Angle-only based collision risk assessment for unmanned aerial vehicles

Examensarbete utfört i Reglerteknik vid Tekniska högskolan i Linköping

av

Fredrik Lindsten

LiTH-ISY-EX--08/4192--SE

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Linköping, 28 November, 2008
This thesis investigates the crucial problem of collision avoidance for autonomous vehicles. An anti-collision system for an unmanned aerial vehicle (UAV) is studied in particular. The purpose of this system is to make sure that the own vehicle avoids collision with other aircraft in mid-air. The sensor used to track any possible threat is for a UAV limited basically to a digital video camera. This sensor can only measure the direction to an intruding vehicle, not the range, and is therefore denoted an angle-only sensor. To estimate the position and velocity of the intruder a tracking system, based on an extended Kalman filter, is used. State estimates supplied by this system are very uncertain due to the difficulties of angle-only tracking. Probabilistic methods are therefore required for risk calculation.

The risk assessment module is one of the essential parts of the collision avoidance system and has the purpose of continuously evaluating the risk for collision. To do this in a probabilistic way, it is necessary to assume a probability distribution for the tracking system output. A common approach is to assume normality, more out of habit than on actual grounds. This thesis investigates the normality assumption, and it is found that the tracking output rapidly converge towards a good normal distribution approximation.

The thesis furthermore investigates the actual risk assessment module to find out how the collision risk should be determined. The traditional way to do this is to focus on a critical time point (time of closest point of approach, time of maximum collision risk etc.). A recently proposed alternative is to evaluate the risk over a horizon of time. The difference between these two concepts is evaluated. An approximate computational method for integrated risk, suitable for real-time implementations, is also validated.

It is shown that the risk seen over a horizon of time is much more robust to estimation accuracy than the risk from a critical time point. The integrated risk also gives a more intuitively correct result, which makes it possible to implement the risk assessment module with a direct connection to specified aviation safety rules.
Abstract

This thesis investigates the crucial problem of collision avoidance for autonomous vehicles. An anti-collision system for an unmanned aerial vehicle (UAV) is studied in particular. The purpose of this system is to make sure that the own vehicle avoids collision with other aircraft in mid-air. The sensor used to track any possible threat is for a UAV limited basically to a digital video camera. This sensor can only measure the direction to an intruding vehicle, not the range, and is therefore denoted an angle-only sensor. To estimate the position and velocity of the intruder a tracking system, based on an extended Kalman filter, is used. State estimates supplied by this system are very uncertain due to the difficulties of angle-only tracking. Probabilistic methods are therefore required for risk calculation.

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Fredrik Lindsten
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Abbreviations

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<tr>
<td>ATC</td>
<td>Air traffic control</td>
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<td>CV</td>
<td>Constant velocity</td>
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<td>EKF</td>
<td>Extended Kalman filter</td>
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<td>ENU</td>
<td>East, north, up (Cartesian coordinate system)</td>
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<td>EO</td>
<td>Electro-optical</td>
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<td>FL</td>
<td>Flight level</td>
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<td>IFOV</td>
<td>Instantaneous field of view</td>
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<td>Instrument flight rules</td>
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<td>Interacting multiple model</td>
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<td>MSC</td>
<td>Modified spherical coordinates</td>
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<td>RMSE</td>
<td>Root mean squared error</td>
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<td>TCAS</td>
<td>Traffic collision avoidance system</td>
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<td>UAV</td>
<td>Unmanned aerial vehicle</td>
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<td>VFR</td>
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State vectors

Absolute Cartesian coordinates

\[ x_{\text{own}}^A = \begin{pmatrix} s_{x|\text{own}}^A & s_{y|\text{own}}^A & s_{z|\text{own}}^A & v_x^A_{|\text{own}} & v_y^A_{|\text{own}} & v_z^A_{|\text{own}} \end{pmatrix}^T \]

\[ x_{\text{int}}^A = \begin{pmatrix} s_{x|^\text{int}}^A & s_{y|^\text{int}}^A & s_{z|^\text{int}}^A & v_x^A_{|\text{int}} & v_y^A_{|\text{int}} & v_z^A_{|\text{int}} \end{pmatrix}^T \]

Absolute Cartesian coordinates relative the own vehicle

\[ x_{\text{rel}}^A = x_{\text{int}}^A - x_{\text{own}}^A = \begin{pmatrix} s_v^A & s_y^A & s_z^A & v_x^A & v_y^A & v_z^A \end{pmatrix}^T = \begin{pmatrix} x_1^A & x_2^A & x_3^A & x_4^A & x_5^A & x_6^A \end{pmatrix}^T \]

Relative Cartesian coordinates

\[ x^R = \begin{pmatrix} s_x^R & v_x^R & v_y^R & v_z^R \end{pmatrix}^T = \begin{pmatrix} x_1^R & x_2^R & x_3^R & x_4^R \end{pmatrix}^T \]

Modified spherical coordinates

\[ z = \begin{pmatrix} 1 & \varphi & \theta & \frac{r}{r} & \varphi \cos \theta & \dot{\theta} \end{pmatrix}^T = \begin{pmatrix} z_1 & z_2 & z_3 & z_4 & z_5 & z_6 \end{pmatrix}^T \]
Chapter 1

Introduction

In this chapter the background of the thesis is presented. Three different problems are formulated stated, together with the objectives that the thesis aims at achieving. The existing work on the same area is summarized to serve as a reference for further studies on the topic. The chapter ends with the outline of the thesis.

1.1 Background

Unmanned aerial vehicles (UAVs) are likely to be a much more common element in aviation in the near future. The interest for UAVs is constantly growing, and there are many possible applications, both military and civil. Amongst others can be mentioned reconnaissance, firefighting, police observation and operations in hazardous environments. Many of these applications will require the UAV to operate in the same airspace as other aircraft, both manned and unmanned. One critical problem that has to be dealt with is therefore how to avoid a collision between the UAV and another aircraft.

A near mid-air collision (NMAC) occurs if the distance between two aircraft ever becomes less than a predefined safety distance. The horizontal safety distance is approximately 150 m (500 ft) and the vertical safety distance is approximately 75 m (250 ft) [29]. NMAC occurs if both the horizontal and vertical safety distances are violated at the same time.

Today, ground-based air traffic control (ATC) is used to provide human pilots with information on how to maneuver their aircraft so that possible NMAC scenarios do not occur. However, in the last decade semi-automatic systems like the traffic collision avoidance system (TCAS) have been put into use. These systems more or less move the responsibility for avoiding conflicts from the ATC to the pilot. The idea with TCAS is that aircraft exchange data on their speed, height and
Introducing new methods. This information can then be used by the system to determine the risk for NMAC, and in the case of too high risk provide the pilots with information on appropriate evasive maneuvers. The problem with these systems, in the case of collision avoidance for UAVs, is that they require all aircraft in the same airspace to be equipped with transponders. In future applications, UAVs are likely to operate on altitudes where small intruding aircraft are present. These aircraft seldom carry transponders, and consequently the requirement is not fulfilled.

One solution to the problem, which will be focused on in this thesis, is to equip the UAV with sensors so that it can measure the state of intruding aircraft, instead of acquiring this information via a data link. Typically we are interested in knowing relative position and velocity. A good sensor for this purpose is radar, which is widely used in applications where the measurements of these states are critical. However, for small UAVs the radar antenna weight and weight of the required power supply are prohibitive [28]. Radars are also expensive, which further adds to the desire to use another type of sensor. Since many UAV applications, such as reconnaissance and surveillance, require the aircraft to be equipped with a camera it would be weight efficient, and therefore desirable, to use this sensor to measure the state of intruding aircraft as well [28]. The work presented in this thesis will be based on the use of passive angle-only sensors, such as digital cameras.

1.2 Problem formulation

The problem of avoiding collision with an intruding aircraft, given information from a passive angle-only sensor, can be divided into the steps shown in Figure 1.1.

The task starts with the detection of a possible threat, i.e. when an intruding aircraft becomes visible to the UAV’s sensor. Two different situations are possible. The first is that the intruder appears literally out of the blue, i.e. it comes close enough to be distinguishable in the sensor. In this case the detection distance is rather large; typically a few kilometers. The second case is that the intruder flies into the cameras field of view, e.g. by appearing from behind or below. In this case the detection distance can be arbitrary small, and the aircraft can hence be very close to each other even when the threat is initially perceived.

When the intruder has been detected the tracking will start. The direction to the intruder is extracted from the camera images and passed to the tracking system. By using this information the system continuously tries to estimate the position and velocity of the target, together with the level of certainty in this estimation. Given this state estimate and certainty measure, the risk calculator’s task is to compute the probability that an NMAC will occur. The calculated probability is used to determine whether the last step, initiating an evasive maneuver, should be triggered or not.
1.2 Problem formulation

Figure 1.1. The different steps of a collision avoidance system

The focus in this thesis will be on the risk calculation. However, to be able to evaluate this step it has to be put in its context. Most important is the interface between the tracking and the risk calculation. To perform the risk calculation, several assumptions about the information provided by the tracking system are made. To be able to determine whether these assumptions are sound or not, the actual outcome of the tracking system must be analyzed.

To divide the evaluation of the collision avoidance system into smaller parts, three different problems are formulated.

1.2.1 Problem 1: Identifying the properties of the tracking system output

The first problem that will be addressed in this thesis is to identify the properties of the tracking system output. The thesis does not aim at finding any new, improved means of performing the tracking. There is a lot of literature on this field (see Section 1.4 for references) and the topic is widely explored. However, the existing
literature tends to focus on evaluating the actual state estimation. This is not strange since this is the main output of the tracking system. Though, to be able to use the output of the tracking as a basis for risk calculation, several other aspects need to be taken into account as well.

These are mainly concerned with the certainty measure of the state estimation. As previously mentioned the tracking does not only estimate the position and velocity of the intruder, but also the level of certainty in this estimation. The certainty measure is given in the form a covariance matrix. Let the estimated state of the intruder be denoted \( \hat{x} \) and the corresponding covariance matrix \( \hat{P} \). When performing probabilistic risk calculation based on these variables, the true state of the intruder is assumed to be a multivariate normally distributed stochastic variable according to

\[
x \sim N(\hat{x}, \hat{P})
\]  

If this assumption is too far from the truth, the risk calculation will evidently fail. It is thus important to study the true distribution and covariance of the intruder’s state, to see if the assumption is applicable.

1.2.2 Problem 2: Defining the probability of NMAC

The second problem addressed is to determine how the probability of NMAC should be defined to get a method for risk calculation which is robust to the large uncertainties associated with camera based tracking. Two different viewpoints will be analyzed, one from which the event of NMAC is considered to occur only momentarily and one where the event of NMAC is considered over a period of time. How the definition of the event of NMAC differs in these two cases is described in the following paragraphs.

Let \( t \) be the absolute time, \( t_p \) the prediction time and \( T_p \) the prediction time horizon. This means that, at every time point \( t = T \) the risk calculation will be performed. Obviously the risk calculator has to make a prediction of the future states of the involved vehicles; otherwise it would not be able to ‘see’ a collision until it actually occurs. The predictions are made at the relative time points \( t_p \) over the interval \( 0 < t_p < T_p \), which are equal to the absolute points of time \( t \) over the interval \( T < t < T + T_p \). The absolute and relative timescales are depicted in Figure 1.2.
The definition of the momentary event of NMAC at time $t_p$ then becomes

$$\text{NMAC}_{t_p} = |s(t_p)| < R$$  \hspace{1cm} (1.2a)

where $s(t_p)$ is the relative distance between the two vehicles at time $t_p$ and $R$ is the radius of the safety zone. Here, the safety zone has been approximated with a sphere instead of a puck, which vastly simplifies the risk calculation. By using $R = 150$ m, i.e. the largest of the horizontal and vertical safety distances, this approximation will be pessimistic in the sense that the calculated collision risk will be somewhat higher than the actual risk. The interesting probability when using this definition of NMAC is the maximum momentary probability over the prediction horizon, i.e.

$$\max_{0 < t_p < T_p} P(\text{NMAC}_{t_p})$$  \hspace{1cm} (1.2b)

In words this probability can be interpreted as; from the current time and during the specified time horizon, how high is the risk of NMAC when the risk is at its peak?

In contrary, the definition of the event of NMAC over a period of time becomes

$$\text{NMAC}_{(0,T_p)} = \left( \min_{0 < t_p < T_p} |s(t_p)| \right) < R$$  \hspace{1cm} (1.3a)

In this case we consider the event of NMAC over the entire time horizon as one event, and the interesting probability simply becomes

$$P(\text{NMAC}_{(0,T_p)})$$  \hspace{1cm} (1.3b)

In words this probability can be interpreted as; from the current time and during the specified time horizon, how high is the risk that NMAC will occur at any time?

The advantage of using (1.2) instead of (1.3) is that the sought probability is easier to calculate, since only one time point has to be treated simultaneously. The problem with this definition, on the other hand, is that it is believed to diminish the probability of NMAC. This is due to the large uncertainties associated with
angle-only tracking. If the state estimate is very uncertain, no distinct peak will be present in the calculated probabilities and the maximum of the momentary probability will not reflect the total risk.

It is expected that the definition of NMAC according to (1.3) does not suffer from this problem to the same extent, since the risk is integrated over time. Whether this is the case or not will be the main objective to investigate when comparing the two different definitions.

1.2.3 Problem 3: Validating an approximate method for risk calculation

If it can be shown that the definition of NMAC according to (1.3) is to prefer, a method for computing the probability is required. This leads to the third problem addressed in this thesis, namely to evaluate the performance of the approximate method for calculating $P(\text{NMAC}(0,T_p))$ presented by Nordlund and Gustafsson [31, 32]. An approximate method is necessary since the calculation of $P(\text{NMAC}(0,T_p))$ using a straightforward approach is computationally intractable.

1.3 Objectives

This thesis aims at examining the outputs of the tracking in a collision avoidance system to see if the assumption according to (1.1) is appropriate to make. The thesis also aims at determining which of the two considered definitions of NMAC that is most suitable to use in the risk calculation step of a collision avoidance system. It furthermore aims at evaluating the approximate method for calculating $P(\text{NMAC}(0,T_p))$ presented by Nordlund and Gustafsson [31, 32], to determine whether this method is appropriate to use in the risk calculation or not.

1.4 Existing work

The collision avoidance problem is widely studied, although the existing work tends to focus on the individual steps of the problem rather than the whole. Especially the problem of angle-only target tracking (also denoted angle-only target motion analysis or passive ranging) is thoroughly examined. The basics of target tracking are covered by Bar-Shalom and Li [9]. Blackman and Popoli cover most aspects of the target tracking problem and dedicates one chapter to angle-only tracking [10]. Holtsberg analyzes different approaches to the problem and how to find an optimal platform maneuver (platform maneuvering is discussed in Section 3.3 of this thesis) [22]. Allen and Blackman have presented an implementation of a target tracking system [6]. Erlandsson has implemented two different kinds of tracking systems and conducted an evaluation of these [15].
When it comes to conflict detection, different approaches are reviewed by Kuchar and Yang [27]. Traditionally, risk assessment is focused on a critical time point (time of closest point of approach, time of maximum risk etc.) as investigated by Chan [13, 14], Krozel and Peters [26] and Prandini et al. [33]. To perform the actual risk calculation, a computational method is necessary. An arbitrarily accurate approach is to apply Monte Carlo approximation, as investigated by Yang et al. [36] as well as Jansson and Gustafsson [23]. This might however not be feasible in a real-time implementation. Nordlund and Gustafsson have examined the collision risk based on the definition of NMAC over a horizon of time and also proposed an approximate method for calculating the collision probability [31, 32].

1.5 Limitations

To autonomously sense and avoid collision with intruding aircraft in mid-air is a very hard problem. A vast number of aspects and different parameters affect the situation. It is therefore necessary to make certain simplifications when conducting simulations on the problem.

This thesis only covers the case with a single intruder encountering the own vehicle at the same time. The intruder is assumed not to make any distinct maneuvers during the time of the collision scenario. To deal with the fact that an aircraft never travels on an exact straight line, noise has been added to the intruder’s acceleration. This causes a drift in the position, which can be seen as way of modeling small maneuvers as well. Intruders that make distinct maneuvers (e.g. rapid turns) can not be tracked by the system used in this thesis. If this case is to be covered, an extension of the tracking is required. This could for instance be accomplished by using an IMM-filter (interacting multiple model) in the tracking system [15].

In a real implementation the field of vision for the camera is typically limited to $\pm 110^\circ$ in the horizontal plane and $\pm 15^\circ$ in the vertical plane [29]. This means that an intruder only can be tracked when it is positioned ahead of the own vehicle. For all simulation scenarios presented in this thesis, as well as for most possible collision scenarios, this is the case. Therefore the intruder is assumed never to fall out of the field of vision.

The problem of target tracking is not pursued in detail in this thesis. The focus has been on the properties of the tracking system output and the interface between the tracking and the risk calculation. The tracking system implemented in the simulation environment is to large parts based on the one proposed by Erlandsson [15]. Since the number of parameters affecting the tracking system is very large, not all of them have been varied in the simulations performed in this thesis. It would be of much interest to perform a more exhaustive evaluation of the tracking system, in which the parameters’ individual effects on the output properties are deduced. This, however, lies beyond the scope of this thesis.
1.6 Outline

Chapter 1 presents the background of the thesis. Three problems are formulated and from these problems the aims of the thesis are derived.

Chapter 2 presents the simulation scenarios that are used throughout the thesis. These scenarios form the basis for the evaluation of the collision avoidance system.

Chapter 3 addresses the first problem, namely to identify the properties of the tracking system output. The chapter introduces the basics of target tracking and discusses the observability problem with which angle-only tracking is afflicted. Two different platform maneuvers are presented, which are used throughout the thesis. The chapter ends with a presentation and analysis of acquired simulation results.

Chapter 4 addresses the second problem, namely to deduce which of the two considered definitions of NMAC that is most suitable to use. The two definitions, along with their assumed differences, are presented at the beginning of the chapter. The results from several simulations are thereafter presented and discussed.

Chapter 5 addresses the third problem, namely to evaluate the approximate method for risk calculation proposed by Nordlund and Gustafsson [31, 32]. The chapter presents the nature of the approximations and discusses the need to simplify the risk calculation. The chapter ends by presenting and analyzing the results from several simulations in which the method is employed.

Chapter 6 summarizes the conclusions from the three different problems and presents ideas for future work on the same topic.
Chapter 2

Simulation scenarios

This chapter presents the collision scenarios that are used to evaluate the collision avoidance system. The chapter starts with a discussion on which factors that should be taken into account when choosing the scenarios. The geometry of a collision scenario and the parameters affecting the scenario are thereafter specified. The chapter ends with the definitions of those scenarios that are used throughout the thesis.

2.1 Principles for choosing scenarios

The performance of the collision avoidance system is evaluated by running a series of simulations. The outputs from the different parts of the collision avoidance systems, e.g. the tracking and the risk calculation, can then be compared with the actual outcome of the simulations. This raises the question of how to choose appropriate simulation scenarios, such that general conclusions can be drawn about the performance of the collision avoidance system.

One solution is to run simulations for all possible collision scenarios. This would guarantee that all interesting scenarios, along with the not so interesting, are covered in the simulations. This approach is however not tractable in practice, neither to implement nor to survey. Far too many different parameters affect the scenario, making the total number of scenarios that need to be considered way too high.

The idea of running simulations for a broad range of scenarios is still applicable though, namely by performing so called Monte Carlo simulations. This means that a nominal scenario is specified, which is of interest to study in more depth. Numerous simulations can be performed on the scenario, each in which the parameters of the scenario (e.g. the intruder’s velocity and the detection distance) are
Simulation scenarios

randomized. The results of the simulations can be analyzed by looking at the average performance of the collision avoidance system. This approach is appealing to use when the general behavior of the system is to be examined. It still requires the nominal scenario to be specified though, and a partial set of interesting scenarios must thus be picked out.

The prime aspect to consider when choosing these scenarios is that they should reflect situations that are likely to occur in reality. If they are not, conclusions drawn from the simulations cannot be applied to a real implementation. If the scenarios are not chosen properly it could for instance be that the system does not become tested for certain critical situations. It could then perform much worse in reality than in the simulations, making it hazardous to use. The opposite could also be a problem, i.e. that the scenarios used are too harsh and very unlikely to occur in reality. The system could then be rejected because of bad performance in the simulations, but in reality it would have operated more than sufficiently well.

