Formation Flying of Spacecrafts for Monitoring and Inspection

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Declaration

I confirm that the work presented in this thesis is, except of the instructions of my tutor, completely on my own achievement and that any resources used in it are properly denoted in the bibliography.

Würzburg, September 15, 2009

(Jian Ma)
Abstract

Formation flying of multiple spacecrafts has been an essential of interest in space technology. Groups of spacecrafts involved in formation can provide higher performance with comparable lower cost in many space applications. This thesis presents the work on studying the formation flying control issues of two satellites with close separation between each other in the LEO orbit under the Leader/Follower architecture. Within this hierarchical structure, Follower is controlled in order to find a fuel-optimal path to the entry of the pre-designed formation orbit, maintain it and maneuver for formation reconfiguration with optimal control - Linear Quadratic Regulator (LQR). Besides, noise and communication failures are taken into consideration which bring uncertainties to the linearized system dynamics provided by the modified Hill-Clo Hessy-Wiltshire (HCW) Equations. Thus nonlinear Sliding Mode Control (SMC) and Kalman filter are necessarily employed for precise requirements. The theoretical results are supported by simulation platform based on MATLAB®.

Keywords: formation flying, simulation, the HCW Equations, optimal control, LQR, SMC, Kalman filter.
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1 Introduction

1.1 State of the Art

Since human’s successful access to space in the 1960s, spacecrafts have been developed elaborately as the main exploration carriers and platforms. From the first artificial satellite Sputnik 1 to the International Space Station (ISS), the largest man-made satellite in Earth orbit, requirement for spacecrafts has increased extensively and dramatically. In order to satisfy multi-task requirements, complex large spacecrafts are designed and manufactured with various kinds of payloads and instruments onboard. Consequentially, single large satellite is common in missions such as scientific observation, weather monitoring, global navigation and civil relay communication.

At the same time, the cost of design and developing life cycle of single large satellites has become a critical problem facing space organizers and engineers, as few nations in the world could afford to build rockets and launch satellites. Some international program such as ISS needs the collaboration of several countries. This significantly constrains the development of space exploration of human. Moreover with the increasing complexity, stability of large spacecrafts decreases. It is not impossible that minor mistake leads to the failure of the whole mission and causes a catastrophe, which was the case with space shuttle Challenger (1986). Additionally, the increase in size and the overlap of orbits raises the possibility of spacecrafts crash. On Feb 11, 2009 at about 17:00 GMT, collision between the Iridium Satellite LLC-operated satellite of the USA and Russian Cosmos-2251 military satellite occurred at about 780 km above the Russian Arctic. The accident draws space researchers’ attention to the safety of large spacecrafts. Another problem is space debris also called space junk resulting from entire spent rocket stages and defunct satellites. On one hand, in mission operation large spacecrafts could generate more space debris; on the other hand, risk rises with increasing of the size of spacecrafts. Thus, since the 1980s research on small spacecrafts and formation flying has been carried out in the aim of finding a possible replacement to single large spacecraft.

Formation flying, defined by researchers from the US National Aeronautics and Space Administration (NASA), means the concept of grouping multiple satellites which can communicate with each other, share data processing, payload and mission functions so as to accomplish the objective of normal single larger satellite usually more expensive than the group satellites. In general, this type of architecture has a distributed nature. Similarly, cluster and constellation, two other terms appear frequently in space technology domain. These three terms should be properly delineated for a better understanding. A satellite cluster is simply a group of two or more satellites located in different or similar orbits and performs a desired mission for a local area. Its main difference from formation flying is that a cluster may not necessarily require special geometry of satellites’ formation. The application of a cluster requires its elements to be distributed and usually the system...
is designed with redundancy. If the cluster is designed with intended shape, the cluster could be referred as formation flying. For example, a space-based radar system can use formation flying because the position of the elements needs to be adjusted to form different aperture. A satellite constellation is also a kind of distributed satellite system, but serves a large or even global area. In the constellation, each satellite is controlled individually without close-loop control for their relative position. A representative example is the Global Positioning System (GPS) of the USA. It uses a constellation of 24 Medium Earth Orbit (MEO) satellites with 4 backup ones. Shape control is not within function requirement of a constellation. In practice, a distributed space system provides more redundancy, reliability, survivability and less cost. Assuming that 4 satellite in a formation group for Earth observation, if one were to fail, the overall capability of the system drops temporarily, but the integrity on the whole is not totally lost.

