes. The purpose of this study was to see if individuals have an optimal alternative in mind, and when they are faced with a number of alternatives that are about equally attractive, if they choose the alternative that is closest in to their optimal alternative in terms of attractiveness.

**Present Study**

To see if individuals have an optimal alternative in mind and prefer or choose the alternative that is most similar to their optimal alternative, this study measured distance with help of two distance functions on two of the presented alternatives. One distance function was the non-weighted Euclidean distance, which has been used in previous studies. However, this
function does not take into account the weights that individuals might give to different attributes (i.e. dimensions) in an alternative. Therefore, the other alternative applied a weighted Euclidean distance function where the weights are included in the distance calculation (proof for this function is found in measurement section). As it will be shown in the proof, the weighted Euclidean distance function generates an alternative that has the same rank order in as in the optimal alternative. With this function, an alternative that had the same rank order between the attribute values as between importance weights of the same attributes was generated. Two other alternatives each applied one particular non-compensatory rule. In addition, we included an alternative that applied no rule. As shown in the study of Montgomery (1983) decision-makers will, at an early stage, exclude alternatives with low or unattractive attribute values. Therefore, to create a similar attractiveness among the alternative towards the optimal alternative, all the presented alternatives had constant MAU and equally attractive attribute values.

For the purpose of the study, first we identified the participants’ optimal alternative with help of think-aloud method. We asked them to think-aloud how they would reason in the hypothesized situation, and then we gave them the different alternatives, where they had to choose one. The following hypotheses were assumed:

H1: When individuals face a decision-making situation, they will have an optimal alternative in mind.

H2: Individuals will choose the alternative that is most similar to their optimal alternative, either in terms of shortest non-weighted Euclidean or in terms of weighted Euclidean (i.e. having the same rank order in the generated alternative as in the optimal alternative, for more details see the proof in the measurement section).

H3: Alternatives that are most similar to the optimal alternative, in terms of the two above-mentioned distances) will receive the highest attractiveness in preference rating of each alternative.

Method

Participants

Totally 83 students from Stockholm University were recruited by setting up information and a sign-up sheet at Stockholm University. Students could upon participation receive course credits or choose to get a cinema ticket. Thirty-three of the participants participated in the pilot testing. Of the remaining 50 participants, five of them were excluded, because of incomplete data. For example, some participants did not write their age. Of the remaining 45 participants, 13 were men and 32 were women. The average age of the participants were 29.6 years (SD = 9.9) with a range of 19 – 67 years. Participants were randomly assigned to two different decision making tasks which resulted in that 25 were presented with a choice of apartments/houses (22 women and 3 men) and 20 were presented with a choice of cars (10 women and 10 men).

Instruments

A computer application was developed in JAVA and a questionnaire. The data from the questionnaire were excluded from the final analysis.

Procedure

Pilot studies
To find out the best method for identifying the optimal alternative, five different methods were tested. For each method, we gave participants three separate tasks. Task 1 was about vacation, task 2 was about apartments/houses, and the last task was about cars. For each task, we told participants that they should imagine that they had been seeing several alternatives for that specific task, e.g., several cars or apartments and had to choose one of them.

The methods were as follows:

Method 1: Participants were asked to think aloud about the given decision-making tasks, imagining that they had been seeing several alternatives for themselves.

Method 2: Participants were asked to imagine that one of their friends had been seeing several alternatives for the given task and had to choose one of the alternatives. They should think aloud how they thought their friend would reason.

Method 3: Participants were provided with a list of decision making steps, where they had to preference rate, from 1-5, how much they used each step in the given decision making task.

Method 4: Participants were asked to write down a list of the decision-making steps they used in the given decision making task.

Method 5: Participants were asked to think aloud a list of the decision-making steps they used in the given decision making task.