It is furthermore preferable to use scenarios where the collision risk is high in some sense. If the system fails to perform satisfying in critical collision scenarios, it can be rejected without further evaluation. If it on the other hand shows good results on the critical scenarios it is more likely to handle the less demanding ones as well. Observe that in this context, a high collision risk does not mean that most scenarios should lead to collision. It simply implies that the collision risk should be of some significance. If all possible scenarios should be seen as a whole, the total collision risk would be extremely low. This is because most aircraft never even come close to each other in mid-air. By focusing on those scenarios where the aircraft actually encounter each other, the performance of a collision avoidance system will be much easier to evaluate.

The above condition is equivalent with that the minimum separation between the aircraft is rather small in all scenarios. An appropriate guideline is that the minimum separation is approximately on the same level as the radius of the safety zone. A scenario in which the two aircraft for instance never come closer than a kilometer apart would only result in a negligible risk for collision. Such a scenario would not be of any interest to study. The range of possible simulation scenarios can thus be reduced by only considering scenarios in which the minimum separation is less than, for instance, three to four times the radius of the safety zone.

2.2 Collision scenario geometry

The geometry of the different collision scenarios are described by the use of a geographic coordinate system. Throughout this thesis the ENU-system is used. This is a Cartesian coordinate system where the axes correspond to the directions east, north and up (hence the name). Absolute coordinates are given by the components $x^A$, $y^A$ and $z^A$ which refer to the three directions of the ENU-system respectively.
The trajectories of the involved aircraft are defined by their initial positions and the angles of their velocity vectors relative to the axes of the coordinate system. In this thesis the own vehicle is always initially placed at the origin of the coordinate system and the intruder starts straight to the east of the UAV. This choice is only made to simplify the illustrations of the geometry, and does not affect the properties of the scenario. The specific features of a scenario do of course only depend on the relations between the involved aircraft, and not on how the scene is rotated.

The headings of the aircraft are specified by the angles of their respective velocity vectors. The own vehicle holds the course given by the angle $\psi$ and descends with the angle $\gamma$ (or ascends if $\gamma$ is negative). The intruder’s course is given by $\alpha$ and the angle of descent is $\beta$. The geometry is depicted in Figure 2.1.

![Figure 2.1. Geometry of a collision scenario.](image1.png)

During the time of the simulations the aircraft will naturally progress along their respective trajectories. When viewing the scenario from the perspective of the own vehicle it can be of interest to let the origin of the ENU-system be fixed to the UAV. This is achieved by considering the relative velocity between the aircraft instead of their individual velocities separately. The relative velocity can be obtained by subtracting the own vehicle’s velocity from the intruder’s velocity, $v_{rel} = v_{int} - v_{own}$, i.e. by placing all velocity on the intruder. Figure 2.2 shows the geometry of the situation, when the origin is fixed to the UAV.

![Figure 2.2. Geometry of a collision scenario where the origin of the coordinate system is fixed to the own vehicle.](image2.png)
As can be seen from Figure 2.2, the heading of the intruder relative to the own vehicle is defined by two angles in a similar way as with the independent velocities. These are denoted $\alpha_{rel}$ and $\beta_{rel}$ respectively. Since the own vehicle is fix to the origin of the coordinate system, the minimum separation of the aircraft is the same as the minimum distance from the origin to the intruder. If both vehicles travel on straight line paths, i.e. $v_{rel}$ is constant, the case of NMAC can be determined by simply considering the initial separation and the angles describing the relative velocity.

2.3 Encounter model

The geometry of a collision scenario, as described in the previous section, declares the parameters of the scenario. These parameters can be used to describe a scenario, but the geometry on its own does not define it. To do this, appropriate values need to be assigned to the parameters. The idea with an encounter model is to reason about the specific properties of a typical near mid-air collision. This forms a basis to build the definitions of the scenarios on. The model used here is derived from physical reasoning, empirical knowledge and statistics of surveyed data, e.g. radar tracks of small sized aircraft. The aspects covered by the model are initial separation, aircraft velocities, climb rates and angle of approach.

The statistics of surveyed data used to build the encounter model in this thesis are presented by Kochenderfer et al. The data comes from radar tracks of aircraft flying under visual flight rules (VFR), taken from 134 sensors with ranges from 60 to 250 nautical miles. A large amount of data has been collected, from which typical aircraft velocities, climb rates and angles of approach are derived [25].

Since the intruder is assumed to move along an approximate straight line path, the encounter model applied in this thesis is solely used to specify the initial states of the intruding and own vehicles. Kochenderfer et al. have presented an encounter model based on Bayesian networks which also covers the aspect of modeling maneuvering aircraft [25].

2.3.1 Initial separation

Since each simulation is assumed to start when the intruder is detected by the UAV’s sensor, the initial separation should be the same as the detection distance. A minimum requirement for the intruder to be detected is that it occupies at least one pixel in the sensor. Whether this is the case or not depends on the distance to, and the size of, the intruder. The detection is furthermore aggravated by the atmospheric transmittance. This is due to scattering and other optical effects of the radiation transmitted by the intruder. When the radiation reaches the sensor the intensity will have decreased. Due to these facts the detection distance can be
2.3 Encounter model

seen as stochastic [21]. Define thus a stochastic variable

\[ X_r = “\text{distance for detection}” \quad (2.1) \]

\( X_r \) can then be seen as gamma distributed; see Appendix A.

\[ X_r \sim \Gamma(k_r, \theta_r) + b_r \quad (2.2) \]

where \( k_r \approx 22.85, \theta_r \approx 83.68 \text{ m} \) and \( b_r = 1200 \text{ m} \). The expectation value and standard deviation of the detection distance are 3112 m and 400 m respectively. Figure 2.3 shows the probability that a target will be detected within a certain range, i.e. \( P(X_r > r) \).

![Probability for detection within a certain distance.](image)

Based on (2.2) an appropriate initial separation can be chosen for each scenario. The distribution can either be used as a guideline to specify a deterministic initial separation, or used to randomize the distance e.g. in the case of Monte Carlo simulations.

### 2.3.2 Aircraft velocities

When it comes to the speeds of the involved aircraft, no physical derivation can be applied in the same way as with the detection distance. If the simulations are performed with a specific UAV platform in mind, the own vehicle’s speed can simply be taken from the specifications of the actual aircraft. This thesis does not focus on any particular underlying platform, but the UAV is assumed to be a medium light aircraft. Based on this, the own vehicle’s speed is set to 50 m/s throughout the thesis. This rather modest speed is reasonable since the UAV is likely to have somewhat limited performance capabilities. Since the own vehicle’s speed is known to the collision avoidance system the choice of speed should not
affect the system’s performance to any significant extent, as long as it is within realistic bounds.

The intruder’s speed, on the other hand, should not be set to any fix value. It is apparent that the UAV can encounter a wide range of different threats in mid-air. This implies an equally wide range of possible intruder velocities. To narrow the scope somewhat, it can be assumed that aircraft flying in the same airspace to some extent have similar speeds. By analyzing surveyed data, it can be seen that aircraft flying under VFR have an average speed of approximately 55 m/s [25]. It can also be seen that the distribution of velocities tends to be skewed right, i.e. the mass of the distribution is focused on values lower than the mean. This behavior is well described by a gamma distribution function, which is why this distribution will be used to model the speed of the intruding vehicle. Define a stochastic variable

\[ X_v = \text{“speed of intruding vehicle”} \]  
\[ X_v \sim \Gamma(k_v, \theta_v) + b_v \]  

where \( k_v = 5 \), \( \theta_v = 8 \) m/s and \( b_v = 15 \) m/s. The expectation value and standard deviation of the intruder velocity become 55 m/s and 17.89 m/s respectively. Figure 2.4 shows the probability density function for \( X_v \).

\[ \begin{align*} 
X_v &= \text{“speed of intruding vehicle”} \\
X_v &\sim \Gamma(k_v, \theta_v) + b_v 
\end{align*} \]

\[ \begin{align*} 
\text{where } k_v &= 5, \quad \theta_v = 8 \text{ m/s } \quad \text{and } b_v = 15 \text{ m/s} 
\text{The expectation value and standard deviation of the intruder velocity become } 55 \text{ m/s and } 17.89 \text{ m/s respectively.}
\end{align*} \]

\[ \text{Figure 2.4. Probability density function for the speed of the intruding vehicle.} \]

\[ \begin{align*} 
2.3.3 \text{ Climb rates} 
\end{align*} \]

The change of altitude of the involved aircraft can be described by their respective angles of descent, \( \gamma \) for the own vehicle and \( \beta \) for the intruder. Appropriate values to use for these parameters are derived from statistics of analyzed data.


2.3 Encounter model

A typical behavior of aircraft in mid-air is apparent in the surveyed data. An aircraft either stays on the same altitude for level flight, or changes its altitude rather rapidly to a new altitude level. To cover this behavior in the encounter model, Kochenderfer et al. suggests that the possible climb rates are divided into a number of bins. Each bin is associated with a probability that the angle of descent lies in this specific bin. When sampling from the distribution, one of the bins is chosen at random. The result is thereafter randomized from a uniform probability distribution within the bin. However, if the bin includes zero in its range, the resulting value is set to zero instead [25]. This is done to model the fact that most aircraft do not change their altitude, and when they do so it is done with a higher climb rate. Table 2.1 presents the bins that are used to sample the intruder’s angle of descent, $\beta$. The own vehicle’s angle of descent, $\gamma$, is set to zero throughout this thesis.

<table>
<thead>
<tr>
<th>Bins (rad)</th>
<th>Probability</th>
<th>$\beta$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>[-0.080, -0.050]</td>
<td>0.02</td>
<td>$\beta \sim U(-0.08, -0.05)$</td>
</tr>
<tr>
<td>[-0.050, -0.015]</td>
<td>0.13</td>
<td>$\beta \sim U(-0.05, -0.015)$</td>
</tr>
<tr>
<td>[-0.015, 0.015]</td>
<td>0.7</td>
<td>$\beta = 0$</td>
</tr>
<tr>
<td>[0.015, 0.050]</td>
<td>0.13</td>
<td>$\beta \sim U(0.015, 0.05)$</td>
</tr>
<tr>
<td>[0.050, 0.080]</td>
<td>0.02</td>
<td>$\beta \sim U(0.05, 0.08)$</td>
</tr>
</tbody>
</table>

The standard deviation of $\beta$ is approximately 0.022 rad. This value is derived in Appendix B where the probability density function for $\beta$ is illustrated as well.

2.3.4 Angle of approach

The angle of approach is the initial bearing of the intruder, i.e. the angle in the horizontal plane at which the intruder is detected. This angle defines the relationship between the two vehicles’ trajectories. For a head on collision the angle of approach is zero and for collisions from the side the angle is nonzero. Since the intruder is assumed to be initially placed straight to the east of the own vehicle, the angle of approach is equal to the own vehicle’s course angle, $\psi$. The distribution of the angles of approach, derived from statistics of VFR aircraft/VFR aircraft encounters, is illustrated in Figure 2.5. The depicted distribution is derived from the encounter model by Kochenderfer et al. [25].
As can be seen from Figure 2.5, it is very rare for intruders to come from behind. The encounters in this thesis will thus be focused on the cases where the intruder is detected straight ahead or with a relatively small angle of approach.

The angle of approach will not be randomized during the Monte Carlo simulations. The angle will instead be set as a deterministic parameter, based on the specified distribution. Since the result of a Monte Carlo simulation is analyzed by determining an average performance, it is preferable if the different simulations all represent instances close to the nominal situation. By allowing the angle of approach to vary during the Monte Carlo simulations, outcomes from very different situations will be combined into a single result. Both head on collisions and situations where the intruder is approaching from behind will for instance be added together. By doing so the combined result of the Monte Carlo simulations will be hard to interpret, and it is thus better not to let the angle of approach vary at random.

For any given set of aircraft speeds and heading of the own vehicle, it is possible to derive the course angle of the intruder, $\alpha$, that is required for a collision to occur. This angle will be called $\alpha_0$ and it can be calculated as

$$\alpha_0 = \arcsin \left( \frac{-v_{\text{own}} \sin \psi \cos \gamma}{v_{\text{int}} \cos \beta} \right)$$  \hspace{1cm} (2.5)

The derivation of $\alpha_0$ is pure geometry which is omitted here.

As stated in Section 2.1 the collision scenarios that are of interest to study are those where the minimum separation between the aircraft is rather small. This can be achieved by letting the intruder’s course angle, $\alpha$, be close to $\alpha_0$. During Monte Carlo simulations $\alpha_0$ will thus be used as the nominal course angle, and deviations from this value are assumed to be uniformly distributed over the interval $-0.04$ to $0.04$. This implies that

$$\alpha \sim \alpha_0 + U(-0.04, 0.04)$$  \hspace{1cm} (2.6)
2.4 Intruder dynamics

To be able to simulate the motion of the intruder, its dynamics has to be modeled in some way. The standard approach to model a dynamic system is by the use of differential equations. Let the state of the intruder at time $t$, i.e. its current position and velocity, be described by the state vector $x(t)$. The propagation of the state over time can then be expressed with the differential equation

$$
\dot{x}(t) = f_c(x(t), u(t), w(t))
$$

(2.7)

$f_c$ is a (possibly nonlinear) function of the current state $x$, the input signal $u$ and the process noise $w$. The input is typically a control signal known to the system, e.g. throttle or control surface position. The process noise is a model of the uncontrollable effects on the system. This could for instance be wind or turbulence. Besides from being uncontrollable, the process noise is also often unknown to the system.

To implement the dynamic model in a computerized simulation environment, it needs to be discretized [4]. In a time discrete model, the state and the signals affecting the system are only considered at specific points of time, $t = t_0, t_1, \ldots, t_k$. Assume that the time steps are equidistant so that $t_k = kT_s$ where $T_s$ is the sample time and $k = 0, 1, 2, \ldots$. Let $x_k$ denote the state vector at time $t_k$, i.e. $x_k = x(t_k)$. The time discrete model can then be expressed as

$$
x_{k+1} = f_d(x_k, u_k, w_k)
$$

(2.8)

Observe that in the general case, $f_c$ is not identical to $f_d$, i.e. the function describing the dynamics is dependent on the discretization.

For a linear system, i.e. if the function $f_d$ is linear, the model can be simplified to

$$
x_{k+1} = Ax_k + Bu_k + B_w w_k
$$

(2.9)

In absolute Cartesian coordinates the state vector of the intruder is

$$
x_k^A = (s^A_x(t_k) \ s^A_y(t_k) \ s^A_z(t_k) \ v^A_x(t_k) \ v^A_y(t_k) \ v^A_z(t_k))^T
$$

(2.10)

By using this state vector the equation of motion for the intruder is linear, which yields the following dynamic model

$$
x_{k+1}^A = A x_k^A + \begin{pmatrix} T_2^2/2 & 0 & 0 \\ 0 & T_2^2/2 & 0 \\ 0 & 0 & T_s^2/2 \end{pmatrix} w_k
$$

(2.11)

The subscripts $c$ and $d$ stand for continuous and discrete respectively.
As can be seen from the model, the control signal \( u_k \) is omitted. The reason for this is that the intruder is assumed not to make any distinct maneuvers during the time of the collision scenario. However, to deal with the fact that small fluctuations in the motion are inevitable during a flight, acceleration has been added in the form of process noise. The trajectory is thus close to a straight line path, but with some deviations. This kind of dynamic model is often denoted a constant velocity (CV) model. \( w_k \) is assumed to be piecewise constant Gaussian white noise. The noise intensity is set to an appropriate value of 0.75 m/s\(^2\) [30].

### 2.5 Scenario definitions

The perhaps most obvious collision scenario, where the collision risk is intuitively high, is when the involved aircraft are approaching each other with a small angle separating their respective trajectories. Since this is a critical situation, it is of great importance that the collision avoidance system can handle it well. This case is therefore suitable to use as an initial scenario for the evaluation of the system. Scenario A and scenario B will both cover the case of an approaching intruder, but with slightly different setups.

Scenario A is intended for Monte Carlo simulations, i.e. where a large number of simulations are performed. Each simulation is randomized around the nominal setup. This scenario is used throughout the entire thesis, in all cases where the average performance of a system is to be evaluated.

Scenario B uses a deterministic setup, i.e. there is no randomization of the initial state of the intruder. This scenario is thus not suitable for Monte Carlo simulations. The scenario is instead used to make individual simulations which are analyzed in more detail. This scenario is used, as an addition to scenario A, in Chapter 4 where the different definitions of NMAC are compared.

The last scenario used in this thesis, scenario C, differs from the other two in the sense that it does not handle a UAV/VFR aircraft encounter. The scenario is instead intended as a replication of a real mid-air collision between two manned aircraft flying under instrument flight rules (IFR). The purpose with this scenario is to see if the ideas used in autonomous collision avoidance could be beneficial in commercial aviation as well.

#### 2.5.1 Scenario A: Approaching intruder with random initial state

Since scenario A is used for Monte Carlo simulations the initial state of the intruder is randomized for each simulation. The randomization is based on the encounter model presented in Section 2.3. The own vehicle is engaged in level flight, i.e. \( \gamma = 0 \). The angle of approach between the aircraft is \( \psi = 20^\circ \). In the nominal
2.5 Scenario definitions

case, the aircraft will collide after approximately 32 s.

Figure 2.6 shows the geometry of the scenario seen from above. Observe that the angle of descent for the intruder, $\beta$, is possibly nonzero. The figure depicts the nominal trajectory of the intruder as well as two other possible trajectories. These are shown simply to illustrate how the intruder’s path might vary in different runs of the Monte Carlo simulations. Since several of the distributions used in the encounter model are unbound, the trajectory of the intruder can of course lie beyond the depicted ones as well.

![Figure 2.6. Geometry for scenario A.](image)

2.5.2 Scenario B: Approaching intruder with deterministic initial state

As previously mentioned, scenario B is similar to scenario A in the sense that they both deal with the case of an approaching intruder. The major difference is that scenario B uses a deterministic initial state of the intruder. It is thus not suitable for Monte Carlo simulations since no randomization takes place.

In scenario B, the course angle, $\psi$, of the own vehicle is zero, i.e. the intruder is detected straight ahead of the UAV. Both $\gamma$ and $\beta$ are zero as well, which means that both vehicles are engaged in level flight on the same altitude. For the case of $\alpha = 0$ there will thus be a head on collision and for growing $\alpha$ the minimum distance between the aircraft will increase. A range of course angles for the intruder is used, namely $0 \leq \alpha \leq 12^\circ$. By the use of different course angles, the collision avoidance system can be evaluated for the full span of collision risks, from definite to negligible.

Three different setups, with respect to initial separation and intruder velocity, have furthermore been used. These are denoted scenario B.1, B.2 and B.3 respectively. In scenario B.2, the intruder starts relatively close to the UAV and travel with a high speed, making the course of events occur rapidly. Scenario B.3 represents the other end of the line, where the initial separation is large and the velocity is quite modest, which makes the progress of the scenario rather slow. Scenario B.1
corresponds to a middle case where both distance and velocity lie between the values used in scenario B.2 and B.3. Table 2.2 summarizes the parameters used to define scenario B and Figure 2.7 depicts the geometry of the scenario.

Table 2.2. Variables defining scenario B

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Initial separation</th>
<th>Intruder velocity</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1</td>
<td>$\mu_r = 3112$ m</td>
<td>$\mu_v = 55$ m/s</td>
</tr>
<tr>
<td>B.2</td>
<td>$\mu_r - \sigma_r = 2712$ m</td>
<td>$\mu_v + \sigma_v = 72.9$ m/s</td>
</tr>
<tr>
<td>B.3</td>
<td>$\mu_r + \sigma_r = 3512$ m</td>
<td>$\mu_v - \sigma_v = 37.1$ m/s</td>
</tr>
</tbody>
</table>

Figure 2.7. Geometry for scenario B.

### 2.5.3 Scenario C: Replication of a real mid-air collision between commercial aircraft

Scenario C is intended to be a replication of a real mid-air collision which occurred over the Brazilian Amazon jungle on September 29, 2006. The two aircraft, one Boeing 737-800 (tail number PR-GTD) and one Embraer Legacy 600 business jet (tail number N600XL), collided in mid air at 4:57 pm Brasilia standard time [12]. The top end of the Legacy’s left wing hit the wing of the Boeing 737, cutting it right off [7]. The Boeing 737 was destroyed by in-flight breakup and impact forces. All 154 passengers and crew were killed in the accident. The pilots of the Legacy managed to safely land their vehicle and the two crew members and five passengers were all uninjured [12]. Figure 2.8 shows an illustration of the two aircraft just before the collision.

The Boeing was flying southbound along airway UZ6 with an altitude of 37000 ft (denoted FL370). The Legacy was flying northbound on the same airway and on the same altitude. The aircraft were thus on a direct head-on collision course. The Legacy was originally scheduled to change altitude from FL370 to FL360 once it passed Brasilia and entered UZ6. However, the Legacy never requested any change
of flight level; neither did the ATC instruct it to descend [12].