1.1.1 Missions Available

The concept of formation flying dates back to the later 1980s and intensive research of it started in the 1990s. Many institutions have shown great interest in this research area including NASA, Jet Propulsion Laboratory (JPL) of California Institute of Technology (Caltech), US Air Force, European Space Agency (ESA), German Space Center (DLR), China National Space Administration (CNSA) and so on. Innovative programs have been proposed for developing and testing of new space technologies for formation flying.

Initiated by American Air Force Research Laboratory (AFRL) in the late 1990s, the joined TechSat-21 program serves as a proof of conceptual mission for a series of innovative design and technology via formation flying of micro-satellites. Actually, the concept of replacement of large single satellite with virtual large satellite via formation of micro-satellites was put forward in 1995 by Air Force New World Vistas Space Technology Panel. It also proposed to explore the technical challenges and benefit in a later program. In 1997, a formation of eight micro satellites (Figure 1.1) was formulated as a core concept by AFRL. They carried RF antennas which formed a multi-mission sparse aperture sensing platform. Then, flight test was launched by AFRL in 2000 consisting of three 150 kg satellites in a 550 km LEO orbit. The significant features of this experiment were autonomous formation maintenance, sparse aperture sensing and validated simulation for broad range of missions and satellite configurations [13]. Unfortunately, due to numerous cost overruns,
The TechSat-21 project was canceled in 2003.

The first-ever formation-flying mission of NASA was successfully demonstrated by the launch of Earth Observation 1 (EO-1) in November 2000. In this mission EO-1 satellite was designed to fly in formation with LandSat-7, an Earth environment satellite launched in 1999. Main task of the two satellites was to collect high-resolution images of the Earth. Those images were forward to United States Geological Survey (USGS) to analyze Earth’s environment. In the mission operation, EO-1 satellite flew only 450 km behind LandSat-7 in the same ground track. The maintain of the separation was 3 minutes guaranteeing that EO-1 met the requirement. Its ground track in the cross-track direction remained within $+/-3$ km of LandSat-7’s at the equator \[14\]. This operation of short distance separation made it possible that EO-1 satellite could observe the same ground region at required period together with LandSat-7 satellite. In this mission, Enhanced Formation Flying (EFF) technology developed by NASA’s Goddard Space Flight Center (GSFC) demonstrated the formation of two independent satellites which could react to each other’s orbit change quickly and efficiently. The requirement for EO-1 was that it could follow LandSat-7 with-

![Figure 1.2: EO-1 formation flying](image)

out breaking the along track separation even after the maneuver of LandSat-7. Additionally, formation flying enabled the chaser satellite EO-1 adjust its relative position to the leader satellite to the ideal formation flying location. Definitely, the success of this mission that was involved in NASA’s New Millennium Program (NMP) was the milestone for formation flying research and development.

Later on, some other successful missions in NMP starting in 1995 were carried out successively to speed up space exploration using leading-edge technologies such as ion thruster, high-efficiency power generation. All these new development of space technology form the backbone of precise formation flying. It was also involved in planned mission such as NASA’s Terrestrial Planet Finder (TPF) mission. In TPF, virtual space interferometer system will be implemented to detect and analyze the light from other stars. The requirements of the mission will need 1 km baseline for telescope operation which is not practical via single spacecraft carrying telescopes. Precise formation flying in future will be developed to meet those requirements, and GPS-like systems and sensor fusion techniques will
be included to measure 3-dimensional positions of spacecrafts in a group.