Each method was tested by two participants. The pilot tests showed that the most suitable method to identify the participant’s optimal alternative and its attributes was the first method. Mainly because it was found that participants found it easier to talk about their optimal alternative when thinking aloud and hence more information about the optimal alternative could be retrieved. In addition, it was found that the vacation task was not a suitable task, primarily because in participants wanted to hear other people’s opinions, for instance a friends or spouses’ opinion. Therefore, the vacation task was only to use as exercise in the think-aloud method. After this pilot phase, a computer application was developed for generating and presenting alternatives, which were based on participant’s optimal alternative. The computer application was tested on 23 participants. This time participants thought aloud according to the first method, their optimal alternative was identified, and then continued on conducting the experiment via the computer application. The application presented the alternatives by giving each attribute in the alternative a value from 0 to 100. However, participants tended to make a calculation of the given values and not see the attributes itself. Therefore, we changed the way the alternatives were presented. A more detailed description of how the alternatives were generated and presented will be given below.

**Main experiment**

Each experiment session involved one participant and the experimenter. Participants where randomly divided into two groups. Group 1 received the decision task about choosing a car. Group 2 received the decision task about choosing an apartment/house. The experiment was conducted in the following steps:

1. **Think aloud.** The nature of the experiment and the method used, in this case the think aloud method was introduced to the participants. They were told that this experiment was part of a study on decision-making. This information was also given on paper. After reading about the experiment, participants could decide if they wanted to continue with their participation. If they decided to continue, they would get an exercise in the think aloud method. After this exercise, the experimenter decided whether the participants had understood the method completely. If the participants needed additional exercise, they
would get that. After the exercise, they received the decision task that was either the car or the apartment/house task. The instructions for the tasks were identical. They were told that they should imagine that they had been seeing several alternatives for that specific task, e.g. several cars or apartments, and had to choose one of them, and think aloud while doing that. When participants indicated that they had finished “thinking aloud”, they were asked whether they thought about an optimal alternative while thinking aloud and if yes, what role this optimal alternative played in the decision making-process. They were also asked whether they had been in this decision making task before. The think aloud data were audio recorded.

2. **Attribute identification.** In order to have time to analyze the think aloud data on the spot and identify important attributes, and camou flage the real purpose of the experiment, participants where given the a questionnaire. Meanwhile the attributes of the optimal alternative was identification. The attributes were transformed to neutral words. For instance if the participant had said that it is important for her/him to live close to the city, then they would get an attribute called location which was supposed to refer to the importance of location. Once identified, the attributes were typed into the computer application, by the experimenter.

3. **Attribute review and ranking.** Afterwards, participants where shown the identified attributes via the computer application. The attributes were presented in the order they were identified. Participants were told that these attributes have been identified as the most important attributes. They could in this phase add or remove attributes if they felt that the attributes were not representative for their optimal alternative. After review of the attributes, they could click on “OK” to go to the next page. Consistently going to the next page was always made by clicking on the “OK” button. On this page participants had to rank their attributes after importance by distributing 100 points among the attributes. They were told that they are not allowed to give same points to two different attributes nor give zero to any of the attributes. If they did so, the application would give them a message reminding them of the constraints. In addition, they could not go to the next page until the sum of the given preference rating values was equal to 100.

4. **Alternative presentation.** The next page was an information page where participants were instructed about what was going to be shown on the coming sections. They were told that they would see five alternatives at the same time and that the alternatives were based on the decision task they were given earlier. The alternatives would consist of the attributes that were identified earlier. The alternative sets would repeat ten times, each time with five different alternatives in each set. In each set, participants were asked to, as fast as possible choose the most promising alternative (Appendix A). Before continuing to this section, participants were shown an example of how a set of alternatives would be presented. Participants could ask whatever question they had in order to understand the presentation of the alternatives in the sets. Once they indicated that they had understood how the alternatives were going to be presented, they could continue to the next page. The attributes were presented in the order that they were identified which in most cases differed from participants ranking order. In this page each attribute in the alternatives was shown as a pie diagram, where a full circle meant the best possible value on that attribute and a half circle meant the half good value and so forth (Appendix C). Each set was timed in order to see how long it took the participant to choose the most promising alternative. Participants were not aware of the timing. The alternatives in each set were generated by the computer application according to the following constraints:

- One alternative had the shortest non-weighted Euclidean distance. This would be equivalent to selecting the alternative which was most similar (in terms of non-weighted Euclidean distance) to the optimal alternative.
- One alternative had the best value on the most important attribute, which would be equivalent to using the lexicographic rule.
- One alternative was most similar to the optimal alternative, in terms of shortest weighted Euclidean distance, which gave us same ranking order as the rank order given by the participants. We call this rule for rank-order rule and the alternative that applies this rule for the rank-order alternative (for proof see calculation further in this paper).
- One alternative had the best overall attribute value, which is equivalent to using the maximin rule.
- One alternative had none of above constraints.