The exact geometry of the collision is of course hard to determine, but no indications of any abnormal maneuvering or attempts to evasive actions were found in the investigation of the accident. It seems thus that the aircraft collided straight head-on. The meteorological conditions in the area of the collisions were good and it can therefore seem strange that no visual acquisition was made by any of the air crew members [12]. However, when an aircraft is flying under IFR it is not common for the pilots to watch the blue sky all the time. They are instead occupied by monitoring their instruments and due to the rapid course of events it would be easy to miss an oncoming threat.

Aircraft flying on altitudes as high as 37000 ft travel much faster than the UAV/VFR aircraft considered in scenarios A and B. The typical cruise speed for the Boeing 737 is around mach 0.785 and for the Legacy about mach 0.8, which at 37000 ft corresponds to approximately 230 m/s and 240 m/s respectively [2][5]. These speeds have therefore been used for the own and intruding vehicles in scenario C. The aircraft are also larger than what has been considered in the UAV/VFR aircraft scenarios. This implies a longer detection distance which in this scenario is set to 4500 m. The aircraft were engaged in level flight which means that $\gamma = \beta = 0$. Since the collision is believed to be head on the angle of approach should be small. In scenario C, $\psi$ is set to $3^\circ$. It is of course possible that the parameter values were somewhat different in the real encounter. The properties of the scenario will however not change for slight modifications of its parameters. It will thus be possible to draw conclusions from the scenario even though the exact values are unknown.
Chapter 3

Tracking

This chapter deals with the subject of target tracking. The chapter starts with a presentation of the topic and a discussion on the problems associated with it. Simulation results are then presented and the performance of the tracking system is evaluated. The analysis of the simulation results is focused on the first problem addressed in this thesis, namely to identify the properties of the tracking system output.

3.1 Angle-only target tracking

The collision avoidance system is triggered when a possible threat, in this chapter denoted the target, is detected by the own vehicle’s sensor. The threat is typically another aircraft, flying in the same airspace as the UAV. After detection, the state of the target, that is the position and velocity, has to be monitored continuously so that the collision avoidance system can determine whether there is a risk for collision or not. This part of the system is called target tracking.

In angle-only target tracking the sensor used to measure the state of the target is a sensor which, as the name suggests, only measures the angles of the target. In most cases this refers to a passive electro-optical sensor (EO-sensor), such as a regular digital video camera. The outputs from the EO-sensor are the azimuth and elevation of the target. The angles are measured in a coordinate system which is fixed to the own vehicle, since this is the case for the actual sensor. However, azimuth and elevation are often defined in an absolute, geographical coordinate system such as the ENU-system instead. Therefore the measurements need to be compensated with the attitude of the own vehicle. Azimuth is then defined as the angle in the horizontal plane and elevation as the angle in the vertical plane, for the vector from the own vehicle to the target. The definitions of azimuth and elevation are illustrated in Figure 3.1.
EO-sensors are very accurate at measuring angles. In the sensor model used in the simulations the sensor output is modeled as

\[ y = \begin{pmatrix} \varphi \\ \theta \end{pmatrix} + \epsilon \]  

(3.1)

where \( \epsilon \) is Gaussian noise with zero mean and a standard deviation of 1 mrad. This level of accuracy is typical for a sensor used in this kind of application [30]. The problem that arises is how to use this accurate angle measurement to make an estimation of the position and velocity of the target.

### 3.2 Observability

As previously mentioned, the task at hand for the target tracking system is to use the information about the angles of the target to make an estimation of the target’s position and velocity. In control theory this is a very common and essential problem which is addressed by introducing an observer. The purpose of the observer is to use information about measurable states of a dynamic system (in this case the angles of the target) to estimate non-measurable states (e.g. range and velocity).

However, to be able to do this, the states that are to be estimated must be observable [17]. In the case where both the own vehicle and the target travel with constant velocities range is unobservable [16]. This fact can easily be realized by considering Figure 3.2.

**Figure 3.1.** Azimuth (\( \varphi \)) and elevation (\( \theta \)).
Three different angle measurements are shown, each illustrated with a line corresponding to possible positions of the target. Two different trajectories are furthermore depicted. Both these trajectories will generate the same measurements, so it is impossible to determine whether the target is close by, travelling with a low speed, or further away, travelling faster.

To be able to use angle-only tracking we need to gain observability in some way. The key to this lies in the condition for unobservability, as stated before. Range is not observable if both the own vehicle and the target travel with constant velocities. By considering Figure 3.2 again, it can be realized that if the own vehicle would maneuver, i.e. travel with non-constant velocity, the two trajectories would no longer generate the exact same angle measurements. We could hence gain observability by performing own platform maneuvers.

### 3.3 Choice of own platform maneuver

As stated in the previous section, the purpose of the own platform maneuver is to gain observability. To accomplish this, a necessary condition is that the dynamics of the own craft is at least one degree higher than that of the target [16]. I.e. if the target is static the own vehicle must have a nonzero velocity, if the target travel with constant velocity the own vehicle must have a nonzero acceleration etc. In this thesis a CV model, i.e. a straight line path, is assumed for the target. The platform maneuver must thus contain a nonzero acceleration. This is however not a sufficient condition to gain observability [16]. By considering a head-on case...
where both vehicles travel directly towards each other, this can be easily realized. Even if the own craft accelerate in the direction of the line of sight, range will not be observable. Another necessary condition to gain observability is thus that the own vehicle has a velocity component perpendicular to the line of sight [15].

Fulfilling these two conditions will, in theory, yield observability in range. In practice, different maneuvers will however perform unequally well. To get an optimal platform maneuver, in the view of achieving as good measurements as possible, it is preferable that the movement perpendicular to the line of sight is of substantial magnitude and that the rate of change in bearing is high [22]. I.e. a large and quick platform maneuver performs better than a small and slow one. In other aspects this can be far from optimal though. A larger maneuver will for instance result in a larger divergence from the mission, the fuel consumption will increase and the time to reach a certain destination will be longer. The choice of platform maneuver must thus be a compromise between gaining observability and interfering with the mission as little as possible.

As mentioned in Section 1.2.1 it is expected that the definition of NMAC according to (1.3) is more robust to large uncertainties in the state estimate than the definition according to (1.2). Due to this it is of interest to investigate whether this method for risk calculation can perform well even for an insignificant platform maneuver. Two different maneuvers will thus be used in this thesis. The first is a so called SS-turn in the horizontal plane; a maneuver that is chosen especially to increase observability in range, but which also entails a substantial interference with the mission. The duration of the maneuver is 12 s, during which the aircraft accelerates in the horizontal normal direction with about 9 m/s² (0.9 g). The total displacement from the original trajectory, caused by the maneuver, is approximately 80 m. The SS-turn is depicted in Figure 3.3.

![Figure 3.3. SS-turn as own platform maneuver.](image-url)


3.4 Tracking filter

The second maneuver is chosen to resemble the natural behavior of the aircraft during a flight. Due to wind, turbulence and other anomalies associated with the aircraft’s dynamics, there will always be some fluctuations in the aircraft's motion. The control system in the UAV, or the pilot in a manned aircraft, normally tries to compensate for these disturbances and steer the aircraft as close to the desired path as possible. Even if it is inevitable that small fluctuations still will be present, these will hardly be sufficient to gain observability. Since the target will suffer from similar variations in its motion, the platform’s movement will not induce any actual advantage. If the control system on the other hand allows the natural oscillations to become more apparent, range might become observable. The maneuver, illustrated in Figure 3.4, can thus be seen as an attempt to use the natural variations in the aircraft’s motion as an own platform maneuver. The properties and magnitude of such a maneuver must of course be configured according to the performance of the tracking in a practical implementation. Here it is chosen as a small sinusoidal movement in the vertical plane. This maneuver has shown decent results in the simulations, and induces a small disturbance to the UAV’s mission. The period of the oscillation is 8 seconds and the amplitude is just over 3 m. The acceleration in the vertical normal direction during this movement is approximately 1.7 m/s$^2$ (0.18 g).

![Figure 3.4. Vertical oscillation as own platform maneuver (observe the different scales on the axes).](image)

3.4 Tracking filter

In target tracking the observer is often referred to as a tracking filter. This filter can be implemented in many different ways, but a common approach is to use a Kalman filter of some sort. The tracking filter used throughout this thesis is a MSC-EKF working at 10 Hz, i.e. an extended Kalman filter working in modified spherical coordinates. This filter is appropriate to use in tracking of non-maneuverable targets [15]. The algorithm used to implement the filter is described
The main output of the tracking filter is the estimate of the target’s state ($\hat{x}$). Though, to be able to use this information in probabilistic risk calculation, it is equally important to know the certainty in the estimation. This is handled by the tracking filter by producing, not only the state estimate itself, but the covariance of this state as well ($\hat{P}$). Observe that the covariance matrix supplied by the tracking filter is an estimate on its own. The meaning of this is that we never can be absolutely sure of how uncertain the state estimation is.

### 3.5 Filter initialization

The MSC-EKF is a recursive filter which means that the previous estimate is used to make a new estimation (see Appendix C). When the tracking starts, no previous estimate is available. It is thus necessary to set up the filter with initial state and covariance matrix estimations. Let these be denoted $\hat{x}_{0|0}$ and $\hat{P}_{0|0}$ respectively\(^1\). It is of course desirable that the initial estimates correspond to the state of the target when the tracking starts, i.e. at detection. If this is to be accomplished, it is necessary for the system to reason about the position and velocity of the target when it is detected.

Let the origin of the absolute Cartesian coordinate system be fixed to the UAV. By using the definitions of azimuth and elevation presented in Section 3.1 the position of the intruder becomes

$$s^A_{rel} = \begin{pmatrix} s^A_x \\ s^A_y \\ s^A_z \end{pmatrix} = \begin{pmatrix} r \cos \varphi \cos \theta \\ r \sin \varphi \cos \theta \\ -r \sin \theta \end{pmatrix} \quad (3.2)$$

where $r$ is the distance to the intruder, $\varphi$ is the azimuth angle and $\theta$ is the elevation.

In the same coordinate system, the relative velocity between the aircraft is $v^A_{rel} = v^A_{int} - v^A_{own}$. Recall the collision scenario geometry presented in Section 2.2. By using the specified angles for the headings of the aircraft, the relative velocity vector becomes

$$v^A_{rel} = \begin{pmatrix} -v_{int} \cos \alpha \cos \beta \\ -v_{int} \sin \alpha \cos \beta \\ -v_{int} \sin \beta \end{pmatrix} - \begin{pmatrix} v_{own} \cos \psi \cos \gamma \\ v_{own} \sin \psi \cos \gamma \\ -v_{own} \sin \gamma \end{pmatrix} \quad (3.3)$$

By combining the position and velocity, the state vector for the intruder in absolute

\(^1\)The subscripts are according to the notation used in Appendix C and refer to that the estimations are made at time 0.
3.5 Filter initialization

Cartesian coordinates is obtained

\[
x_{rel}^A = \begin{pmatrix}
 r \cos \varphi \cos \theta \\
 r \sin \varphi \cos \theta \\
 -r \sin \theta \\
 -v_{int} \cos \alpha \cos \beta - v_{own} \cos \psi \cos \gamma \\
 -v_{int} \sin \alpha \cos \beta - v_{own} \sin \psi \cos \gamma \\
 -v_{int} \sin \beta + v_{own} \sin \gamma
\end{pmatrix}
\] (3.4)

The state vector is thus defined by a set of variables, of which some are known \((\varphi, \theta, v_{own}, \psi, \gamma)\) and other unknown \((r, v_{int}, \alpha, \beta)\). To be able to initialize the tracking filter, the system needs to guess appropriate values for the unknown variables. In a similar way, the initialization of the covariance matrix requires the system to reason about the certainty of the initial guess. This is done by specifying assumed standard deviations together with the guesses.

For a real implementation, this reasoning requires empirical knowledge about what kind of aircraft we are most likely to encounter as well as information on typical detection distances etc. To acquire this knowledge, actual flight tests need to be performed and analyzed thoroughly. In this thesis the initialization is based on the encounter model presented in Section 2.3. Observe that the filter initialization is not the same thing as the encounter model. Initializing the filter is something that needs to be done by the tracking system, regardless if it is to be used in a simulation or a real application. The encounter model on the other hand is a way of describing the reality, so that realistic simulations can be performed. The two problems are of course closely connected; they both concern reasoning about typical collision situations. This is why the same variables are present on both topics.

The variables that are known to the system or measured by the sensor are given in Table 3.1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
<th>Standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\varphi)</td>
<td>From sensor</td>
<td>1 mrad</td>
</tr>
<tr>
<td>(\theta)</td>
<td>From sensor</td>
<td>1 mrad</td>
</tr>
<tr>
<td>(v_{own})</td>
<td>50 m/s</td>
<td>0 m/s</td>
</tr>
<tr>
<td>(\psi)</td>
<td>Scenario specific</td>
<td>0 rad</td>
</tr>
<tr>
<td>(\gamma)</td>
<td>0 rad</td>
<td>0 rad</td>
</tr>
</tbody>
</table>

The tracking system’s initial guesses for the unknown variables and their assumed standard deviations are summarized in Table 3.2. The variables are typed with a hat (\(\hat{\cdot}\)) above their names to indicate that they are the tracking system’s reasoning about the state of the intruder.
Table 3.2. Unknown variables

<table>
<thead>
<tr>
<th>Variable</th>
<th>Initial guess</th>
<th>Assumed standard deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\hat{r}$</td>
<td>3112 m</td>
<td>400 m</td>
</tr>
<tr>
<td>$\hat{v}_{int}$</td>
<td>55 m/s</td>
<td>17.89 m/s</td>
</tr>
<tr>
<td>$\hat{\alpha}$</td>
<td>$\hat{\alpha}_0$</td>
<td>0.023 rad</td>
</tr>
<tr>
<td>$\hat{\beta}$</td>
<td>0 rad</td>
<td>0.022 rad</td>
</tr>
</tbody>
</table>

The course angle of the intruder is assumed to be

$$\hat{\alpha}_0 = \arcsin \left( -\frac{v_{own} \sin \psi \cos \gamma}{\hat{v}_{int} \cos \hat{\beta}} \right) = \arcsin \left( -\frac{50 \sin \psi}{55} \right)$$  
(3.5)

This is the angle which is believed to lead to a collision. The reason for assuming this value is that situations actually leading to collision are the most critical. It is therefore desirable that the tracking is as good as possible during these cases.

At the start of every simulation the tracking can be initialized by determining the initial state vector from equations (3.5) and (3.4), using the assumed valued for all variables. The initial covariance matrix is calculated with the propagation of uncertainty formula presented in Appendix E. Example 3.1 presents the resulting initialization for scenario A.

--- Example 3.1: Tracking filter initialization in scenario A ---

Recall scenario A presented in Section 2.5.1. The course angle of the own vehicle is $\psi = 20^\circ \approx 0.349$ rad which yields

$$\hat{\alpha}_0 = \arcsin \left( -\frac{50 \sin 0.349}{55} \right) \approx -0.316 \text{ rad}$$  
(3.6)

If the initial angle measurements are correct, i.e. $\varphi = \theta = 0$, the initial state vector becomes

$$\hat{x}_{0|0} = \begin{pmatrix} 3112 & 0 & 0 & -99.3 & 0 & 0 \end{pmatrix}^T$$  
(3.7)

The corresponding covariance matrix is calculated with the propagation of uncertainty formula presented in Appendix E which result in

$$\hat{P}_{0|0} = \begin{pmatrix} 160000 & 0 & 0 & 0 & 0 & 0 \\ 0 & 9.68 & 0 & 0 & 0 & 0 \\ 0 & 0 & 9.68 & 0 & 0 & 0 \\ 0 & 0 & 0 & 354 & 0.477 & 0 \\ 0 & 0 & 0 & 0.477 & 1.46 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1.43 \end{pmatrix}$$  
(3.8)
3.6 Tracking performance

When evaluating the performance of the tracking system, it is possible to view the problem from two different perspectives and the result can thus be interpreted in different ways. The most obvious is to consider the quality of the state estimation, i.e. to ascertain how well the tracking system manages to estimate the state of the target. This can be done by examining the estimation error. If \( \hat{x}(t) \) is the estimate of the true state of the target, \( x(t) \), the estimation error is formed as \( e(t) = x(t) - \hat{x}(t) \). It is evident that the accuracy of the state estimation coincides with the magnitude of the error; if the estimation is good the error will be small and vice versa.

The quality of the estimation is the most common thing focused on in existing work on the subject of target tracking. To achieve a small estimation error is however not the sole important thing to consider. When the tracking filter output is used in probabilistic risk calculation, the true state is assumed to follow the multivariate normal distribution

\[
x(t) \sim \mathcal{N}(\hat{x}(t), \hat{P}(t))
\]

(3.9)

where \( \hat{P}(t) \) is the certainty measure produced by the filter in the form of a covariance matrix. The same assumption can be stated in terms of the estimation error,

\[
e(t) \sim \mathcal{N}(\mathbf{0}, \hat{P}(t))
\]

(3.10)

where \( \mathbf{0} \) is the null vector. If this assumption is too far from the truth, the risk calculation will fail no matter how accurate the estimation is. It is thus important to analyze, not only the magnitude of the estimation error, but also how well the true characteristics of the error are covered by the assumption (3.10).

Since the main purpose of this thesis is to study the risk calculation step of the collision avoidance system, the accuracy of the estimate is not the most critical. The collision probability can of course be calculated even if the variance of the underlying stochastic variable is high. The important thing is that the tracking filter supplies estimates of the interesting states of the target, and that these estimates are realistic to the behavior of a real up-and-running tracking filter. The filter used in the simulations should produce estimates with the same degree of certainty and correlation, as could be expected from a practical implementation. However, there is no need to tweak all filter parameters to get the maximum performance out of the tracking. The tracking evaluation presented in this thesis will thus be focused on determining the level of correctness in assumption (3.9).

This section covers the evaluation of the state and variance estimations. The accuracy of the estimate, i.e. the magnitude of the estimation error, is examined to see how well the tracking system actually performs for different choices of own platform maneuvers. The variances, extracted from the estimated covariance matrix, are compared with the spread in observed data to determine if the variance
is estimated correctly. In Section 3.7 the distribution of the true state estimates is analyzed to see if the assumption of normality according to (3.9) is adequate.

### 3.6.1 Measures of evaluation

The tracking filter is evaluated by running a series of simulations according to scenario A presented in Section 2.5.1. The target’s initial position is randomized for each simulation. The tracking starts at the detection of the target and is thereafter performed continuously for 20 seconds. The estimated state of the target is compared with its true state to get the estimation error at every point of time and for each simulation. Assume that \( N \) simulations have been performed. Let \( \hat{x}_j^i(t) \) be the estimate of the true state component \( x_j^i(t) \) in simulation \( j \), where \( j = 1, 2, 3, \ldots, N \). The subscript \( i \) means that the state component \( x_j^i(t) \) is the \( i \):th element of the state vector \( x_j(t) \).

The estimation error in element \( i \) and from simulation \( j \) becomes \( e_j^i(t) = x_j^i(t) - \hat{x}_j^i(t) \). To determine how well the tracking filter performs and to find the properties of the estimation, a statistical analysis is performed on the range of observed estimation errors.

Relative Cartesian coordinates, as described in Appendix D, have been used to analyze the result of the tracking. The state vector is defined as

\[
x^R(t) = \begin{pmatrix} s^R_x(t) & v^R_x(t) & v^R_y(t) & v^R_z(t) \end{pmatrix}^T
\]

(3.11)

The four components of the state vector are the range to the target \( s^R_x \), the relative velocity along line of sight \( v^R_x \) and two perpendicular velocity components orthogonal to line of sight \( v^R_y \) and \( v^R_z \). The displacement of the target relative to line of sight (i.e. what could be seen as \( s^R_y \) and \( s^R_z \)) is neglected, since the angle measurements are so accurate. For the rest of this chapter the superscript \( R \) (relative coordinates) is neglected to simplify the notation.

To examine the quality of the range estimation, the relative error in range is used as well. This measure is more intuitive than the absolute error in range, since it does not get exaggerated at very long distances. For example, an absolute estimation error in range of 300 m is much worse if the underlying distance is just 500 m, than it is if the distance is 3000 m. By using the relative range, the former case will yield a much higher error than the latter, which coincides with the intuitive interpretation of the range error. The relative error in range is defined as

\[
i_{rel}^j = \frac{s_j^R - \hat{s}_j^R}{s_j^R} = \frac{e_j^1}{x_j^1}
\]

(3.12)

To get an average performance measure of the range estimate the so called root mean squared error (RMSE) is calculated. The RMSE of the relative range is defined as

\[
\text{RMSE}(t) = \sqrt{\frac{\sum_{j=1}^{N}(i_{rel}^{j}(t))^2}{N}}
\]

(3.13)
3.6 Tracking performance

This measure accumulates the total error in the estimation, regardless the character of the error (i.e. if it is due to bias or variance). The RMSE in relative range is suitable to use as a general performance measure for the tracking filter.