The alternatives were generated in a way so that they did not fall into other alternative constraints. In addition, they also had MAU that was 80% of the optimal alternative. The generation and position of the presented alternatives were random. Participants could not change their choice once made.

5. **Think aloud while choosing an alternative.** When finished with the ten sets, the participants could go to next page where they received another set of 5 alternatives. However, this time they had to think aloud while choosing the most promising alternative. The data from this task are not reported in the present paper.

6. **Preference rating.** After finishing thinking aloud, participants were again shown two sets. This time the alternatives were shown one at a time and participants had to separately preference rate how attractive from 0 to 100 each alternative appealed to them. Here 0 meant not attractive at all and 100 meant extremely attractive.

7. **Permutation presentation.** Finally participants were given all possible pair permutation of the attributes. They were told to imagine optimal values on the given pair permutation and judge between 0-5, were 0 meant not likely and 5 meant very likely, how likely they thought it was for that specific pair permutation to exist in reality (Appendix B). The data from this task are not reported in the presented paper.

**Measurement**

Similarity was measured by inverting the following two distance functions: (1) Non-weighted Euclidean distance (distance = sum of squared deviations between a given alternative and the optimal alternative across a number of attributes, with no regard to the importance of attributes). This is equal to calculating the shortest Euclidean distance alternative. (2) Weighted Euclidean distance, (distance = sum of squared products of importance weights and deviations between a given alternative and optimal alternative across attributes) this was equivalent to having an alternative that had the same rank-order as the optimal alternative. The non-weighted Euclidean distance function was used because this function has been used in calculating distances in metrics and could be used to calculate distance to the optimal alternative. The weighted Euclidean distance was used because it is assumed that the weights or the rank-orders that are given to different attributes in an optimal alternative are important, and should therefore be included in the calculation. Participants themselves in relation to what they considered as an optimal alternative defined the attributes. The optimal alternative was assumed to consist of maximally positive values (e.g. maximally positive location, size, and rent). Other collected data were the time it took each participant to come to decision, participants decision-making styles (DMS), preference rating, whether participants thought about an optimal alternative (TOOA), and whether if they had been in the given decision making situation before.

Except application of the above-mentioned similarity measures, and the application of decision-making rules, the alternatives had adjusted MAU value. The reason for this was to keep the alternatives similar so that they would differ only in decision making
rules and measured similarity. In order to apply MAU values that were 80% of the optimal alternative, we first calculated MAU for the optimal alternative, and then we reduced MAU for the generated alternatives with 20%. The following function. The MAU for the optimal alternative (MAU$_{opt}$) was calculated by:

$$MAU_{opt} = \left( \sum_{i=1}^{n} o w_i \right)$$

Where $o$ is the optimal values that are considered always be 100 and $w_i$ is the given attribute weights by the participants. In order to get the desired MAU ($MAU_D$) that each generated alternative should have which is 80% of the optimal MAU$_{opt}$, we used the following function:

$$MAU_D = 0.8 \times MAU_{opt}$$

We ask the computer to generate random numbers $R_i$ with mean=80 (80% of the optimal value 100) and SD=10. Our generated alternative has MAU:

$$MAU_R = \sum_{i=1}^{n} R_i w_i$$

$MAU_R$ will most likely not be equal to $MAU_D$, we therefore modify $R_i$ such that $MAU_R = MAU_D$. We do this compensation by multiplying each $R_i$ by a constant $c$, to get the desired MAU. The constant is:

$$c = \frac{MAU_D}{MAU_R}$$

So we will now have attribute values $a_i$, where $a_i = R_i c = R_i \frac{MAU_D}{MAU_R}$

We repeat generating random numbers to ensure that $0 \leq a_i \leq 100$. We verify that our alternative will have MAU equal to $MAU_D$:

$$MAU_D = c \times MAU_R = \sum_{i=1}^{n} a_i w_i = c \sum_{i=1}^{n} R_i w_i$$

The shortest non-weighted Euclidean distance ($d$) was calculated by applying an extended Pythagorean Theorem as shown:

$$d = \sqrt{\sum_{i=1}^{n} (I - a_i)^2}$$

The weighted Euclidean Distance (WED) between an alternative $j$, (i.e. the rank-order alternative) and the optimal alternative is defined as:

$$WED_j = \sqrt{\sum_{i=1}^{n} w_i^2 (I - a_i)^2}$$

(1)

Where $w_i$ is the importance weight of attribute $i$, $a_i$ is the attractiveness value on attribute $i$, and $I$ is the attractiveness value on any attribute of the optimal alternative ($I$ assumed to be the same for all attributes).