To further analyze the filter output a statistical analysis is performed on the estimation errors in the four states of the relative state vector. This consists of determining bias and variance of the estimates. The calculated quantities can then be compared with the estimated covariance matrix to see if the true properties of the estimation error can be described by the estimated covariance. A more in-depth analysis is presented in Section 3.7 where the distribution of the estimates is examined.

The bias of the estimation error can be studied by calculating the sample mean of \( e_i(t) \) over the sample \( j = 1, 2, 3, \ldots, N \) according to

\[
\bar{e}_i(t) = \frac{1}{N} \sum_{j=1}^{N} e_i^j(t)
\]

(3.14)

If the estimation is unbiased, the sample mean should be close to zero. The spread of the sample can be determined in a similar way by calculating the sample variance \( \varsigma_i^2(t) \) and sample standard deviation \( \varsigma_i(t) \) as

\[
\varsigma_i^2(t) = \frac{1}{N-1} \sum_{j=1}^{N} (e_i^j(t) - \bar{e}_i(t))^2
\]

(3.15a)

\[
\varsigma_i(t) = \sqrt{\varsigma_i^2(t)}
\]

(3.15b)

The sample standard deviation can be used in the evaluation of the tracking filter output in two different ways. First of all, since it is a measure of how much the estimation error varies around its mean it can be used to determine the accuracy of the state estimation. In this way the sample standard deviation is very similar to the RMSE, as the same kind of information can be deduced from the two measures. Besides from this, the sample standard deviations can also be used in the evaluation of the covariance estimation. The estimated covariance matrix of the state vector in relative Cartesian coordinates is given by

\[
\hat{P}(t) = \begin{pmatrix}
\sigma_1^2 & \rho_{12}\sigma_1\sigma_2 & \rho_{13}\sigma_1\sigma_3 & \rho_{14}\sigma_1\sigma_4 \\
\rho_{12}\sigma_1\sigma_2 & \sigma_2^2 & \rho_{23}\sigma_2\sigma_3 & \rho_{24}\sigma_2\sigma_4 \\
\rho_{13}\sigma_1\sigma_3 & \rho_{23}\sigma_2\sigma_3 & \sigma_3^2 & \rho_{34}\sigma_3\sigma_4 \\
\rho_{14}\sigma_1\sigma_4 & \rho_{24}\sigma_2\sigma_4 & \rho_{34}\sigma_3\sigma_4 & \sigma_4
\end{pmatrix}
\]

(3.16)

The average estimated standard deviation in state \( i \) can be extracted according to

\[
\bar{\sigma}_i = \sqrt{\frac{\sum_{j=1}^{N} \hat{P}_{ij}(i,i)}{N}}
\]

(3.17)

It is desirable that \( \bar{\sigma}(t) \approx \varsigma(t) \). If this is the case the estimated standard deviations, i.e. the certainty measures supplied by the tracking filter, are proper.

\[2\text{The sample variance is an unbiased estimator of the true variance, whereas the sample standard deviation is afflicted with a bias [24]. This is however negligible for large sample sizes.} \]
3.6.2 Estimation error and estimated standard deviation

To see how the choice of platform maneuver influences the quality of the range estimation, three simulations have been performed on scenario A. In one of the simulations no platform maneuver at all is used. In the other two, the different maneuvers presented in Section 3.3 are used. Figure 3.5 shows the RMSE in $\hat{r}_{rel}$ for the three simulations.

![Figure 3.5. RMSEs in $\hat{r}_{rel}$ when an SS-turn (solid line), a small sinusoidal movement (dashed line) and no platform maneuver (dash-dotted line) are used respectively.](image)

When the SS-turn is used the estimate converges fast during the maneuver. After the maneuver (approximately at $t = 12$ s) the error starts to increase though. When the maneuver no longer is performed, range once again becomes unobservable. For the small sinusoidal platform maneuver the estimate converges, but not as fast as when the SS-turn is used. Even though the sinusoidal maneuver is carried out during the entire simulation period, the error never becomes as small as when the SS-turn is used. When no platform maneuver at all is used, range is unobservable and the relative error increases from the start. The platform maneuver seems thus to have the expected effect on the state estimation.

To examine the correctness of the covariance matrix estimates, the average standard deviations ($\bar{\sigma}$) are compared with the sample standard deviations ($\varsigma$). Figures 3.6, 3.7 and 3.8 each show these measures together with the biases$^3$ of the four states of the relative Cartesian state vector. The three figures correspond to the cases where an SS-turn, a sinusoidal maneuver and no platform maneuver is used respectively.

$^3$The absolute values of the sample means, $|\bar{e}_i|$, are used to illustrate the biases.
3.6 Tracking performance

Figure 3.6. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is an SS-turn.

Figure 3.7. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is a small sinusoidal movement in the vertical plane.
Figure 3.8. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when no platform maneuver is used.

It is apparent in these figures as well, that the estimation gets better when applying an own platform maneuver. This is especially noticeable in the first two states (range and velocity along line of sight). When the SS-turn is used the convergence is faster than for the other two cases. The biases are also much lower. The cases with a sinusoidal maneuver and without any maneuver show similar characteristics, but it is still clear that the platform maneuver yields a somewhat faster convergence and smaller bias.

The estimated standard deviations on the other hand seem to be rather accurate for all three cases. It is also noticeable that the biases are subordinate to the standard deviations in all cases. As previously mentioned it is not necessary to have accurate state estimates for the sake of risk calculation. If the calculation is robust to large uncertainties in the estimates, the risk can be computed correctly as long as the covariance is estimated properly.

According to this, the case without any platform maneuver could work just as well as the other two cases, even though range is unobservable. This seems to be a bit too good to be true, and it is. The problem lies in the initialization of the state vector and covariance matrix. During the simulations presented above, the initialization coincides exactly with the true situation. Observe that this does not mean that the initial state estimate is correct, just that it is unbiased and that the
3.6 Tracking performance

Variance reflects the true situation\(^4\). By performing extensive studies on actual flight test data, one can hope to obtain good values for the initialization, but it can never be guaranteed that it is error-free. To see how the tracking performs when the initialization is erroneous, the same simulations as presented above have been performed again. This time the standard deviations of the true states are 50% higher than the values used in the initialization. The comparisons between sample and average standard deviations are shown in Figures 3.9, 3.10 and 3.11.

![Graphs showing standard deviations and biases](image)

**Figure 3.9.** Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is an SS-turn. The standard deviations are initialized too low.

\(^4\)The biases apparent in the above figures are not due to incorrectness in the initialization, but are instead accumulated during the first few samples of the simulations. This is due to abnormalities in the distributions used in the encounter model and the geometry of the scene.
Figure 3.10. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is a small sinusoidal movement in the vertical plane. The standard deviations are initialized too low.

Figure 3.11. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when no platform maneuver is used. The standard deviations are initialized too low.
The problems with unobservability now become more apparent. When a platform maneuver is used, the standard deviation estimates converge towards the true values. The fastest convergence is again obtained for the SS-turn, but it is evident for the sinusoidal maneuver as well. When no platform maneuver is used the convergence seems to be very slow, if present at all. This is especially evident in the perpendicular velocity components ($v_y$ and $v_z$) where the errors actually increase as time goes by. It is worth noticing that the state components all are dependent of each other, and it is therefore necessary to consider them all simultaneously to be able to draw any conclusions of the result.

Except for these simulations, in which the initial variance is too low, simulations have been performed with too high initial variance as well as with a biased initial state. The outcomes from these simulations are presented in Appendix F, and they all point to the same result as stated in this section.

It is clear that a platform maneuver must be used to get a reliable tracking. The case without any maneuver is thus not considered further on in this thesis. The best results are obtained when an SS-turn is used. When a small sinusoidal movement in the vertical plane is used, the estimation does not converge as fast. The question whether this maneuver is sufficient or not will be pursued further during the rest of this thesis.

### 3.7 Distributions of estimated states

The evaluation of the tracking filter output have this far been focused on the magnitude and spread of the estimation error. The correspondence between the average estimated standard deviation and the sample standard deviation have been deduced. This gave an insight in the general performance of the tracking filter, and how well the certainty measure (the covariance matrix) was estimated.

To further examine the filter output this section presents a more in-depth statistical analysis. In this examination the multivariate normality assumption is considered as a whole, i.e. not solely by studying the marginal standard deviations. Due to this, the analysis presented here is more comprehensive and mathematically correct, but also less intuitive.

The problem of assessing multivariate normality is widely explored. There exist many methods and different strategies for this purpose, each with their individual strengths and drawbacks. Gnanadesikan stated that

> "With multiresponse data it is clear that the possibilities for departure from joint normality are indeed many and varied. One implication of this is the need for a variety of techniques with differing sensitivities to the different types of departures; seeking a single best method would seem to be neither pragmatically sensible nor necessary." [18]
Thode proposes a protocol of several tests, all suitable to use when trying to assess multivariate normality [35]. Three general techniques can be identified:

1. Assessing normality in all the marginal distributions, using univariate techniques. This method makes use of the necessary, but not sufficient, condition that a multivariate normal distribution must be normal in each of its marginal distributions.

2. Reducing the dimension of data in such a way that the result has some simple structure which can be identified.


However, in the case of tracking filter output the assessment is further complicated. To be able to test the output for normality, a sample from the assumed normal distribution is of course necessary. In this case the sample consists of the Monte Carlo simulation result. Recall that \( N \) simulations were performed. At every point of time and for each simulation, \( j = 1, 2, 3, \ldots, N \), the tracking filter delivers an estimate, \( \hat{x}^j(t) \), of the true state \( x^j(t) \) together with a covariance matrix, \( \hat{P}^j(t) \).

The normality assumption reads

\[
x^j(t) \sim N(\hat{x}^j(t), \hat{P}^j(t))
\]  

(3.18)

The problem that arises is that \( \hat{x}^j(t) \) and \( \hat{P}^j(t) \) varies, not only over time, but also with \( j \). i.e. every sample from the Monte Carlo simulation has a unique distribution. To use the samples in a distribution test, it is of course essential that they all come from the same distribution.

To overcome this problem the sample needs to be normalized. This is done by calculating the scaled residuals of the observations according to

\[
\zeta^j(t) = (\hat{P}^j(t))^{-1/2}(x^j(t) - \hat{x}^j(t))
\]  

(3.19)

Under the normality assumption, the scaled residuals themselves become normal with a standard normal distribution [35].

\[
z^j(t) \sim N(0, I)
\]  

(3.20)

The sample of scaled residuals can thus be used in the distribution analysis instead of the original variables. Based on this, two of the listed techniques are applied, namely to conduct marginal normality assessment and to reduce dimension. The third technique, to perform direct multivariate normality assessment, is not used since the scaled residuals are supposedly independent.
3.7 Distributions of estimated states

3.7.1 Marginal normality of scaled residuals

To examine the marginal normality of the scaled residuals, quantile-quantile-plots (QQ-plots) are used. This is the preferred graphical technique to use for this purpose [24]. The QQ-plots compares the observed quantiles of a sample with the theoretical quantiles of a standard normal distribution.

Let the sample of the $i$:th scaled residual be $z_i(t) = \{z_{1i}(t), z_{2i}(t), \ldots, z_{Ni}(t)\}$. The quantiles of the sample are the same observations, but ordered from the smallest to the largest\(^5\). These are denoted by $z_{1i}^{(1)}(t) \leq z_{2i}^{(2)}(t) \leq \cdots \leq z_{Ni}^{(N)}(t)$. For each quantile, $z_{i}^{(n)}(t)$, a $p$-value, indicating the position of the quantile, is given by $p = (n - 0.5)/N$. By using these $p$-values, the quantiles of a theoretical normal distribution are defined as

\[
Q^*(p) = z \quad (3.21)
\]

where $z$ is such that

\[
P(Z \leq z) = p \quad (3.22)
\]

An equivalent definition of the quantiles is

\[
Q^*(p) = \inf \{z \in \mathbb{R} : p \leq \Phi(z)\} \quad (3.23)
\]

where $\Phi(z)$ is the cumulative distribution function for the standard normal distribution.

A QQ-plot is obtained by plotting the observed quantiles versus the theoretical ones. If the sample actually comes from a standard normal distribution, the QQ-plot is expected to be a straight line through the origin with slope 1 [24].

QQ-plots for one of the scaled residuals are illustrated in Figures 3.12 and 3.13. The figures correspond to the cases where the platform maneuver is an SS-turn and a small sinusoidal movement respectively (see Section 3.3). Similar QQ-plots for the remaining three scaled residuals are show in Appendix G.

---

\(^5\)The quantiles are sometimes referred to as the order statistics of the sample.
Figure 3.12. QQ-plot for one of the scaled residuals when the own platform maneuver is an SS-turn.

Figure 3.13. QQ-plot for one of the scaled residuals when the own platform maneuver is a small sinusoidal movement in the vertical plane.
The observed quantiles show very good normality correspondence for both platform maneuvers. Slight deviations from the straight lines can be seen in the upper left plots, i.e. at the beginning of the simulation \( t = 0 \). The quantiles have a somewhat concave shape, which indicate a small asymmetry in the observed sample [24]. As time goes on the quantiles converge towards the straight line rather rapidly. Only a few stragglers tend to deviate from the lines in the plots corresponding to later points of time. This typically indicates the presence of a few outliers. Based on the marginal distributions the normality assumption seems thus to be adequate.

### 3.7.2 Pearson’s \( \chi^2 \) test on squared Mahalanobis distance

One way of assessing multivariate normality, other than focusing on the marginal distributions, is to reduce the dimension of the variables. If this can be done in such a way that the resulting quantity has some characteristic properties, the normality test can be performed by analyzing the resulting variable which has fewer dimensions. Healy suggest to use the squared Mahalanobis distance [19]

\[
(d^2(t))^2 = (z^2(t))^T (z^2(t))
\]  (3.24)

Under multivariate normality, the squared Mahalanobis distance is known to be \( \chi^2_m \)-distributed [35]. The number of degrees of freedom, \( m \), is the same as the dimension of \( z \). In this case, \( d(t)^2 \) should thus follow a \( \chi^2_4 \)-distribution. The normality assessment can thus be performed by testing the sample \( (d^2(t))^2 \) for \( \chi^2_4 \).

To do this, the well known Pearson’s \( \chi^2 \) goodness of fit test is used. The null hypothesis in this test states that an observed sample comes from a certain distribution, i.e.

\[
H_0 : \text{The sample \( \{(d^2(t))^2, \ldots, (d^N(t))^2\} \) comes from a \( \chi^2_4 \)-distribution.}
\]

To perform the test, the sample space is divided into \( K \) cells. The frequency of observations at time \( t \) that falls into cell \( k \) is denoted \( N_k(t) \). Obviously, the cell frequencies add up to the sample size

\[
\sum_{k=1}^{K} N_k(t) = N \quad \forall \, t
\]  (3.25)

Let \( p_k \) be the probability of falling into cell \( k \) if the null hypothesis is true. The expected number of samples in each cell is then \( Np_k \). The Pearson test statistic is defined as

\[
X^2_P(t) = \sum_{k=1}^{K} \frac{(N_k(t) - Np_k)^2}{Np_k}
\]  (3.26)

Under the null hypothesis, \( X^2_P(t) \) is well approximated as a \( \chi^2_{K-1} \)-distributed variable [34]. If \( X^2_P(t) \) is larger than the 100\( \alpha \)% point of the \( \chi^2_{K-1} \)-distribution, the null hypothesis can be rejected with a significance level of 100\( \alpha \)%.
A problem when using the Pearson’s test is to choose an appropriate number of cells and how to select the boundaries for them. Thode suggests that each cell should be equiprobable, i.e. \( p_k = 1/K \) for all \( k \) \([35]\). In this case the average expected cell frequency should be at least 2, which implies that \( N \geq 2K \) \([34]\). By choosing

\[
K = \lfloor N/2 \rfloor \tag{3.27}
\]

the boundaries can be calculated from the \( \chi^2_{K-1} \)-distribution function, such that all cells become equiprobable.

Figures 3.14 and 3.15 shows \( X^2_P(t) \) over time when the own platform maneuver is an SS-turn and a small sinusoidal movement respectively. The figures also show the 99 % significance level (dashed line) and the mean (dotted line) of the \( \chi^2_{K-1} \)-distribution.

![Figure 3.14](image1.png)

**Figure 3.14.** Test statistic (solid) and 99 % significance level (dashed) when the own platform maneuver is an SS-turn.

![Figure 3.15](image2.png)

**Figure 3.15.** Test statistic (solid) and 99 % significance level (dashed) when the own platform maneuver is a small sinusoidal movement in the vertical plane.

A fast convergence towards the null hypothesis is evident in both cases. From \( t \approx 2 \) s and forward the test statistic is well within the confidence bound at most points of time.
3.8 Conclusions

The result of the tracking highly depends on the platform maneuver used. This was expected due to the observability problems associated with this kind of tracking. It is clear that a platform maneuver must be used to get a reliable tracking system. The case without any maneuver is thus not considered further on in this thesis. The best results are obtained when an SS-turn is used. When a smaller sinusoidal movement in the vertical plane is used, the estimation does not converge as fast. Not only does the smaller maneuver result in a, in general, higher estimation error. The certainty measure produced by the tracking filter tends to be less accurate as well.

Both platform maneuvers will be used during the examination of the risk calculation step of the collision avoidance system. The reason for this is to see how the two definitions of NMAC differ for different levels of certainty in the estimation.

The distribution of the tracking filter output shows good fit to normality. The two tests for multivariate normality both indicated that the observed sample was of normal character. A deviation from the assumed distribution was evident at the beginning of the simulations, but the observations converged within a few seconds. No distinct differences in the tracking filter output distributions could be seen for the two investigated platform maneuvers. The normality assumption made in the risk calculation step of the collision avoidance system is thus considered to be well motivated.
Chapter 4

Defining the probability of NMAC

This chapter covers the second problem addressed in this thesis, namely to compare the two considered definitions of NMAC. The aim of the chapter is to deduce which of the two definitions that is most suitable to use in the risk calculation step of a collision avoidance system. The chapter starts by introducing the two definitions and how they can be used in the risk calculation. The definitions are then compared in two subsequent steps. In the first step the risk calculation is detached from the rest of the collision avoidance system to see how the definitions perform under idealized circumstances. In the second step the risk calculation is put in its context and the collision avoidance system is evaluated as a whole.

4.1 Risk assessment concepts

By using the state estimate and the covariance matrix supplied by the target tracking system, the risk calculator is supposed to determine the probability of NMAC. As stated in Section 1.2.2, the first problem that has to be dealt with is to decide how the event of NMAC should be defined. Recall the two different definitions proposed. First the maximum of the momentary probability over a given time horizon.

\[
\max_{0< t_p< T} P(NMAC_{t_p}) \tag{4.1a}
\]

where

\[
NMAC_{t_p} = |s(t_p)| < R \tag{4.1b}
\]

Since this definition deals with the momentary probability of NMAC, it will further on be denoted the momentary definition.
Secondly the probability that NMAC will occur at any time during a given time horizon.

\[ P(\text{NMAC}(0,T_p)) \]  

where

\[ \text{NMAC}(0,T_p) = \left( \min_{0<t_p<T_p} |s(t_p)| \right) < R \]  

Since this definition considers the case of NMAC over the entire time horizon as one event, it will further on be denoted the time horizon definition. \( R = 150 \) m is the radius of the safety zone and the time horizon \( T_p \) is set to 45 s throughout this thesis.

The need for probabilistic methods for analyzing the collision risk is of course due to the stochastic nature of the state estimation. Let \( \hat{x} \) be the estimated state of the intruder and \( \hat{P} \) the corresponding covariance matrix, both supplied by the tracking filter. We can then describe the true state of the intruder as dependent on the estimate according to

\[ x \sim N(\hat{x}, \hat{P}) \]  

This stochastic variable forms the basis of the risk calculation. However, calculating the exact probabilities according to (4.1) and (4.2) is in general intractable.

The advantage of using (4.1) instead of (4.2) is that the sought probability is easier to calculate, since only a single point of time has to be treated at the same time. The problem with this definition, on the other hand, is that it is likely to diminish the probability of NMAC. This is due to the large uncertainties associated with angle-only tracking. If the state estimate is very uncertain, no distinct peak will be present in the calculated probabilities and the maximum probability will not reflect the total risk. Example 4.1 illustrates this problem in a one-dimensional case but the same kind of reasoning holds for higher dimensions as well.