**Proof for the ranking rule**

We will show, for a number of alternatives that all have the same level of MAU for a given set of importance weights $w_1$ to $w_n$, that $WED_j$ is minimized for an alternative $j$ for which it is true for all attributes $i$ that
\[ w_j(I - a_{ij}) = K, \quad \text{(2)} \]

where \( K \) is a constant. Equation 2 implies that the higher the value of \( a_{ij} \) the higher will the value of \( w_j \), which is equivalent to the ranking rule. To understand why Eq. 2 must be true, assume a case where Eq. 2 holds for the two attractiveness levels \( a_1 \) and \( a_2 \) for a given alternative \( j \), implying that

\[ w_1(I - a_1) = w_2(I - a_2). \quad \text{(3)} \]

We will now show that Eq. 3 is true when WED is minimized, given that MAU is constant across alternatives. The assumption of constant MAU implies that

\[ w_1a_1 + w_2a_2 = C, \quad \text{(4)} \]

where \( C \) is a constant. To simplify the proof we make the following definitions.

\[ I_1 = w_1 I \quad \text{(5a)} \]
\[ I_2 = w_2 I \quad \text{(5b)} \]
\[ X_1 = w_1 a_1 \quad \text{(5c)} \]
\[ X_2 = w_2 a_2 \quad \text{(5d)} \]

Eq. 4 can now be written

\[ X_1 + X_2 = C. \quad \text{(6)} \]

Solving for \( X_1 \) in Eq. 7 yields

\[ X_1 = k - X_2. \quad \text{(7)} \]

Inserting equations 5a, 5b, and 7 in the WED equation (in line with Eq. 1, squared version) for alternatives with constant MAU (Eq. 7.) varying on two attributes 1 and 2 yields:

\[ \text{WED}^2 = (I_1 - (C - X_2))^2 + (I_2 - X_2)^2 \quad \text{(8)} \]

Reducing Eq. 8 yields
\[ \text{WED}^2 = I_1^2 + C^2 + X_2^2 - 2CX_2 - 2I_1C + 2I_1X_2 + I_2^2 + X_2^2 - 2I_2X_2 \quad (9) \]

Derivating Eq. 9 as a function of \( X_2 \) yields

\[ \text{WED}^2 = 2X_2 - 2C + 2I_1 + 2X_2 - 2I_2. \quad (10) \]

Rearranging the terms in Eq. 10 yields

\[ \text{WED}^2 = 4X_2 - 2C + 2(I_1 - I_2). \quad (12) \]

To minimize \( \text{WED}^2 \) we let \( \text{WED}^2 = 0 \) implying that

\[ 0 = 4X_2 - 2C + 2(I_1 - I_2). \quad (13) \]

Inserting definitions 5a, 5b, 5c, and 5d and Equation 6 (constant MAU) into Equation 13 yields after reducing the equation

\[ 0 = 4w_2a_2 - 2w_1a_1 - 2w_2a_2 + 2w_1I - 2w_2I. \quad (14) \]

Reformulating and canceling terms in Equation 14 yields after division by 2

\[ 0 = w_2a_2 - w_1a_1 + w_1I - w_2I \quad (15) \]

Rearranging and simplifying Eq. 15 yields

\[ w_1(I - a_1) = w_2(I - a_2). \quad (16) \]

Q.E.D.