--- Example 4.1: Illustration of risk calculation in one dimension ---

Assume that the scene is one-dimensional, i.e. that all motion takes place along a single line. The relative distance between the aircraft, \( s(t_p) \), is assumed to have the probability density function \( p_s(s) \). The probability of NMAC according to (4.1) can then be calculated as

\[ P(\text{NMAC}_{t_p}) = \int_{-R}^{R} p_s(s) \, ds \]  

Now, consider Figure 4.1 where the probability density function is depicted at three different prediction time points, \( t_p = 1, 2 \) and 3. The figure shows two instances of the function, one when the uncertainty is high and one when it is low.
4.1 Risk assessment concepts

It is apparent that the collision risk calculated according to (4.4) will be different depending on which one of the two probability density functions that is used. Possible values of the integral are shown in Figure 4.2.

Figure 4.1. The probability density functions for distance between the aircraft at three different points of time. Two density functions are depicted, one where the uncertainty is high (dashed line) and one where it is low (solid line).

Figure 4.2. $\int_{-R}^{R} p_s(s) \, ds$ for two probability density functions, one with high and one with low variance.
As can be seen from Example 4.1 the maximum probability will be lower when the uncertainty is large, even though the cumulated collision risk might be high. It is expected that the definition of NMAC according to (4.2) do not suffer from this drawback since the entire time horizon is taken into account in the probability calculation. The problem with this definition is of course that probability is harder to calculate. An interesting fact is that

\[ \int_{0}^{T_p} P(\text{NMAC}_{t_p}) \, dt \neq P(\text{NMAC}_{(0,T_p)}) \] (4.5)

The collision risk seen over an interval of time can thus not be calculated in any simple manner based on the momentary collision risk.

It is of course possible to use the momentary risk in other manners than to take the maximum of the probability over the time horizon. Prandini et al. have considered this, most notably through different weighted averages of \( P(\text{NMAC}_{t_p}) \) over the horizon \([0, T_p]\), and found that these measures of criticality all are less effective than the maximum probability [33].

### 4.2 Monte Carlo approximation

One straightforward approach to calculate the probabilities for a specific flight scenario is to use Monte Carlo approximation. This should not be confused with the Monte Carlo simulations that are used in the evaluation of the collision avoidance systems. The underlying idea is very similar though, namely to sample numerous times from a distribution to determine the average outcome.

In Monte Carlo approximation a large number \( N_{rc} \) of samples are drawn from (4.3). For each of these samples the events of NMAC are evaluated according to (4.1) and (4.2). The probabilities can then be approximated with the outcome of the sampling according to

\[ P(\text{NMAC}_{t_p}) \approx \frac{1}{N_{rc}} \sum_{i=1}^{N_{rc}} (|s_i(t_p)| < R) \] (4.6)

and

\[ P(\text{NMAC}_{(0,T_p)}) \approx \frac{1}{N_{rc}} \sum_{i=1}^{N_{rc}} \left( \min_{0 < t_p < T_p} |s_i(t_p)| < R \right) \] (4.7)

\(^1\)The subscript rc, (risk calculation) is used to distinguish the number of samples used in the risk calculation from the number of runs in a Monte Carlo simulation.
This way of computing the probabilities can be made arbitrary accurate by choosing a sufficiently large $N_{rc}$. The problem is that for an online application, sufficiently large often means too large. To get an appropriate accuracy for this specific problem it is required to use $N_{rc} \geq 90000$ [31]. Such a large $N_{rc}$ would most certainly entail a too high computational load. An offline application on the other hand, does not depend on performing the calculations fast and efficiently. Monte Carlo approximation will therefore be used when dealing with the second problem addressed in this thesis, namely to compare the two different definitions of NMAC. During this evaluation $N_{rc} = 90000$ is used for all risk calculations.

### 4.3 Risk calculation as part of a collision avoidance system

The purpose of the risk calculation in a collision avoidance system is to determine the risk for collision if a possible threat is detected in mid-air. If the collision probability is considered to be too high, the system should trigger an evasive maneuver. The existing TCAS which is applied in commercial aviation can detect and avoid NMAC, given a collision scenario, with a probability which is approximately 0.91 [8]. It is therefore reasonable to require at least the same from the UAV’s collision avoidance system.

However, even if the risk calculation would indicate that the probability for collision is greater than 0.09 for the current states of the own and intruding vehicles, it is not always desirable to initiate a maneuver. It could very well be that this risk decreases in the near future, for instance if better measurements become available or if the intruder changes its course. It would then be unnecessary to initiate the maneuver which would introduce a disturbance in the UAV’s mission. A better approach is to reason like this; if the evasive maneuver should be triggered now, then what would the probability of NMAC be? If this probability is too high, initiate the maneuver, otherwise keep to the mission. How this reasoning could work in a certain flight scenario is illustrated in Figure 4.3.

The threshold at which the evasive action is initiated can be seen as an acceptance level for the actual outcome. Given that the collision probability is calculated correctly, it is expected that the obtained NMAC frequency coincides with the critical probability. The choice of threshold needs to be a tradeoff between obtaining robustness in the system and to keep the NMAC frequency low. If the threshold is set too low, the system can be seen as being paranoid. Evasive maneuvers will be triggered even when it is unnecessary, which is an undesired and possibly dangerous behavior.

By using the TCAS level of 0.09 as a point of reference, an appropriate threshold can be set. This value should however be seen as a total outcome acceptance. Since there are other aspects affecting the performance of the collision avoidance system, such as inaccuracies in the tracking and missed detections, the critical
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Figure 4.3. Risk calculation at three subsequent points of time. The evasive maneuver is not triggered until the collision risk, given the evasive action, becomes greater than some predefined threshold. In this case the maneuver is initiated in the last view, where \( P(\text{NMAC}|\text{evasion}) \geq 0.02 \).

A threshold of 0.02 will thus be used throughout this thesis.

4.4 Simulation results: Idealized circumstances

The different definitions of NMAC are evaluated in two natural steps. The first is to compare them under idealized circumstances. This means that the risk calculation is detached from the rest of the collision avoidance system. Instead of feeding the risk calculation with information from a tracking filter, the calculation is based on exact knowledge of uncertainty of the intruder’s state. This information is extracted directly from the surrounding simulation environment. In the second step the risk calculation is reconnected with the rest of the collision avoidance system. The output from the tracking system is then used as the source of information for the risk calculation. In this step the entire collision avoidance system is evaluated as a whole, which of course make it more realistic and closer to a real implementation.
The advantage of using this strategy, to divide the evaluation into two steps, is that the definitions can be compared without any disturbance from external error sources. The theoretical potentials of the two definitions can then be deduced. When the risk calculation later is put into its context, the final result is easier to interpret. It can for instance be determined if an error is due to deficiencies in the underlying NMAC definition or simply due to shortcomings of the tracking.

This section deals with the comparison of the definition under idealized circumstances, whereas the next covers the evaluation of tracking based risk calculation.

4.4.1 Exact knowledge of uncertainty

If the risk calculation is to be performed with exact knowledge of uncertainty, this knowledge needs to be obtained in some way. Recall that the risk calculation assumes the state of the intruder to be stochastic according to

$$x \sim N(\hat{x}, \hat{P})$$

(4.8)

The meaning of ideal conditions is that this assumption is true. I.e. in each simulation, the state of the intruder should be perfectly described by (4.8) where $\hat{x}$ and $\hat{P}$ are known to the system.

To accomplish this, a few modifications need to be done in the definitions of the simulation scenarios. The encounter model otherwise used in this thesis (presented in Section 2.3) randomizes the intruder’s initial state from several different distributions. Due to this, the normality assumption can never be exact at the beginning of a simulation. If the risk calculation is to be tested under idealized circumstances, the encounter model can consequently not be used.

During these simulations the initial state of the intruder is instead randomized from a multivariate normal distribution. The mean and covariance are still the same as for the encounter model, which ensures that the scenario is approximately the same as when the model is used. The mean and covariance of the initial state of the intruder are derived in the same manner as for the initialization of the tracking filter; see Section 3.5 for details.

Now, let the initial state of the intruder be $x_0$ which is drawn from a normal distribution

$$x_0 \sim N(\hat{x}_0, \hat{P}_0)$$

(4.9)

The normality assumption is obviously fulfilled at this point in time. However, since the intruder progresses along its trajectory as the simulation goes on, so will the distribution of its state. Recall the dynamic model from Section 2.4 which describes the motion of the intruder. If the process noise is neglected the model becomes

$$x_{k+1} = Ax_k$$

(4.10)
which entails that
\[ x_1 = A x_0 \sim N(A \hat{x}_0, A \hat{P}_0 A^T) \Rightarrow x_2 \sim N(A^2 \hat{x}_0, A^2 \hat{P}_0 (A^T)^2) \Rightarrow \]
\[ \cdots \Rightarrow x_k \sim N(A^k \hat{x}_0, A^k \hat{P}_0 (A^T)^k) \]  \hspace{1cm} (4.11)
By using (4.11) the exact distribution of the intruder’s state can thus be derived at all points of time, simply by knowing the initial distribution.

### 4.4.2 Scenario A: Risk calculation with evasive maneuver

To compare the two definitions of NMAC, Monte Carlo simulations have been performed on scenario A presented in Section 2.5.1. However, to be able to idealize the conditions for the risk calculation, the encounter model otherwise adopted in this thesis was omitted. The initial state of the intruder in each Monte Carlo run was instead drawn from a normal distribution, as described in Section 4.4.1.

Since the risk calculation in this step of the evaluation is independent of the tracking, the own platform maneuver has no impact on the result. To keep the scenario as intact as possible, a platform maneuver is still used though. In these simulations the small sinusoidal movement in the vertical plane, presented in Section 3.3 was applied.

At the start of each simulation the intruder’s state is randomized from (4.9). The aircraft then progress along their respective trajectories. At every point of time the risk calculation is carried out by calculating the probability according to either (4.6) or (4.7). When the indicated risk, given an evasive action, becomes larger than the threshold of 0.02, the evasive maneuver is initiated. As stated in Section 4.3, it is believed that the outcome frequency of NMACs should coincide with the critical probability if the risk is calculated correctly. Figure 4.4 shows the NMAC frequency as a function of the number of Monte Carlo runs.

![Figure 4.4. NMAC frequencies for the two definitions under idealized circumstances and for an increasing number of Monte Carlo runs. The dashed line is the momentary definition, (4.1), and the solid line is the time horizon definition, (4.2).](image-url)
4.5 Simulation results: Tracking based risk calculation

It is evident that the definition of NMAC according to (4.2) performs much better than the definition according to (4.1). When the time horizon definition is used the frequency converges to the threshold level, whereas the momentary definition produces a result which is much higher. This is not very surprising since the definition of NMAC over an interval of time intuitively is more correct than the momentary definition.

Two important conclusions can still be drawn from this simulation though. First of all, and most importantly, is that the time horizon definition works as intended. By using this definition the outcome frequency will be the same as the specified threshold level, given that the risk calculation is correct. This is an appealing property since it makes it possible to implement the risk calculation with a direct connection to specified aviation safety rules. If the observed outcome frequency does not correspond to the threshold level, as with the momentary definition of NMAC, it is necessary to obtain additional, empirical knowledge of the problem. I.e. if a certain acceptance level for the NMAC frequency is stated, the designer of the collision avoidance system needs to find another value for the threshold which will yield the desired result. To find such a value, and to be able to guarantee its correctness, is likely to be a very hard problem.

The second thing that can be concluded from the simulation is that the momentary definition of NMAC yields a result which is much higher than the critical probability. In this case, the frequency seems to converge towards a value more than three times the threshold level. Even if it would be possible to find another threshold that could guarantee that the critical probability was not exceeded, this level is likely to be very low. To find out how low, the same simulations as presented above have been performed once again. This time the threshold, at which the evasive maneuver is triggered, is pushed down until the critical 2% outcome frequency is reached. This resulted in a threshold of 0.006. To use such a low value in a real implementation would make the system fragile since an evasive action very easily is initiated. It is for instance possible that the sole presence of noise in the risk calculation would trigger the evasive maneuver even when no actual threat was available.

4.5 Simulation results: Tracking based risk calculation

In this section the two definitions are compared under more realistic circumstances. The simulation results presented here come from simulations where the encounter model from Section 2.3 is used. The risk calculation is furthermore based on tracking filter output. This data is both noisy and more or less incorrect, i.e. the certainty measure used by the risk calculator to determine the collision probability does not coincide exactly with the underlying situation.

The results presented in this section come from simulations on both scenario B and
scenario A. The former has been used to analyze the calculated probabilities in a single simulation. This evaluation shows how the probabilities according to the two definitions differ and how they are affected by the noisy environment induced by the use of a tracking filter.

Scenario A is used in a similar way as was presented in the previous section. From these simulations it can be seen how much the NMAC outcome frequencies are affected by the changed circumstances.

4.5.1 Scenario B: Risk calculation using deterministic initial state

Recall scenario B, presented in Section 2.5.2. The intruder is detected straight ahead and is approaching the own vehicle. The course angle of the intruder, $\alpha$, is separating their respective trajectories. The simulation is run for a range of angles, $0 \leq \alpha \leq 12^\circ$. Three different setups (B.1, B.2 and B.3), with respect to initial separation and intruder velocity are used. The parameters defining the three instances of the scenario are recapitulated in Table 4.1.

<table>
<thead>
<tr>
<th>Scenario</th>
<th>Initial separation (m)</th>
<th>Intruder velocity (m/s)</th>
</tr>
</thead>
<tbody>
<tr>
<td>B.1</td>
<td>3112</td>
<td>55</td>
</tr>
<tr>
<td>B.2</td>
<td>2712</td>
<td>72.9</td>
</tr>
<tr>
<td>B.3</td>
<td>3512</td>
<td>37.1</td>
</tr>
</tbody>
</table>

The purpose of using a full range of angles is to test the system for both high and low collision risks. By using this range, the probability of NMAC will span the entire interval from definite to negligible. To maintain this variety in the collision risks no evasive maneuver is considered during the simulations on scenario B.

In scenario B.1, the initial state of the intruder is equal to the initialization of the tracking filter. A good state estimation can thus be expected, even if it is worsened to some extent by the process and measurement noises. If the SS-turn, presented in Section 3.3, is used as an own platform maneuver the certainty of the state estimation will improve rather rapidly. In this case the risk calculator is expected to determine an accurate probability of NMAC. The calculated probabilities according to the two definitions are illustrated in Figure 4.5. The probabilities are shown for the range of angles and at different points of time. An SS-turn has been used as an own platform maneuver.

The true outcome of the scenario changes from a positive case of NMAC to a negative case when $\alpha$ becomes greater than approximately $6^\circ$, which is illustrated with a vertical line. For a sufficiently large angle, the intruder will pass outside
4.5 Simulation results: Tracking based risk calculation

Figure 4.5. Calculated probabilities according to the two definitions, momentary (dashed) and time horizon (solid), for the range of separating angles. The figure shows how the probabilities develop over time. An SS-turn is used as an own platform maneuver.

of the own vehicle’s safety zone, and vice versa. In the ideal case, the calculated probabilities should follow this step function. Due to the uncertainties in the estimate, this is of course not the case, but the true outcome can be seen as a guideline for what the calculated probabilities should look like.

Two things are noticeable in this simulation. First of all, the calculated probabilities converge towards the true outcome as time goes on. The upper left plot shows the probabilities 1 s after the intruder is detected. The collision risks are very inconsistent with the true outcome. It is thus reasonable to require at least a few seconds of tracking before any conclusions can be drawn from the risk calculation. The convergence that can be seen as time goes on is not surprising since the risk calculation becomes more and more certain. This is partly due to the fact that the state estimation from the tracking gets better, but also because it becomes more obvious whether NMAC will occur or not as the aircraft come closer to each other.

The second thing that can be concluded from the simulation is that the momentary probability of NMAC tends to be smaller than the probability seen over a horizon of time. This result is consistent with how the two definitions are expected to differ, as stated in Section 4.1, i.e. that the momentary definition of the probability of NMAC is believed to diminish the actual collision risk. This drawback with the momentary definition is expected to be even more evident when the uncertainty
Defining the probability of NMAC

of the state estimate is high. In the simulation presented above an SS-turn, which yields a rather certain estimate, was used as an own platform maneuver. It would therefore be of interest to see how the two definitions handle the case of a small sinusoidal platform maneuver instead. In this case the uncertainty of the state estimate is higher. Figure 4.6 shows the results of a simulation on scenario B.1 when the smaller platform maneuver is used.

![Figure 4.6](image)

**Figure 4.6.** Calculated probabilities according to the two definitions, momentary (dashed) and time horizon (solid), for the range of separating angles. The figure shows how the probabilities develop over time. A small sinusoidal movement in the vertical plane is used as an own platform maneuver.

The result is as expected; the momentary probability is further diminished while the probability over a horizon of time is less affected. This is most evident after a few seconds when the state estimate has converged in the case of an SS-turn. It can also be concluded that the overall performance is better when the SS-turn is applied than it is for the small sinusoidal maneuver. This is of course due to the higher level of accuracy in the tracking filter output which forms the basis for the risk calculation.

The results from simulations on scenario B.2 and B.3 are presented in Appendix H. These results show great similarities to the ones obtained from scenario B.1 and no additional conclusions can thus be drawn from them.
4.5 Simulation results: Tracking based risk calculation

4.5.2 Scenario A: Risk calculation for random flight scenario with evasive maneuver

To further analyze the two definitions of NMAC, Monte Carlo simulations have been performed on scenario A. These simulations are similar to those performed under idealized circumstances presented in Section 4.4.2. The purpose of this section is to determine how much the NMAC outcome frequency is affected by the scarce information provided by the tracking filter, as opposed to when the system has exact knowledge of uncertainty.

These simulations are closer to a real collision avoidance situation than the ones performed on scenario B, since they take an evasive maneuver into account. In scenario B the calculated probabilities spanned the entire range from 0 to 1. When the risk calculation is performed with respect to an evasive action, the probabilities will be much lower. In these cases it is most important that the risk calculation performs well for probabilities close to the critical threshold level of 0.02.

To compare the methods, a large number of Monte Carlo simulations have been performed. In each simulation the own vehicle uses one of the two considered platform maneuvers, either an SS-turn or a small sinusoidal maneuver. The risk is calculated according to one of the two definitions. Figure 4.7 shows the resulting NMAC frequencies for the different setups and for an increasing number of Monte Carlo runs. The threshold at which the evasive action is triggered is 0.02 in all simulations.

![Figure 4.7. NMAC frequencies for the two definitions when the own platform maneuver is either an SS-turn or a small sinusoidal movement. The risk calculation is based on tracking filter output.](image)
Defining the probability of NMAC

The first thing to notice in this simulation is that the outcome frequencies have deviated from the threshold level. Recall from Section 4.4.2 that the time horizon definition converged towards the threshold when the simulations were performed under idealized circumstances. This is no longer the case; the observed frequency is higher than the critical probability for both definitions and regardless of which platform maneuver that is used. This is due to the inaccuracies brought to the system by introducing noise and erroneous estimations in the simulations. The risk calculation no longer has absolute knowledge of uncertainty and the probability of NMAC can therefore never be calculated with exact correctness. How large the offset is, and whether the observed NMAC frequency will be higher or lower than the threshold, is dependent on the properties of the tracking filter output. If a maximum acceptance level is specified it is thus necessary to tune the parameters of the filter to obtain the desired result.

One interesting observation that can be made from Figure 4.7 is that the NMAC frequencies obtained with the time horizon definition appear to be approximately the same for the two platform maneuvers. This is not the case for the momentary definition where the NMAC frequency is higher for the small sinusoidal maneuver than it is for the SS-turn. When the smaller maneuver is used, the uncertainty in the state estimate is higher than for the SS-turn. The performance of the risk calculation seems thus to be affected differently when the uncertainty increases, depending on the underlying NMAC definition. When the momentary definition is used the performance is worsened, while the time horizon definition appears to be more robust to the different levels of certainty.

Since the observed outcome frequencies are stochastic, the significance in the above statement is determined in Example 4.2. Readers not interested in the derivation of the significance level are encouraged to skip this example. The conclusion drawn from the example is that the NMAC frequencies for the momentary definition are significantly different while the frequencies for the time horizon definition are not.