Finally, it will be shown how the proof described above can be generalized to cases involving more than two attributes (cf. Eq. 2). Assume a case where a given set of alternatives are characterized on \( n \) attributes, where all alternatives \( i \) have the same MAU, i.e.

\[ w_1a_{1i} + w_2a_{2i} + \ldots + w_na_{ni} = C, \]

where \( C \) is a constant. In line with Eq. 2 it is assumed that \( b_j(I - a_{ij}) \), i.e., the weighted distance to \( I \) on a particular attribute, is the same across all attributes. Consider now any pair \( w_ix_j \) and \( w_jx_j \) of weighted attributes sampled from the \( n \) attributes. Going through the steps described in Eqs. 3 to 16 for this pair it can be seen that when MAU is kept constant for these two attributes, WED is minimized when \( w_i(I - a_i) = w_j(I - a_j) \), that is when the weighted distance to \( I \) is the same for the two attributes. This means that any deviation from Eq. 3 for any pair of attributes, which occurs without changing the weighted sum (MAU) of the two, attributes leaves, implies that the over all WED will increase. This means
in turn that in the case when MAU is the same for all alternatives, WED will be minimized when Eq. 3 holds for all attributes that characterize the alternatives.

**Results**

In order to identify the existence of an optimal alternative a content analysis was performed on the transcribed think aloud protocols (Svenson, 1989). First, the protocols were divided into statements (e.g. sentence with one main idea). Each statement was coded into one category. The question for the analysis was: Do individuals think of an optimal alternative when they are faced with a multi-alternative decision making situation? Therefore, one category corresponded to reference to an optimal alternative. In all, the following six categories were identified.

- **Reference to an optimal alternative.** The concept of optimal alternative was operationalized by looking for words or phrases that indicated that the participant had one or more specific traits or attributes that desired. This category was the only one that had to do with the optimal alternative. One example of statement in this category is, “...*Okaaay, when I am buying a house, I think that it should have good location...*”

- **Negative aspects.** This category contained words or phrases that mention negative aspects of attributes or alternatives. This category does not directly inform about the optimal alternative. Instead, it informs about attribute or alternatives that will not be considered in the decision-making. One example of statement in this category is, “...*But this one is much smaller and ....*”

- **Decision strategies.** This category contained words or phrases that not directly relate to the optimal alternative, but indicate the strategies used to make a decision. For instance, they can say, “I would remove alternatives that have bad attributes”. One example of statement in this category is, “...*One should not buy a car when it is dark, standing in someone’s garage because then you will not find any flaws....*”

- **Influence of others.** This category consisted of words or phrases that indicated that the participant took great care of what other thought about their way of thinking, deciding and of the made decision. For instance they could say “of course I care about what my family thinks.....”. One example of statement in this category is, “...*I want to have my father with me because I don’t know much about cars....*”

- **Experience related.** This category contained words or phrases about experiences and previous knowledge. A typical phrase in this category could be “once I bought a car that......”. One example of statement in this category is, “...*I have made similar decision three times in my life so....*”

- **Not task-related.** In this category, we put statements or words that did not have anything to do with the task or were general sentences. One example of statement in this category is, “...*this task was difficult ....*”

Table 1 shows the frequencies of the different categories. Verifying our first hypothesis, it can bee seen that the reference to an optimal alternative category was by far the most mentioned category.

<table>
<thead>
<tr>
<th>Category</th>
<th>Frequency</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reference to an optimal alternative</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Negative aspects</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Decision strategies</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Influence of others</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Experience related</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Not task-related</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1. Frequency of the different categories in the think aloud protocols
Table 2 shows that the rank-order alternative clearly was most frequently chosen in accordance with our second hypothesis. A one-way repeated measures ANOVA on the choices shows that the choice frequencies differed significantly (F(4,176) = 16.716, p<0.005).

Table 2. Choice frequency table for the chosen alternatives out of 450 alternatives.

<table>
<thead>
<tr>
<th>Chosen alternative</th>
<th>Frequency</th>
<th>%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank-order</td>
<td>187</td>
<td>41.5</td>
</tr>
<tr>
<td>Lexicographic</td>
<td>77</td>
<td>17.1</td>
</tr>
<tr>
<td>Maximin</td>
<td>80</td>
<td>17.8</td>
</tr>
<tr>
<td>Euclidean</td>
<td>59</td>
<td>13.1</td>
</tr>
<tr>
<td>No specified rule</td>
<td>47</td>
<td>10.4</td>
</tr>
</tbody>
</table>

N=45.