--- Example 4.2: Test for significance ---

Assume that the true probabilities for NMAC, for one of the different definitions, are $p_1$ and $p_2$ for the two platform maneuvers respectively. If $N$ Monte Carlo simulations have been performed, the outcome frequencies will follow

$$ F_1 \sim \text{Bin}(N, p_1) \approx N(Np_1, \sqrt{Np_1(1 - p_1)}) $$

$$ F_2 \sim \text{Bin}(N, p_2) \approx N(Np_2, \sqrt{Np_2(1 - p_2)}) $$

where the last approximation is valid when $Np(1 - p) \gtrsim 10$ [1]. Since $N = 10^4$ and $p \approx 0.05$ in this case, the approximation is assured to be very accurate.

Let the null hypothesis be that the two observed outcome frequencies, for one of the two definitions, come from equiprobable experiments, i.e.

$$ H_0 : p_1 = p_2 = p $$
The implication of the null hypothesis is that there is no significant difference between the observed NMAC outcome frequencies for the two platform maneuvers. Let the observed frequencies be $f_1$ and $f_2$ where $f_1 \leq f_2$. The probability of observing a result, at least as extreme as the pair $f_1$, $f_2$, under the null hypothesis is

\[
P(E) = P((F_1 \leq f_1 \cap F_2 \geq f_2) \cup (F_2 \leq f_1 \cap F_1 \geq f_2)) =
\]

\[
= \Phi_{F_1}(f_1) (1 - \Phi_{F_2}(f_2)) + \Phi_{F_2}(f_1) (1 - \Phi_{F_1}(f_2))
\]

\[
= \frac{1}{2} \left( 1 + \text{erf} \left( \frac{f_1 - Np}{\sqrt{2Np(1-p)}} \right) \right) \left( 1 - \text{erf} \left( \frac{f_2 - Np}{\sqrt{2Np(1-p)}} \right) \right)
\]

(4.14)

$E$ is the event that one of the variables $F_1$ and $F_2$ is at least as small as $f_1$ and the other variable is at least as large as $f_2$, where $F_1$ and $F_2$ are drawn from (4.12) and (4.13) respectively.

Let the significance level for the test be 99% which means that the null hypothesis can be rejected if the maximum value of (4.14) over $p \in [0,1]$ is smaller than 0.01.

For the momentary definition, the NMAC frequencies for the SS-turn and the small sinusoidal maneuver were 0.0579 and 0.0710 respectively (see Figure 4.7). The number of NMACs in 10000 simulations were thus $f_1 = 579$ and $f_2 = 710$. This yields

\[
\max_{p \in [0,1]} P(E) \approx \cdots \approx 3.0 \cdot 10^{-5} \ll 0.01
\]

(4.15)

which means that the null hypothesis can be rejected with 99% significance. It can therefore safely be concluded that the momentary definition is affected negatively by an increased uncertainty.

For the time horizon definition, the NMAC frequencies were 0.436 and 0.394 which yields $f_1 = 394$ and $f_2 = 436$. This gives

\[
\max_{p \in [0,1]} P(E) \approx \cdots \approx 0.041 > 0.01
\]

(4.16)

The null hypothesis can hence not be rejected, and there is no reason to believe that the time horizon definition is affected in any way by the increased uncertainty.
4.6 Conclusions

One major difference between the two definitions of NMAC is evident in all simulations; the momentary definition, (4.1), tends to deliver a smaller collision risk than the time horizon definition, (4.2). When the uncertainty becomes large, the risk according to the momentary definition is further diminished. In angle-only target tracking the state estimate is typically afflicted with a high level of uncertainty. When the risk calculation is based on this kind of tracking there will thus be a significant difference between the two definitions of NMAC.

When testing the definitions under idealized circumstances it has been proven that the time horizon definition is intuitively correct. I.e. the specified threshold level is reflected in the observed NMAC outcome frequency when this definition is used. This is not the case for the momentary definition. If a certain acceptance level for the NMAC outcome frequency is specified, it is thus not possible to use this value as threshold when the momentary definition is used. It is instead necessary to set another value to obtain the desired result. To choose this value appropriately and to be able to guarantee its correctness is likely to be a very hard problem.

It is of course possible to use the momentary probability of NMAC in other ways than to take the maximum value over the given time horizon, as is done in this thesis. The probability could for instance be integrated over time, which would yield a much more conservative risk calculation. The same problem with this definition would still remain though; the observed NMAC frequency would not be the same as the specified threshold level.

The two definitions have also been tested in a more realistic environment, where the risk calculation is based on tracking filter output. In these simulations it was shown that the time horizon definition is more robust against large uncertainties in the estimation of the intruder’s state. This suggests that the UAV for instance could use a smaller platform maneuver to gain observability in the tracking, if the time horizon definition is used.

Due to the differences between the considered definitions presented above, it is concluded that the time horizon definition is the most appropriate to use in the risk calculation step of a collision avoidance system.
Chapter 5

Approximate method for risk calculation

This chapter deals with the third problem addressed in this thesis, namely to evaluate the approximate method for calculating $P(\text{NMAC}(0,T_p))$ presented by Nordlund and Gustafsson [31, 32]. The approximate method is designed to calculate the probability of NMAC according to the time horizon definition. The momentary definition of NMAC will thus not be considered throughout this chapter. The chapter starts with a discussion on the need for simplifying approximations and a short presentation of the method that is used. The outcomes from two different simulations are thereafter presented together with a discussion on the result.

5.1 The need for simplifying approximations

This far the probability of NMAC according to the time horizon definition (4.2) has been calculated with Monte Carlo approximation. As stated in Section 4.2 this method for computation can be made arbitrarily accurate by increasing the number of samples in the approximation. However, this improved accuracy comes with the cost of an increased computational load.

In a real implementation of a collision avoidance system the risk calculation needs to be performed as often as possible. This means that the calculation should be executed at every output from the tracking filter, i.e. as soon as new information is available. In this thesis the tracking filter is assumed to work at 10 Hz, which is reasonable for a real implementation as well [30]. The probability of NMAC needs thus to be determined ten times every second.

If Monte Carlo approximation is to be used, it is required to use at least 90000 samples to get an appropriate accuracy for this kind of problem [31]. This means
that 90000 samples need to be drawn, evaluated and added together in 0.1 s which is likely to induce a much higher computational load than can be accepted solely from the risk calculation.

The approximate method proposed by Nordlund and Gustafsson will perform the risk calculation roughly 1000 times faster than the corresponding Monte Carlo approximation [31]. If it can be deduced that the level of accuracy in these approximations is sufficient, they would thus induce a great benefit to the risk calculation step of a collision avoidance system.

5.2 Nature of approximations

The approximate method for risk calculation will not be thoroughly reviewed in this thesis. All details concerning the method can instead be found in the articles by Nordlund and Gustafsson [31, 32]. To still get some acquaintance with the nature of the approximations the underlying idea will however briefly be presented.

The approximations can be divided into two parts, one geometrical and one numerical. Due to this, the method will further on be denoted the geometric numerical method. In Figure 5.1 a possible collision geometry is depicted in relative Cartesian coordinates, as described in Appendix D. To simplify the illustration the geometry is shown in two dimensions, but the reasoning holds for the three-dimensional case as well.

![Figure 5.1. Geometry of a collision scenario in relative Cartesian coordinates.](image)

The intruder is fixed to the origin of the system and the x-axis is aligned with the line of sight. The predicted trajectory of the own vehicle, relative the intruder, is seen as a piecewise linear path. Since the intruder is assumed to have a constant velocity, all breakpoints in the predicted trajectory are due to maneuvering of the own vehicle. The case of NMAC seen over the prediction horizon is equivalent with that any of the line segments cut the safety zone of the intruder (i.e. a sphere with radius 150 m placed at the origin of the system).
5.2 Nature of approximations

The geometrical approximation consists of replacing the sphere with a disc perpendicular to the line of sight. The case of NMAC can then be seen as the event that any of the line segments cut the disc. Figure 5.2 shows the geometry when this approximation is made.

![Figure 5.2. Geometry of a collision scenario in relative Cartesian coordinates when the geometrical approximation is made.](image)

Note that the placement of the disc is arbitrary. In Figure 5.2 it was placed at the origin of the system, i.e. across the center of the sphere. If it were to be placed closer to the own vehicle, at for instance $s_x = R$, the approximation would be more conservative.

The advantage of making the geometrical approximation is that the individual events for each line segment to cut the disc are mutually exclusive. Assume that the path of the own vehicle, during the prediction time horizon, can be divided into $J$ linear segments. The breakpoints of the path occur at times $T_j$ where $j = 0, 1, \ldots, J$. Under the geometric approximation, the probability of NMAC can hence be simplified to

$$P(\text{NMAC}(0, T)) = \sum_{j=0}^{J-1} P(\text{NMAC}(T_j, T_{j+1}))$$

It is thus only necessary to treat each linear segment separately, which vastly simplifies the calculation.

The individual probabilities can however still not be expressed in closed analytic form. Due to this numerical approximations are necessary as well. The main part of the numerical approximations consists of using an iterative algorithm which converges towards the sought probability [31, 32].

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1This statement is only true if the relative velocity component perpendicular to the line of sight, i.e. $v_x$, do not change sign during the prediction. This condition is likely to be fulfilled in all interesting possible collision situations.
5.3 Simulation results

The geonumerical approximate method is evaluated through simulations in which it is compared with the result from Monte Carlo approximation. Two scenarios are used, one with respect to a UAV application and one in which a self-contained collision avoidance system is applied in commercial aviation.

5.3.1 Scenario A: Risk calculation with evasive maneuver

To determine the correctness in the geonumerical approximation, simulations have been performed on scenario A presented in Section 2.5.1. As was proven in Chapter 4, the time horizon definition of NMAC is robust to different choices of platform maneuver. It is thus sufficient to use a single maneuver for the evaluation of the geonumerical approximation method. For the simulations presented in this section, the small sinusoidal maneuver in the vertical plane (see Section 3.3) is employed.

To conduct the evaluation the probability of NMAC over a given time horizon is computed with both Monte Carlo approximation and the geonumerical method. If the calculation is correct, the two methods should produce roughly the same result. Figure 5.3 shows the calculated probabilities given an evasive action, i.e. $P(\text{NMAC}(0,T_p) | \text{evasion})$, for one simulation on scenario A. The figure shows how the probabilities according to the two computational methods develop over time.

![Figure 5.3. Calculated probabilities according to Monte Carlo approximation (solid) and geonumerical approximation (dashed) for one simulation on scenario A.](image)

When the threshold level of 0.02 is reached, the evasive maneuver is triggered. In
this case the avoidance was successful since the probabilities of NMAC decrease towards zero once the evasive action is initiated. As can be seen in the figure, there is a good correspondence between the calculated probabilities according to the two methods. The result from the geonumerical approximation follows the Monte Carlo result closely throughout the entire simulation. To analyze the magnitude of the error, the difference between the calculated probabilities according to the two methods is depicted in Figure 5.4.

![Figure 5.4](image-url)

**Figure 5.4.** Difference between the calculated probabilities according to Monte Carlo approximation and geonumerical approximation for one simulation on scenario A. The dotted lines show the $3\sigma$-levels for the Monte Carlo approximation.

It is worth noticing that the Monte Carlo method is an approximation in itself and should thus not be seen as the true collision risk in the comparison. It is however easy to derive the accuracy of the Monte Carlo approximation for a given number of samples. A confidence interval for the true underlying probability can therefore be obtained from the Monte Carlo result. In Figure 5.4 the $3\sigma$-levels for the Monte Carlo approximation is depicted together with the difference between the calculated probabilities. These levels provide a 99.7% confidence interval for the true probability, i.e. the actual collision risk can be asserted to lie between the dotted lines in Figure 5.4 with 99.7% certainty.

Due to this the Monte Carlo approximation cannot be said to be significantly better than the geonumerical approximation, as long as the latter resides within the confidence interval. As long as the geonumerical approximation lies within the boundaries it is thus safe to assume that it is just as accurate as the Monte Carlo approximation, using 90000 samples. As can be seen from Figure 5.4 this is the case for large parts of the simulation. Around the absolute time, $t = 20$ s, the probability stretches outside the boundaries though. From Figure 5.3 it can be seen that this point of time corresponds to where the collision probability reaches its peak and hits the threshold level.
The geonumerical approximation seems hence to be very accurate (just as accurate as the Monte Carlo approximation) when the underlying probability is close to zero. When the probability increases towards the threshold level it becomes visible that the geonumerical method indeed differs from the true probability. This difference is however still rather small; the absolute error between the calculated probability and the $3\sigma$-level is less than $1.4 \cdot 10^{-3}$ (0.14 percentage points). The relative error is less than 0.08 during the entire simulation.

Even though this result shows that the probability according to the geonumerical method is determined with high accuracy (a relative error less than 0.08), the consequence of the approximation error needs to be considered. As stated above, the maximum error was obtained when the probability was close to the threshold level. This is a troublesome result since the important thing for the risk calculator is to determine when the threshold is exceeded. In the end, the output from the risk calculation can be cut down to two discrete cases; either to initiate an evasive action or not. A small error in the calculated risk, when it is close to the threshold level, could cause the evasive maneuver to be triggered somewhat later than it would have otherwise. Even a small delay, such as a single sample time (in this case 0.1 s), could lead to an increased NMAC frequency.

To see whether this is the case or not, Monte Carlo simulations have been performed on scenario A. Figure 5.5 show the obtained NMAC frequencies for two sets of simulations, one where the geonumerical approximation method is used and one where Monte Carlo approximation is used. The own platform maneuver is a small sinusoidal movement in the vertical plane for both sets of simulations.

![Figure 5.5. NMAC frequencies for the geonumerical approximation and for Monte Carlo approximation.](image)

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2The simulation results where the risk calculation is based on Monte Carlo approximation are the same as presented in Section 4.5.2.
For the geonumerical approximation 525 of the 10000 Monte Carlo runs resulted in NMAC, i.e. the outcome frequency is 0.0525. When the risk calculation is based on Monte Carlo approximation the corresponding number of NMACs is 394. To determine if these frequencies are significantly different a hypothesis test is conducted in the same way as was described in Example 4.2. Recall that the null hypothesis, stating that the observed frequencies come from equiprobable experiments, can be rejected if the sample statistic is less than 0.01. The observed frequencies yield

\[
\max_{p \in [0,1]} P((F_1 \leq 394 \cap F_2 \geq 525) \cup (F_2 \leq 394 \cap F_1 \geq 525)) \approx \ldots
\]

\[
\ldots \approx 1.6 \cdot 10^{-6} \ll 0.01
\]

(5.2)

It can thus be said with 99 % significance that the geonumerical method gives a different result than when the risk calculation uses Monte Carlo approximation.

As previously stated, this result is likely to depend on the error in the risk calculation when the NMAC probability is close to the threshold level. The result presented in Figure 5.5 shows that the geonumerical method gives a higher NMAC frequency than the Monte Carlo approximation. This indicates that the geonumerical approximation tends to underestimate the collision probability. If the probability is underestimated the evasive maneuver will be triggered somewhat later, leading to a higher collision frequency.

The error in the calculated NMAC probability does not have to be large for this effect to take place. For the simulation illustrated in Figure 5.3 the relative error was less than 0.08 during the entire scenario. This can however still be sufficient for the evasive action to be initiated at a later time point. Even a very small delay, such as a single sample time, might cause many of the simulations otherwise clear of NMAC to fall on the wrong side of the safety zone.

It is easy to realize that this problem can be dealt with if the accuracy of the geonumerical method can be ascertained. If it for instance would be known that the relative error never exceeds 0.1 the threshold can simply be lowered with 10 %, in this case from 0.02 to 0.018. It is then guaranteed that the trigger point never becomes delayed. The problem of finding proper error boundaries for the geonumerical method is however not pursued in this thesis.

5.3.2 Scenario C: Self-contained collision avoidance in commercial aviation

The collision avoidance system evaluation made in this thesis has been focused on a UAV application. It is however possible to find other potential scopes of use, such as commercial aviation. Simulations have therefore been performed on scenario C, which is a replication of a real mid-air collision between two manned commercial aircraft (see Section 2.5.3). This section does not aim at presenting an exhaustive investigation of the possible use for this kind of collision avoidance
in commercial aviation. The purpose of the section is simply to bring the idea to mind and to serve as a stepping stone for further exploration of the topic.

Recall scenario C from Section 2.5.3. The two aircraft, one Embraer Legacy 600 and one Boeing 737, collided in mid-air as a result of a head-on encounter. Both aircraft were equipped with TCAS which is the standard collision avoidance system to use in commercial aviation. TCAS uses information shared directly between the aircraft via transponders. In this particular case, the transponder on the Legacy did however not communicate with the Boeing 737, either because it was malfunctioning or turned off by mistake [12]. Due to this, neither of the two aircraft’s collision avoidance systems were working. This is a drawback with TCAS which could be handled by equipping the aircraft with self-contained collision avoidance systems, i.e. systems not dependent on communicating with each other. A tracking based system could for instance serve as a complement to TCAS and hence increase the safety when the primary system fails.

When dealing with commercial aircraft, such as the Legacy and the Boeing 737, the circumstances are different from the case with a small UAV. Most notably is that the cruise speeds of the aircraft are much higher. In scenario C the own vehicle travels at 230 m/s and the intruder at 240 m/s, which are typical speeds for the Boeing 737 and the Legacy respectively [2][5]. The guess, \( \hat{v}_{\text{int}} \), used to initialize the tracking filter is set to the same as the own speed, i.e. 230 m/s. The aircraft are also larger than what has been considered in the UAV/VFR aircraft scenarios. This implies a longer detection distance. In scenario C, the intruder is detected at 4500 m whereas the range guess used to initialize the tracking filter, \( \hat{r} \), is set to 5000 m. All other parameters used to initialize the tracking filter are the same as for the previous scenarios (see Section 3.5).

No platform maneuver is used in the simulations on scenario C, since this would be impractical for a manned aircraft. One possible solution to the unobservability problem is to use a range measuring sensor, such as the radar. For aircraft of this magnitude, the sensor weight and power supply would no longer be a problem. This option is however not considered in this thesis. Instead, the same kind of passive angle-only sensor as used for the UAV evaluation is employed to see if this is applicable in this case as well.

Figure 5.6 shows the geometry of a simulation on scenario C. The collision risk is computed with the geonumerical approximate method. When the risk exceeds the threshold of 0.02 an evasive maneuver is initiated. The maneuver consists of an 8 m/s\(^2\) (0.8g) dive with a 1 s delay, i.e. it takes 1 s after the maneuver is triggered until the acceleration is applied.
The minimum separation between the aircraft is 205 m and NMAC is thus avoided, which is a very positive result. The simulation suggests that the use of a simple self-contained collision avoidance system could have averted the accident.

One problem with this result though is that the evasive maneuver is triggered very soon after the intruder is detected; merely 1.4 s. This is due to the very rapid course of events in scenario C; if no avoidance is performed the aircraft will collide in just below 10 s. To obtain a robust collision avoidance system it is necessary to give the system more time before the evasive action is triggered. This is also the case if the risk calculation is intended to serve as a warning system for the pilot. If the pilot is to initiate the evasive maneuver manually there will be a delay from the time of the warning until the maneuver is performed.

To improve the robustness of the collision avoidance system several additional simulations have been performed on scenario C with an increasing threshold level. If the threshold is set to a higher value, the system will wait longer before any action is taken. This will allow it to become more certain that a collision actually will occur, before initiating any evasive maneuver. Figure 5.7 shows the minimum separation between the aircraft as function of the threshold level. The figure also shows the time between detection and initiation of the evasive action.
When the threshold is raised above approximately 0.07 the minimum separation becomes less than 150 m and there will be an NMAC. However, even if the threshold is set as high as 0.9 the minimum separation is 54 m, i.e. an NMAC occurs but the collision is avoided. In this case the evasive action is triggered 5 s after the detection, which is roughly half the time to collision. This result suggests that the use of a self-contained collision avoidance system could serve as a backup for a primary system like TCAS. By using a high threshold level the system will be more robust and it can still be enough to avoid a collision that would have occurred otherwise.

5.4 Conclusions

When evaluating the geonumerical approximation on a single case, it was shown to coincide well with the corresponding Monte Carlo approximation. During large parts of the simulation the geonumerical method was within the confidence bounds and the relative error never exceeded 0.08. To see if this result is general, additional simulations need to be performed to further evaluate the method.

The geonumerical approximation was also used in Monte Carlo simulations. It was shown that the obtained NMAC outcome frequency indeed differed from when Monte Carlo approximation was used. This result indicates that the geonumerical approximation tends to underestimate the probability of NMAC. It is likely that this behavior is caused by the placement of the disc in the geometrical part of the approximation. Recall from Section 5.2 that the approximation will be more
5.4 Conclusions

If the disc is placed at a positive x-coordinate, for instance at $s_x = R$. If the disc is placed at $s_x = 0$, as is the case in this thesis, the geometrical approximation is likely to underestimate the NMAC probability to some extent. The increased NMAC outcome frequency does however not mean that the approximation error is large. Even a small error could induce a somewhat delayed trigger point for the evasive maneuver, which in turn would yield a higher NMAC frequency. If the error boundaries for the geonumerical approximation can be found, this problem is easily coped with by compensating for the maximum relative error.

This result also indicates that many of the simulations clear of NMAC when the Monte Carlo approximation is used lies very close to the safety zone. It is shown that a small delay for the trigger point of the evasive action, such as a single sample time (0.1 s), moves cases otherwise clear of NMAC to within the safety zone. 0.1 s might seem to be an insignificant delay, but since the aircraft come approximately 10-15 m closer during this time, it is apparent that cases close to the safety zone will fall on the wrong side. Negative cases of NMAC are thus transformed into positive cases of NMAC, which shows in the outcome frequency.

Due to this, just looking at the NMAC outcome frequencies might be misleading. The use of a definite safety zone divides the result into two discrete cases, either NMAC or not, without consideration of how close to the boundary the underlying result is. It would perhaps be more adequate to use a gray zone close to the boundary, or simply to study the sample statistics of the minimum separations for all Monte Carlo simulations. It is recommended that this is done in further work on the topic.

To conclude the evaluation of the geonumerical approximation, it has been show that the method computes the NMAC probability with high accuracy but with some tendencies to underestimate the risk. A more conservative approximation could be obtained by making a different placement of the disc in the geometrical part of the approximation. An important part in the future development of this method is to find error boundaries. Given these it would be a simple task to compensate for the approximation errors induced by the method, and thus obtain the desired result from the collision avoidance system.
Chapter 6

Closure

In this chapter the thesis work is concluded and further work on the same topic is proposed. Three problem have been studied, each dedicated a chapter in this thesis (Chapter 3, Chapter 4 and Chapter 5 respectively). The conclusions presented in this chapter summarize the three problems and relate them to each other. For deeper discussions on the individual problems, see the corresponding chapters.

6.1 Conclusions

The problem of sensing and avoiding intruding objects in mid-air is large and complex. To be able to handle the problem it is commonly divided into different steps or modules. The work presented in this thesis has been focused on one link in the chain, namely the risk calculation. It is however crucial that the interface between this other modules is proper. To ascertain this, the risk calculation was put in its context and an evaluation was performed both on the tracking system output, which is the basis for risk assessment, and on the risk calculation module itself.

The tracking system output consist of a state estimate, \( \hat{x} \), and a corresponding covariance matrix, \( \hat{P} \). To be able to calculate the probability of NMAC, the true state of the intruder is assumed to be normally distributed with mean \( \hat{x} \) and covariance \( \hat{P} \). By examining the sample statistics from a large number of Monte Carlo simulations, the correctness of this assumption was investigated.

Since angle-only target tracking is reliant on a platform maneuver to gain observability in range, two platform maneuvers of different magnitude have been used in the evaluation. It is expected that a large and quick maneuver will yield a more accurate tracking than a smaller maneuver [22]. This is consistent with what have been observed in the evaluation. However, if the risk calculation is robust to large
uncertainties it is not important that the state estimate is accurate, merely that the normality assumption is correct.

By conducting statistical hypothesis tests and applying well-known visual techniques for assessing normality, it has been shown that the tracking output indeed can be seen as normally distributed. A slight divergence was seen at the beginning of the tracking, but the output converged to normality within a few seconds. This result was independent on which of the two platform maneuvers that was used, which suggests that a normally distributed output is a general feature for this kind of tracking systems.

When evaluating the actual risk assessment module, the focus has been on how the probability of NMAC should be defined to get a method for risk calculation which is robust to the large uncertainties afflicting angle-only tracking. Two different definitions have been analyzed; the more traditional maximum of the momentary risk has been compared with the cumulated risk over a critical time horizon. Two major differences have been found between the methods.

The first is that the time horizon definition, as opposed to the definition using maximum momentary risk, supplies a direct connection between the threshold parameter and the actual outcome observed in the simulations. This property means that the designer of a collision avoidance system can set a threshold with respect to specified aviation safety regulations, and know that this parameter is reflected in the actual performance of the system. In a real implementation, the high noise level together with other inaccuracies will ruin this property to some extent, as was seen in Section 4.5.2. Due to this it might seem like an irrelevant feature, since the reality always will be noisy. The important thing though, is that the time horizon definition on its own works as intended instead of being an additional potential source of errors. If the time horizon definition is used, the development of the collision avoidance system can be focused on e.g. the tracking to get this step as accurate as possible. It is then known that the risk calculation will yield the desired result as long as its input is correct. For the definition using maximum momentary risk, this is not the case; even if the tracking output could be made exact the result of the risk calculation would still be unknown.

The second major difference between the two definitions of NMAC is that the time horizon definition was shown to be much more robust to state uncertainty than the momentary definition. This is an appealing property since the level of uncertainty in the state estimate is likely to vary much between different applications and often be of considerable magnitude. Statistical hypothesis tests have been conducted and confirm the conclusion that maximum momentary risk is affected negatively from an increased uncertainty, whereas the cumulated risk is more robust.

Due to these two differences it is recommended that the time horizon definition is used for risk assessment. However, without an efficient numerical algorithm, its practical use is limited. For that reason, a recent geometrical–numerical (geometrical) approximation was evaluated [31, 32]. The method was shown to perform well and was mostly within the statistical confidence margins when compared with
a corresponding Monte Carlo approximation. The relative error never exceeded 0.08. However, when the method was used in Monte Carlo simulations it became evident that even such a small error affected the average result. To cope with this, it would be of great interest to derive the error bounds for the geonumerical method. If these were to be found, it would be a simple thing to compensate for the inaccuracies introduced by the approximations.

In addition to the collision avoidance evaluation towards a UAV application conducted in this thesis, a real mid-air collision has been studied. The purpose with this investigation was to see if a self-contained collision avoidance system, using a simple EO camera, could be applicable in commercial aviation as well. The result was that a tracking based collision avoidance system very well could serve as a backup for a primary system like TCAS. By using a secondary avoidance system, severe accidents involving manned traffic aircraft could possibly have been averted.

### 6.2 Further work

Further work on the same topic should mainly be concerned with finding error boundaries for the geonumerical approximate method. To do so it is perhaps necessary to look at the two parts of the approximations, the geometrical and the numerical, individually. For the numerical parts it may be possible to find analytic expressions which restrain the approximation error. It is worth noticing that this part of the approximation can be made arbitrary accurate by increasing the number of iterations in the method, of course at the expense of an increased computational load. To analyze the consequences of the geometrical part a simulation study, focused on the errors induced by this approximation, could be performed. It would also be of interest to compare the two parts of the approximations with each other to see how they contribute to the total error. This could serve as guidance when choosing the numbers of iterations for the numerical part.

It would also be of interest to further develop the simulation environment and to perform additional simulations. This could corroborate the results presented in this thesis, making it easier to draw general conclusions. The expansion of the simulation environment could for instance be an implementation of a more advanced tracking filter, such as the IMM filter which is able to track maneuvering targets [15]. The different parameters of the tracking system could be varied to a larger extent than what has been made in this thesis, to see how this affects the final result. During additional simulations it is recommended that the minimum separations between the involved aircraft are considered in addition to whether NMAC occurs or not.

Finally, as the next step in the evaluation of the collision avoidance system it would be desirable to feed it with actual flight test data. This way the circumstances are guaranteed to be as realistic as possible.
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Appendix A

Detection distance as a stochastic variable

A minimum requirement to be able to detect a target is that it occupies at least one pixel in the sensor. Whether this is the case or not depends on the distance to, and the size of, the intruder. By using a purely geometrical approach, the number of pixels that the intruder occupies across a single dimension can be calculated as

\[ N_{px} = \frac{L}{l} \]  \hspace{2cm} (A.1)

where \( N_{px} \) is the sought number of pixels, \( l \) is the width covered by each pixel at the distance of the intruder (\( r \)) and \( L \) is the minimum length of the target visible to the camera. The latter will of course differ very much between different targets; a skydiver is somewhat smaller than a Boeing 747. Since we still need to make a guess on the size of the target, it is preferable to assume that it is relatively small. This way we are more likely to assume that a target is closer than it actually is, instead of the other way around. A good choice is thus \( L = 2 \) m, which is based on empirical knowledge of what kind of targets we are most likely to encounter [30]. Figure A.1 shows the geometry of the situation.

Figure A.1. The geometry of a target detection situation.
By studying Figure A.1 we realize that

\[ l \approx r \phi_{IFOV} \]  \hspace{1cm} (A.2)

since \( l \ll r \). \( \phi_{IFOV} \) is the camera’s instantaneous field of view, i.e. the angle covered by each pixel in the camera. For a typical camera used in this kind of applications this is approximately 0.3 mrad. With values inserted we get an expression for the number of pixels as a function of the distance to the target.

\[ N_{px}(r) = \frac{2 \cdot 10^4}{3r} \]  \hspace{1cm} (A.3)

The detection distance can however not be calculated by sole geometrical means. When the radiation emitted by the target reaches the sensor the intensity will have decreased, which further aggravates the detection. This is due to optical effects associated with the lens, the detector, the atmospheric transmittance etc. Each of these subsystems contributes to the damping of the intensity, and the magnitude of the damping can be described by their respective modulation transfer functions (MTF). The MTF of an optical system is the gain of the modulation, or contrast, as a function of the spatial frequency. Since a high spatial frequency correspond with small objects visible to the sensor, it can be realized that the spatial frequency is proportional to the distance of the target. The MTFs can therefore be seen as functions of \( r \) instead. Each MTF has a zero-frequency gain of 1 and is monotonically decreasing for higher frequencies. I.e. when there is no distance to the target no damping will occur, and when the distance increases so does the damping.

As mentioned before, there are many individual components contributing to the damping in the system, and each component has its own MTF. The total MTF of the system can however be calculated in a simple manner, as the product of the MTFs of the involved subsystems [20]. For a typical UAV application the total MTF could depend on \( r \) as shown in Figure A.2.

![Figure A.2. The total MTF is a monotonically decreasing function of \( r \).](image-url)
The number of pixels occupied in the sensor can now be modified with the MTF of the system according to

\[ N_{cy}(r) = N_{px}(r) MTF(r) \] (A.4)

where \( N_{cy} \) is called the number of cycles across the target [30]. Based on the number of cycles, the detection distance can be seen as stochastic. We define a stochastic variable

\[ X_{cy} = "the number of cycles required for detection" \] (A.5)

By empirical means this variable has been shown to have the cumulative distribution function [21]

\[ P(X_{cy} < N_{cy}) = \frac{(2N_{cy})^E}{1 + (2N_{cy})^E} \] (A.6)

where

\[ E = 2.7 + 1.4N_{cy} \] (A.7)

We are however more interested in the detection distance than we are in the number of cycles required for detection. Define thus a new stochastic variable

\[ X_r = "distance for detection" \] (A.8)

Using the relation between \( r \) and \( N_{cy} \) given by (A.4), the probability density function for \( X_r \) can be determined. Unfortunately, this function is rather complicated and inconvenient to work with. It would therefore be desirable to find a standard distribution which approximates the source distribution.

By analyzing the density function visually it can be seen that it is unimodal and right-skewed. The distribution can therefore be well approximated with a gamma distribution which has the same properties. The approximate distribution is given by

\[ X_r \sim \Gamma(k_r, \theta_r) + b_r \] (A.9)

where \( k_r \) is a shape parameter, \( \theta_r \) is a scale parameter and \( b_r \) is an offset. To achieve an as good approximation as possible, the mean, variance, skewness and excess kurtosis are determined numerically for the source distribution. The obtained values are given in Table A.1.

<table>
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<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Mean (( \mu_r ))</td>
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</tr>
<tr>
<td>Variance (( \sigma_r^2 ))</td>
<td>1.600 \cdot 10^5 m^2</td>
</tr>
<tr>
<td>Skewness (( \gamma_{1r} ))</td>
<td>0.343</td>
</tr>
<tr>
<td>Excess kurtosis (( \gamma_{2r} ))</td>
<td>0.525</td>
</tr>
</tbody>
</table>
Since the translated gamma distribution has three parameters, three of the source
distribution’s properties can be matched. The mean, variance, skewness and excess
kurtosis for the gamma distribution are given by

\[ \mu_r = k_r \theta + b_r \quad (A.10a) \]
\[ \sigma^2_r = k_r \theta^2 \quad (A.10b) \]
\[ \gamma_1 = \frac{2}{\sqrt{k_r}} \quad (A.10c) \]
\[ \gamma_2 = \frac{6}{k_r} \quad (A.10d) \]

The match is made by setting \( k_r \) to the average of the two values suggested by
(A.10c) and (A.10d). \( \theta \) is thereafter set according to (A.10b) and finally, \( b_r \) is
set to match (A.10a). For the values given in Table A.1 the parameters become
\( k_r = \left( \frac{1912}{400} \right)^2 \approx 22.85 \), \( \theta_r = \frac{400^2}{1912} \approx 83.68 \) m and \( b_r = 1200 \) m.

The density functions of the source distribution and the fitted gamma distribution
are shown in Figure A.3.

\[ \begin{align*}
0 & \quad 500 & \quad 1000 & \quad 1500 & \quad 2000 & \quad 2500 & \quad 3000 & \quad 3500 & \quad 4000 & \quad 4500 & \quad 5000 \\
0 & \quad 0.2 & \quad 0.4 & \quad 0.6 & \quad 0.8 & \quad 1 & \quad 1.2 \\
0 & \quad 500 & \quad 1000 & \quad 1500 & \quad 2000 & \quad 2500 & \quad 3000 & \quad 3500 & \quad 4000 & \quad 4500 & \quad 5000 \\
0 & \quad 0.2 & \quad 0.4 & \quad 0.6 & \quad 0.8 & \quad 1 & \quad 1.2 \\
\end{align*} \]

**Figure A.3.** Probability distribution for detection distance (solid line) and fitted gamma
distribution (dashed).
Appendix B

Probability distribution for $\beta$

$\beta$ is randomized by selecting a bin at random, in which the value is taken from a uniform distribution. If the bin containing zero in its range is picked, $\beta$ is set to zero. Table B.1 specifies the bins and their corresponding probabilities.

<table>
<thead>
<tr>
<th>Bins (rad)</th>
<th>Probability</th>
<th>$\beta$ (rad)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$[-0.080, -0.050]$</td>
<td>0.02</td>
<td>$\beta \sim U(-0.08, -0.05)$</td>
</tr>
<tr>
<td>$[-0.050, -0.015]$</td>
<td>0.13</td>
<td>$\beta \sim U(-0.05, -0.015)$</td>
</tr>
<tr>
<td>$[-0.015, 0.015]$</td>
<td>0.7</td>
<td>$\beta = 0$</td>
</tr>
<tr>
<td>$[0.015, 0.050]$</td>
<td>0.13</td>
<td>$\beta \sim U(0.015, 0.05)$</td>
</tr>
<tr>
<td>$[0.050, 0.080]$</td>
<td>0.02</td>
<td>$\beta \sim U(0.05, 0.08)$</td>
</tr>
</tbody>
</table>

Table B.1 completely defines the distribution of $\beta$. By using a Dirac delta function to represent the high density at zero, the distribution function becomes as illustrated in Figure B.1.
Since $\beta$ has a symmetric distribution, the mean is zero. The variance of $\beta$ can be calculated as

$$\sigma_{\beta}^2 = E(\beta^2) = \int_{-\infty}^{\infty} b^2 p_{\beta}(b) \, db =$$

$$= \int_{-0.05}^{0.05} \frac{0.02}{0.08 - 0.05} b^2 \, db + \int_{0.05}^{0.15} \frac{0.13}{0.05 - 0.015} b^2 \, db +$$

$$+ \int_{-0.015}^{0.015} \frac{0.7}{0.015 + 0.015} b^2 \delta(b) \, db +$$

$$+ \int_{0.015}^{0.15} \frac{0.13}{0.05 - 0.015} b^2 \, db + \int_{0.08}^{0.05} \frac{0.02}{0.08 - 0.05} b^2 \, db =$$

$$= 2 \cdot \frac{0.13}{0.05 - 0.015} \left[ \frac{b^3}{3} \right]_{0.015}^{0.05} + 2 \cdot \frac{0.02}{0.08 - 0.05} \left[ \frac{b^3}{3} \right]_{0.05}^{0.08} \approx$$

$$\approx 4.7317 \cdot 10^{-4} \text{ rad}^2$$

The standard deviation becomes

$$\sigma_{\beta} \approx \sqrt{4.7317 \cdot 10^{-4}} \approx 0.022 \text{ rad}$$
Appendix C

Tracking filter

An EKF (extended Kalman filter) is a time discrete filter working in two steps, one time update and one measurement update. What is special about the MSC-EKF is that the filter works with two coordinate systems, both absolute Cartesian and modified spherical. The advantage with this is that both update steps become linear.

Recall the absolute Cartesian state vector, describing the state of the intruder

\[
x^A_{rel} = (s^A_x \ s^A_y \ s^A_z \ v^A_x \ v^A_y \ v^A_z)^T = (x_1^A \ x_2^A \ x_3^A \ x_4^A \ x_5^A \ x_6^A)^T
\] (C.1)

To simplify the notation, this state vector will be denoted \( x \) in this appendix. Using this state the equations of motion become linear (see Section 2.4). The measurement, \( y = (\phi \ \theta)^T \), on the other hand is nonlinear in the Cartesian state vector. To cope with this, modified spherical coordinates are introduced. Let

\[
z = \left( \frac{1}{r} \ \phi \ \theta \ \frac{\dot{r}}{r} \ \Omega \ \hat{\theta} \right)^T = (z_1 \ z_2 \ z_3 \ z_4 \ z_5 \ z_6)^T
\] (C.2a)

where

\[
\Omega = \dot{\phi} \cos \theta
\] (C.2b)

The measurement \( y \) is a linear combination from this state vector. Another advantage of using MSC in the filter is that the unobservable state \( r \) is decoupled from the observable states, and will not risk to degrade the filter [15].

The dynamic system becomes

\[
\begin{align*}
x_{k+1} &= Ax_k + \bar{u}_k + \bar{w}_k \\
y_k &= Cz_k + \epsilon_k \\
x &= f_X(z) \\
z &= f_Z(x)
\end{align*}
\] (C.3)
where $A$ describes the dynamics of the system (see Section 2.4), $\tilde{u}_k$ is an input signal (e.g. an own platform maneuver) and $\tilde{w}_k$ is the process noise. The measurement is obtained from

$$C = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

with an additive measurement noise, $\epsilon_k$. The functions $f_X$ and $f_Z$ transform the state vector between the two involved coordinate systems. Recall from Section 2.4 that the process noise was assumed to be piecewise constant Gaussian acceleration with intensity 0.75 m/s$^2$. This yields the covariance matrix for $\tilde{w}_k$

$$Q_\sigma = \text{var}(\tilde{w}_k) = \text{var}(B_w w_k) = \sigma_q^2 B_w B_w^T =$$

$$= \sigma_q^2 \begin{pmatrix} T_s^4/4 & 0 & 0 & 0 & 0 \\ 0 & T_s^4/4 & 0 & 0 & 0 \\ 0 & 0 & T_s^3/2 & 0 & 0 \\ 0 & 0 & 0 & T_s^2 & 0 \\ 0 & 0 & 0 & 0 & T_s^2 \end{pmatrix}$$

where $T_s = 0.1$ s is the sample time and $\sigma_q = 0.75$ m/s$^2$. The measurement noise, $\epsilon_k$, has the covariance matrix

$$R_\sigma = \begin{pmatrix} \sigma^2 & 0 \\ 0 & \sigma^2 \end{pmatrix}$$

where $\sigma = \sigma = 0.001$ rad.

Let $\hat{x}_{k|k}$ denote the state estimate at time $t_k$ given measurements up to time $t_k$. In the same manner, $\hat{x}_{k|-1}$ is the estimate at time $t_k$ based on measurements up to time $t_{k-1}$. A similar notation is used for other involved variables.