In the preference-rating task, participants were presented with alternatives taken from two sets, meaning that participants had to preference rate each rule twice. As shown in table 3, the rank-order alternative got the highest ratings which is substantiated by a strong significant effect found in a within-subject ANOVA (F(4,120) = 3674.471, p<0.001). This verifies our third hypothesis. The table also shows the minimum (Min), maximum (Max), mean (Mean) and standard deviation (SD) for the preference ratings. One-way repeated measures ANOVA showed that there was no difference between the alternatives from the two sets.

Table 3. Minimum, maximum, mean and standard deviation table for the preference ratings.

<table>
<thead>
<tr>
<th>Alternative</th>
<th>Min.</th>
<th>Max.</th>
<th>Mean</th>
<th>SD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Rank-order</td>
<td>30</td>
<td>100</td>
<td>65.5</td>
<td>14.3</td>
</tr>
<tr>
<td>Lexicographic</td>
<td>7</td>
<td>100</td>
<td>55.9</td>
<td>20.2</td>
</tr>
<tr>
<td>Maximin</td>
<td>5</td>
<td>100</td>
<td>59.2</td>
<td>19.6</td>
</tr>
<tr>
<td>Euclidean</td>
<td>1</td>
<td>95</td>
<td>50.6</td>
<td>21.5</td>
</tr>
<tr>
<td>No specified rule</td>
<td>3</td>
<td>90</td>
<td>53.2</td>
<td>18.6</td>
</tr>
</tbody>
</table>

N=45
Out of the 45 participants, when asked whether they thought about an optimal alternative, 34 (75.6 %) said yes. Out of these 15 were given the car task and 19 were given the apartment/house task. Eleven of the participants (24.4 %) said that they did not think about an optimal alternative. Of the 45 participants, when asked whether they had been in the same type of decision-making task situation before, 23 (51.1 %) said yes and 22 (48.9 %) said no. An independent sample t-test showed that there was no significant relationship between whether participants thought about an optimal alternative and at the same time chose the rank-order alternative. Neither was there a significant relationship between the chosen alternative and decision-making task, gender, thought of optimal alternative, and whether participants had been in the decision-making situation before. However there was a significant positive correlation between age and number of times participants chose the rank-order alternative ($r=0.261$, $n=45$, $p<0.05$, one-tailed).

There was a significant positive correlation between “thought of an optimal alternative” and whether participants had been in the same decision-making task situation before ($r=0.271$, $n=45$, $p<0.05$, one-tailed). No significant correlations were found between thought of optimal alternative, decision-making task, age, gender, and as chosen alternatives: rank-order or Euclidean.

**Discussion**

One of our main hypotheses we had was that individuals think of an optimal alternative when they are in decision-making situations. Therefore, we gave participants a decision-making situation where they had to think aloud while making a decision. Directly afterwards we asked them if they thought about an optimal alternative, and analyzed their think aloud protocols. When asked whether they thought about an optimal alternative, the majority of the participants reported that they did. In addition, 73 % the results from the content analysis was about the optimal alternative. This suggests, firstly that individuals think of an optimal alternative and secondly that the method used for identifying the optimal alternative was adequate.

The support of the notion that the participants did think of an optimal alternative gave the opportunity to test the second hypothesis of this study, that individuals choose the alternative that is closest to their optimal alternative. Therefore, participants were presented with different alternatives with different constraints. Two of these alternatives were supposed to be closest to the optimal alternative. Results showed that the most frequent chosen alternative was the alternative that had the same rank order between the attributes as the optimal alternative. This alternative was the one that was closest to participant’s optimal alternative, in terms of shortest weighted Euclidean distance. However, few participants chose alternatives that applied the non-weighted Euclidean distance. These results, not only support the second hypothesis, but it also suggests that the weights that individuals give to different attributes are important. The importance of the weights indicates that individuals prefer to base their decision not only on all the attributes but also on their internal relationships, in terms of the distribution of the weights of the attributes. These results are contradictory to the results from Zakay and Dil (1984), where they measured the distance without using weights and still received results. According to other decision-making approaches, usually, one scans one attribute at a time and does not consider the whole alternative. For example, imagine that you want to buy an apartment and the most important attributes for you are size, price, and location, in that order. According to lexicographic rule, it is enough if you know which of the alternatives that has the best values on size. However, one alternative having the best size can have very bad location and price. According to present findings, you probably want to know the values of the other attributes as well. Consistent with Festinger (1964), this alternative is also consistent with existing beliefs and preferences. The optimal alternative is an already set-
up preference, so choosing an alternative that is as close as possible to the optimal alternative decreases potential dissonance in beliefs and preferences.