Let furthermore $J_X$ and $J_Z$ be the Jacobians of functions $f_X$ and $f_Z$ respectively. These are derived by Erlandsson [15]. This gives the MSC-EKF according to Algorithm 1.
Algorithm 1 MSC-EKF

1. Transform the state vector to Cartesian coordinates
\[ \hat{x}_{k|k-1} = f_X(\hat{z}_{k|k-1}) \]

2. Time update
\[ \hat{x}_{k|k-1} = A\hat{x}_{k-1|k-1} + \tilde{u}_k \]
\[ \hat{P}^{MSC}_{k|k-1} = \Phi_{k|k-1} \hat{P}\dot{k}_{k-1|k-1} \Phi^T_{k|k-1} + Q^{MSC}_{k-1} \]
where \( \Phi \) is given by the chain rule for derivatives
\[ \Phi_{k|k-1} = \left. \frac{\partial z_k}{\partial \hat{z}_k} \right|_{\hat{z}_k} = \frac{\partial z_k}{\partial x_k} \frac{\partial x_k}{\partial \hat{x}_{k-1}} \frac{\partial \hat{x}_{k-1}}{\partial z_k} = J_Z(\hat{x}_{k|k-1})A X(\hat{z}_{k-1|k-1}) \]
and, for \( Q \) given in Cartesian coordinates
\[ Q^{MSC}_{k-1} = J_Z(\hat{x}_{k|k-1})Q J_Z(\hat{x}_{k|k-1})^T \]

3. Transform the state vector to MSC
\[ \hat{x}_{k|k-1} = f_Z(\hat{z}_{k|k-1}) \]

4. Measurement update
\[ \varepsilon_k = y_k - C \hat{z}_{k|k-1} \]
\[ \hat{x}_{k|k} = \hat{x}_{k|k-1} + K \varepsilon_k \]
\[ \hat{P}^{MSC}_{k|k} = \hat{P}^{MSC}_{k|k-1} - K C \hat{P}^{MSC}_{k|k-1} \]
where
\[ K = \hat{P}^{MSC}_{k|k-1} C^T S^{-1} \]
\[ S = C \hat{P}^{MSC}_{k|k-1} C^T + R \]

The covariance matrix in Cartesian coordinates is given by
\[ \hat{P}_{k|k} = J_X(\hat{x}_{k|k}) \hat{P}^{MSC}_{k|k} J_X(\hat{z}_{k|k})^T \]
Appendix D

Relative Cartesian coordinates

For the purpose of risk calculation it is appropriate to use relative Cartesian coordinates as described in this appendix. The same coordinate system is used by Nordlund and Gustafsson in the derivation of the geonumerical approximate method for calculating the probability of NMAC [31, 32]. To compute the collision risk with this method it is thus necessary to provide the risk calculator with information of the intruder’s state in relative Cartesian coordinates. Since the tracking filter works in modified spherical coordinates and absolute Cartesian coordinates (see Appendix C) a transformation of the tracking filter output is required.

In relative Cartesian coordinates the intruder is fixed to the origin of the system and all velocity is placed on the own vehicle. By doing so the safety zone of the intruder becomes a sphere centered at the origin with a radius of 150 m. At every instance of time the system is rotated so that the x-axis coincides with the line of sight. The own vehicle is placed at the positive x-coordinate \( s^R_x \). Since the angle measurements used for tracking the intruder are so accurate it can be assumed that \( s^R_y \equiv s^R_z \equiv 0 \) which reduces the dimension of the state vector from six to four states. The geometry of a collision situation in relative Cartesian coordinates is illustrated in Figure D.1. The geometry is depicted in two dimensions to simplify the illustration.
Relative Cartesian coordinates (state vector $x^R$) should not be confused with absolute Cartesian coordinates where the origin is fixed to the own vehicle (state vector $x^A_{rel}$). The latter coordinate system does not rotate, and the axes of the system always correspond to the same directions (e.g. east, north and up). The subscript $rel$ on the state vector indicates that the state of the intruder is relative to the own vehicle which is fixed to the origin of the system.

Transformation from absolute Cartesian coordinates\(^1\)

$$x^A_{rel} = \begin{pmatrix} s^A_x \\ s^A_y \\ s^A_z \\ u^A_x \\ u^A_y \\ u^A_z \end{pmatrix} = \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \\ x_6 \end{pmatrix} \quad (D.1)$$

to relative Cartesian coordinates

$$x^R = \begin{pmatrix} s^R_x \\ s^R_y \\ s^R_z \\ v^R_x \\ v^R_y \\ v^R_z \end{pmatrix} = \begin{pmatrix} x^R_x \\ x^R_y \\ x^R_z \\ x^R_x \end{pmatrix} \quad (D.2)$$

\(^1\)The superscripts $A$ on the state components of the absolute state vector have been neglected throughout this appendix to simplify the notation.
can be made according to

\[
\begin{align*}
  x^R_1 &= \sqrt{x^2_1 + x^2_2 + x^2_3} \\
  x^R_2 &= \frac{x_1x_4 + x_2x_5 + x_3x_6}{\sqrt{x^2_1 + x^2_2 + x^2_3}} \\
  x^R_3 &= \frac{-x_2x_4 + x_1x_5}{\sqrt{x^2_1 + x^2_2}} \\
  x^R_4 &= \frac{-x_3(x_1x_4 + x_2x_5)}{\sqrt{(x^2_1 + x^2_2 + x^2_3)(x^2_1 + x^2_2 + x^2_3)}} - x_6 \sqrt{x^2_1 + x^2_2} \\
\end{align*}
\]

The covariance matrix in absolute Cartesian coordinates, \( \hat{P}^A \), can be transformed to a covariance matrix in relative Cartesian coordinates, \( \hat{P}^R \), according to

\[
\hat{P}^R = J_{A,R} \hat{P}^A J_{A,R}^T
\]

where \( J_{A,R} \) is the Jacobian for the transformation function from \( x^A_{rel} \) to \( x^R \), i.e.

\[
J_{A,R} = \begin{bmatrix}
\frac{\partial x^R_1}{\partial x_1} & \cdots & \frac{\partial x^R_1}{\partial x_a} \\
\vdots & \ddots & \vdots \\
\frac{\partial x^R_a}{\partial x_1} & \cdots & \frac{\partial x^R_a}{\partial x_a}
\end{bmatrix}
\]

Let \( \rho = \sqrt{x^2_1 + x^2_2} \) and \( r = \sqrt{x^2_1 + x^2_2 + x^2_3} \) which gives the partial derivatives

\[
\begin{align*}
  \frac{\partial x^R_1}{\partial x_1} &= \frac{x_1}{r} \\
  \frac{\partial x^R_1}{\partial x_2} &= \frac{x_2}{r} \\
  \frac{\partial x^R_1}{\partial x_3} &= \frac{x_3}{r} \\
  \frac{\partial x^R_1}{\partial x_4} &= \frac{\partial x^R_1}{\partial x_5} = \frac{\partial x^R_1}{\partial x_6} = 0
\end{align*}
\]
\[
\begin{align*}
\frac{\partial x_1^R}{\partial x_1} &= -\frac{x_1^2 x_4 + x_1 x_2 x_5 + x_1 x_3 x_6 - r^2 x_4}{r^3}, \\
\frac{\partial x_2^R}{\partial x_2} &= -\frac{x_1 x_2 x_4 + x_2^2 x_5 + x_2 x_3 x_6 - r^2 x_5}{r^3}, \\
\frac{\partial x_3^R}{\partial x_3} &= -\frac{x_1 x_3 x_4 + x_2 x_3 x_5 + x_3^2 x_6 - r^2 x_6}{r^3}, \\
\frac{\partial x_4^R}{\partial x_4} &= \frac{x_1}{r}, \\
\frac{\partial x_5^R}{\partial x_5} &= \frac{x_2}{r}, \\
\frac{\partial x_6^R}{\partial x_6} &= \frac{x_3}{r}.
\end{align*}
\]

(D.6b)

\[
\begin{align*}
\frac{\partial x_1^R}{\partial x_1} &= -\frac{x_5 \rho^2 + x_1 x_2 x_4 - x_1^2 x_5}{\rho^3}, \\
\frac{\partial x_2^R}{\partial x_2} &= -\frac{-x_4 \rho^2 + x_2^2 x_4 - x_1 x_2 x_5}{\rho^3}, \\
\frac{\partial x_3^R}{\partial x_3} &= 0, \\
\frac{\partial x_4^R}{\partial x_4} &= \frac{-x_2}{\rho}, \\
\frac{\partial x_5^R}{\partial x_5} &= \frac{x_1}{\rho}, \\
\frac{\partial x_6^R}{\partial x_6} &= 0.
\end{align*}
\]

(D.6c)

\[
\begin{align*}
\frac{\partial x_1^R}{\partial x_1} &= -\frac{\rho^2 + r^2}{\rho^3} x_1 x_3 (x_1 x_4 + x_2 x_5) + \frac{x_4 x_4}{\rho r} - \frac{x_1^2 x_4}{\rho^3}, \\
\frac{\partial x_2^R}{\partial x_2} &= -\frac{\rho^2 + r^2}{\rho^3} x_2 x_3 (x_1 x_4 + x_2 x_5) + \frac{x_3 x_5}{\rho r} - \frac{x_2^2 x_6}{\rho^3}, \\
\frac{\partial x_3^R}{\partial x_3} &= \frac{\rho}{r^3} (x_1 x_4 + x_2 x_5) + \frac{\rho}{r^3} x_3 x_6, \\
\frac{\partial x_4^R}{\partial x_4} &= \frac{x_1 x_3}{\rho r}, \\
\frac{\partial x_5^R}{\partial x_5} &= \frac{x_2 x_3}{\rho r}, \\
\frac{\partial x_6^R}{\partial x_6} &= -\frac{\rho}{r}.
\end{align*}
\]

(D.6d)
Appendix E

Propagation of uncertainty

To initialize the tracking filter the state vector, \( \hat{x}_{0|0} \), and the prediction covariance matrix, \( \hat{P}_{0|0} \), need to be set with appropriate values. This is achieved by reasoning about the current situation and making initial guesses for the variables describing the scenario. Recall the state vector of the intruder in absolute Cartesian coordinates, where the origin is fixed to the own vehicle.

\[
x_{rel}^{A} = \begin{pmatrix} r \cos \varphi \cos \theta \\ r \sin \varphi \cos \theta \\ -r \sin \theta \\ -v_{int} \cos \alpha \cos \beta - v_{own} \cos \psi \cos \gamma \\ -v_{int} \sin \alpha \cos \beta - v_{own} \sin \psi \cos \gamma \\ -v_{int} \sin \beta + v_{own} \sin \gamma \end{pmatrix} \tag{E.1}
\]

where

\[
\alpha = \alpha_0 + \tilde{\alpha} = \arcsin \left( \frac{-v_{own} \sin \psi \cos \gamma}{v_{int} \cos \beta} \right) + \tilde{\alpha} \tag{E.2}
\]

The state vector can be seen as a function of the describing variables, i.e.

\[
x_{rel}^{A} = f(p) \tag{E.3}
\]

where

\[
p = \begin{pmatrix} r & \varphi & \theta & v_{int} & \tilde{\alpha} & \beta \end{pmatrix}^T \tag{E.4}
\]

is the parameter vector describing the collision scenario when the tracking is to be initialized. Note that \( v_{own} \), \( \psi \) and \( \gamma \) are considered to be known to the system with exactness and are hence treated as constants in the expression above.

The initial state vector can be determined simply by making an initial guess for the parameter vector, \( p = \hat{p} \), and calculating \( \hat{x}_{0|0} = f(\hat{p}) \). To determine the covariance
matrix, the uncertainty in \( \hat{p} \) needs to be propagated to \( \hat{x}_{0|0} \). This can be done approximately by using Gauss’ propagation of uncertainty formula [11].

\[
\hat{P}_{0|0} = \text{var}(f(p)) \approx \text{var} \left( f(\hat{p}) + J_{f|\hat{p}}(p - \hat{p})J_{f|\hat{p}}^T \right) = J_{f|\hat{p}} \Sigma_p J_{f|\hat{p}}^T \quad (E.5)
\]

where \( J_{f|\hat{p}} \) is the Jacobian for the function \( f \) at the point \( \hat{p} \) and \( \Sigma_p \) is the covariance matrix for the parameter vector \( p \). The approximation is simply a first order Taylor expansion of the function \( f \) around the point \( \hat{p} \).

The different variables describing the scenario are assumed to be independent, and consequently

\[
\Sigma_p = \text{diag}(\sigma_r^2, \sigma_\phi^2, \sigma_\theta^2, \sigma_\alpha^2, \sigma_\beta^2) = \text{diag}(400^2, 0.001^2, 0.001^2, 17.89^2, 0.023^2, 0.022^2) \quad (E.6)
\]

where the numerical values are based on the encounter model adopted in this thesis (see Section 2.3).

The Jacobian is obtained by differentiating the function \( f \), i.e.

\[
J_f = \begin{bmatrix}
\frac{\partial f_1}{\partial p_1} & \cdots & \frac{\partial f_1}{\partial p_6} \\
\vdots & \ddots & \vdots \\
\frac{\partial f_6}{\partial p_1} & \cdots & \frac{\partial f_6}{\partial p_6}
\end{bmatrix} \quad (E.7)
\]

where

\[
f_1(p) = r \cos \varphi \cos \theta = p_1 \cos p_2 \cos p_3 \\
f_2(p) = r \sin \varphi \cos \theta = p_1 \sin p_2 \cos p_3 \\
f_3(p) = -r \sin \theta = -p_1 \sin p_3 \\
f_4(p) = -v_{\text{int}} \cos \alpha \cos \beta - v_{\text{own}} \cos \psi \cos \gamma = \\
\quad = -p_4 \cos \left( \arcsin \left( -\frac{v_{\text{own}} \sin \psi \cos \gamma}{p_4 \cos p_6} \right) + p_5 \right) \cos p_6 - C_1 \quad (E.8)
\]

\[
f_5(p) = -v_{\text{int}} \sin \alpha \cos \beta - v_{\text{own}} \sin \psi \cos \gamma = \\
\quad = -p_4 \sin \left( \arcsin \left( -\frac{v_{\text{own}} \sin \psi \cos \gamma}{p_4 \cos p_6} \right) + p_5 \right) \cos p_6 - C_2
\]

\[
f_6(p) = -v_{\text{int}} \sin \beta + v_{\text{own}} \sin \gamma = -p_4 \sin p_6 + C_3
\]

and \( C_1, C_2 \) and \( C_3 \) are constants. This yields the partial derivatives
\[
\begin{align*}
\frac{\partial f_1}{\partial p_1} &= \cos p_2 \cos p_3 \\
\frac{\partial f_1}{\partial p_2} &= -p_1 \sin p_2 \cos p_3 \\
\frac{\partial f_1}{\partial p_3} &= -p_1 \cos p_2 \sin p_3 \\
\frac{\partial f_1}{\partial p_4} &= \frac{\partial f_1}{\partial p_5} = \frac{\partial f_1}{\partial p_6} = 0
\end{align*}
\]

(E.9a)

\[
\begin{align*}
\frac{\partial f_2}{\partial p_1} &= \sin p_2 \cos p_3 \\
\frac{\partial f_2}{\partial p_2} &= p_1 \cos p_2 \cos p_3 \\
\frac{\partial f_2}{\partial p_3} &= -p_1 \sin p_2 \sin p_3 \\
\frac{\partial f_2}{\partial p_4} &= \frac{\partial f_2}{\partial p_5} = \frac{\partial f_2}{\partial p_6} = 0
\end{align*}
\]

(E.9b)

\[
\begin{align*}
\frac{\partial f_3}{\partial p_1} &= -p_1 \\
\frac{\partial f_3}{\partial p_2} &= 0 \\
\frac{\partial f_3}{\partial p_3} &= -p_1 \cos p_3 \\
\frac{\partial f_3}{\partial p_4} &= \frac{\partial f_3}{\partial p_5} = \frac{\partial f_3}{\partial p_6} = 0
\end{align*}
\]

(E.9c)

\[
\begin{align*}
\frac{\partial f_4}{\partial p_1} &= \frac{\partial f_4}{\partial p_2} = \frac{\partial f_4}{\partial p_3} = 0 \\
\frac{\partial f_4}{\partial p_4} &= -\cos \alpha \cos p_6 + \frac{v_{own}}{p_4} \sin \alpha \sqrt{1 - \left(\frac{v_{own}}{p_4} \sin \psi \cos \gamma \cos p_6\right)^2} \\
\frac{\partial f_4}{\partial p_5} &= p_4 \sin \alpha \cos p_6 \\
\frac{\partial f_4}{\partial p_6} &= p_4 \cos \alpha \sin p_6 - \frac{v_{own} \sin \alpha}{\sqrt{1 - \left(\frac{v_{own}}{p_4} \sin \psi \cos \gamma \cos p_6\right)^2}} \sin \psi \cos \gamma \tan p_6
\end{align*}
\]

(E.9d)
\[ \frac{\partial f_5}{\partial p_1} = \frac{\partial f_5}{\partial p_2} = \frac{\partial f_5}{\partial p_3} = 0 \]
\[ \frac{\partial f_5}{\partial p_4} = -\sin \alpha \cos p_6 - \frac{v_{own}}{p_4} \cos \alpha \frac{\sin \psi \cos \gamma}{\sqrt{1 - \left( \frac{v_{own}}{p_4} \sin \psi \cos \gamma \cos p_6 \right)^2}} \]
\[ \frac{\partial f_5}{\partial p_5} = -p_4 \cos \alpha \cos p_6 \]
\[ \frac{\partial f_5}{\partial p_6} = p_4 \sin \alpha \sin p_6 + v_{own} \cos \alpha \cos \gamma \frac{\sin \psi \cos \gamma \tan p_6}{\sqrt{1 - \left( \frac{v_{own}}{p_4} \sin \psi \cos \gamma \cos p_6 \right)^2}} \]

\[ \frac{\partial f_6}{\partial p_1} = \frac{\partial f_6}{\partial p_2} = \frac{\partial f_6}{\partial p_3} = 0 \]
\[ \frac{\partial f_6}{\partial p_4} = -\sin p_6 \]
\[ \frac{\partial f_6}{\partial p_5} = 0 \]
\[ \frac{\partial f_6}{\partial p_6} = -p_4 \cos p_6 \]

where
\[ \alpha = \arcsin \left( -\frac{v_{own}}{p_4} \sin \psi \cos \gamma \cos p_6 \right) + p_5 \]
Appendix F

Tracking performance plots

This appendix presents additional standard deviation and bias plots for scenario A, to serve as a complement for the ones presented in Section 3.6.2. The plots given in this appendix come from simulations where the standard deviations are initialized too high, as well as where the initial state is biased.

Figure F.1. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is an SS-turn. The standard deviations are initialized too high.
Figure F.2. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is a small sinusoidal movement in the vertical plane. The standard deviations are initialized too high.

Figure F.3. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when no platform maneuver is used. The standard deviations are initialized too high.
Figure F.4. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is an SS-turn. The initial state is biased.

Figure F.5. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when the own platform maneuver is a small sinusoidal movement in the vertical plane. The initial state is biased.
Figure F.6. Sample standard deviations (solid lines), average estimated standard deviations (dashed lines) and bias (dash-dotted lines) in the four states of relative Cartesian coordinates when no platform maneuver is used. The initial state is biased.
Appendix G

QQ-plots for scaled residuals

This appendix shows QQ-plots for the remaining three scaled residuals, not presented in Section 3.7.1.

Figure G.1. QQ-plot for the second scaled residual when the own platform maneuver is an SS-turn.
Figure G.2. QQ-plot for the third scaled residual when the own platform maneuver is an SS-turn.

Figure G.3. QQ-plot for the fourth scaled residual when the own platform maneuver is an SS-turn.
Figure G.4. QQ-plot for the second scaled residual when the own platform maneuver is a small sinusoidal movement in the vertical plane.

Figure G.5. QQ-plot for the third scaled residual when the own platform maneuver is a small sinusoidal movement in the vertical plane.
Figure G.6. QQ-plot for the fourth scaled residual when the own platform maneuver is a small sinusoidal movement in the vertical plane.
Appendix H

Simulation results from scenario B.2 and B.3

This appendix gives the probabilities according to the two considered definitions of NMAC for a range of angles in scenario B.2 and B.3.

Figure H.1. Calculated probabilities according to the two considered definitions, momentary (dashed) and time horizon (solid), for the range of separating angles in scenario B.2. An SS-turn is used as an own platform maneuver.
Figure H.2. Calculated probabilities according to the two considered definitions, momentary (dashed) and time horizon (solid), for the range of separating angles in scenario B.2. A small sinusoidal movement in the vertical plane is used as an own platform maneuver.

Figure H.3. Calculated probabilities according to the two considered definitions, momentary (dashed) and time horizon (solid), for the range of separating angles in scenario B.3. An SS-turn is used as an own platform maneuver.
Simulation results from scenario B.2 and B.3

Figure H.4. Calculated probabilities according to the two considered definitions, momentary (dashed) and time horizon (solid), for the range of separating angles in scenario B.3. A small sinusoidal movement in the vertical plane is used as an own platform maneuver.
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