The present results and the explanations are based on alternatives that have constant MAU, which in this sense were equally attractive. Therefore one counterargument to the rank-order rule can be that individuals will not choose this alternative if it has unattractive values, while other alternatives have very attractive values. It is not believed that if decision makers have alternatives with extremely high attribute values and a rank-order alternative with extremely low attribute values, the rank-order alternative will be the preferred one. However, as shown in several studies, in most situations, decision makers, will at an early stage exclude alternatives that have unattractive attribute values.

Not only was the rank-order alternative the most chosen alternative, it also received the highest values when participants separately preference rated each alternative. This, not only supports the third hypothesis but it also strengthens the assumption that a comparison is made between the given alternatives and the optimal alternative, and not between the alternatives. In addition, this also suggests that it does not matter if the alternatives are compensatory or non-compensatory.

The generalizability of the present study may be criticized because the findings were in context of the way the alternatives were presented, namely pie-diagrams. This way of presentation could have had an effect on the choices and the outcome could have differed if the alternatives were presented differently.

During this study, two questions came to mind: Do individuals have only one optimal alternative? Do individuals have optimal alternatives no matter what the decision-making situation is? I believe that individuals have several optimal alternatives but they can only have one active at a time. In addition, it is believed that in most situations individuals has an image of what the optimal alternative is. The aim is to investigate these two questions in future studies. It is also not known how individuals reason when there are no rank-order alternatives present in a decision-making situation. Therefore, for future studies I also aim to give participants a set of alternatives where they do not have the rank-order alternative, to see what the outcome will be.

In summary, the hypotheses of the present study were supported. Strong evidence for the existence of an optimal alternative was found. It was also found that optimal alternatives are present when one is in a decision-making situation, suggesting that that they have an important impact in decision-making processes.

References


Appendix A - Screen Shots of the computer application

Participants were shown their most important attributes.
Participants were told to distribute 100 points among the attributes.

If they sum of the points was not 100 or gave same point or 0, they got this message.

Instructions about what is going to happen in the coming sections
An example of the given presentation of the alternatives.

Message about the next page, which is going to be think aloud.
Instruktioner


Instructions about the attractiveness pages.
Example of a preference rating page.
Instruktioner

På nästa sida får du ett antal egenskaper som presenteras parvis. Du ska föreställa dig *dina optimala* värden på egenskaperna. Ange hur väl det stämmer att dessa egenskaper kan förekomma tillsammans i verkligheten (Se 1-5 skalan nedan).

Exempelvis kan ett par egenskaper om semester vara <Våder, Pris>.
Eftersom det finns många länder där både optimalt pris och optimalt väder finns samtidigt, så är kanske sannolikheten stor att dessa egenskaper förekommer tillsammans. Då kan du välja <5> som betyder 'Stämmer precis'. Däremot är kanske <idpunkt, vinteraktivitet> inte så sannolikt om ditt optimala värde på väder är sommar eftersom det är svårt att äka skidor under sommaren och då kan den få <2> - Stämmer ganska dåligt.

Skala:
1. Inte alls sannolikt
2. Stämmer ganska dåligt
3. Stämmer delvis, delvis inte
4. Stämmer ganska väl
5. Stämmer precis

Instruction about the permutation section.
Hur pass bra stämmer det att följande par med dina optimala värden finns:

(Price), (Location)

Skala:
1 - Inte alls sannolikt
2 - Stämmer ganska dåligt
3 - Stämmer delvis, delvis inte
4 - Stämmer ganska väl
5 - Stämmer precis

Example of the permutation presentation

Message indicating the end of the experiment.
Appendix B – User example of the presented alternatives

This picture was shown to the users, to show them how the alternatives were going to be presented. They were told that the attributes were about a vacation. Each column represented one alternative. Each pie diagram was a given value for a specific attribute in that alternative.
Appendix C – Template on the different values on the pie diagrams.

This Picture was given to the users as template of the different levels on the pie diagrams.