Meanwhile, European Space Agency (ESA) has proposed space programs for testing formation flying technology, with the On-Board Autonomy 3 project (PROBA-3) as its first step [15]. The PROBA-3 mission was proposed in December 2005 with the coverage of design, development and in-flight operation of a group of small satellites in order to test and validate ESA’s formation flying technology. As a demonstrative mission expected to be launched in 2010, a solar corona observer will be carried. In general, the mission can be decomposed into three phases, i.e., experimental/demonstration, non-experimental and deployment. Recently, PROBA-3 has completed the preparatory phase and two independent satellites with 3-axis stabilization will be included later. In the experiment phase, coronagraph and formation flying maneuver will be executed around the apogee, where the space environment is quiet for each orbit at HEO orbit. Additionally, ESA proposed the Darwin mission to look for the origins of life. Four or five spacecrafts will search for Earth-like planets and analyze chemical components in their atmosphere. In the formation group, one leading satellite will act as a central communication hub. The other four will serve as collectors reflecting the light to the central one [16]. In this mission, satellites will be placed in the opposite direction from the Sun, where Lagrangian Point 2 (L2) locates.

The Swarm mission is recently proposed by ESA to study the Earth’s magnetic field. Its concept consists a constellation of three satellites, whose orbit is on the polar orbit within 400 to 550 km in altitude. The primary research objectives are to study the core dynamics, geo-dynamo processes and the core-mantle interaction. In order to map the entire Earth magnetic field, a satellite formation is required to distinguish among different magnetic field with different magnetic activity. One pair of satellites (Swarm A and B) fly side-by-side in near-polar circular orbits with 1 to 1.5-degree separation in between, and one higher satellite (Swarm C) flies in a circular orbit of 530 km altitude.

In 2002, a pair of satellites in the Gravity Recovery and Climate Experiment project (GRACE) supported by NASA and DLR were launched. The mission’s aim was to measure Earth’s gravity field and its temporal variations with high accuracy as well as some other information about the vertical temperature distribution in the near Earth space. Both satellites weight 485 kg each with 500 km altitude and 90 minute orbital period. The distance between the two is around 220 km in the same LEO orbit. When orbiting the Earth, the leader satellite is first pulled away from the follower by passing areas with slightly stronger gravity. When the follower passes above the same areas, it is pulled
towards the leader. The variation of the vertical distance is measured by an extremely precise microwave ranging instrument onboard. Meanwhile, satellite Global Positioning System (GPS) receivers determine the exact position of the two satellites in formation.

On September 25, 2008, Shenzhou-7, China’s third manned spaceflight was launched. In the mission, an extra-vehicular activity was carried out by astronauts with a small companion satellite (BX-1) being one of the highlights. Shenzhou-7 is a large spacecraft with 9 m in length and 8,000 kg mass, while BX-1 being a micro cube satellite with 40 cm of each side and 40 kg mass. BX-1 was released from Shenzhou-7 space capsule after the space walk, and its onboard camera took images of Shenzhou-7. The functions of BX-1 was reported to be test for formation flying technology and health monitoring in future missions. The space capsule was in a near-circular orbit about 330 km altitude with an inclination of 42.4 degrees. BX-1 was supposed to be placed in an elliptical orbit with an apogee 4 km higher than the capsule’s orbit and a perigee 4 km lower with the same period.
of capsule [18]. In this condition, BX-1 would be circling around the capsule ranging from 4 km to 8 km. With the help of micro gas thrusters, BX-1 could perform orbit maneuver, but was not able to reach a higher level of accuracy in relative orbit control.

Figure 1.6: Shenzhou-7 spacecraft and BX-1 companion satellite [6]

Above all, extensive scientific researches on formation flying of group satellites have been carried out for years by various institutes, universities, research organizations in different countries. Software simulations and ground-based test beds have been developed and successful demonstration missions have also been performed to validate the simple formation of identical spacecrafts and to test the comprehensive technologies in formation flying keeping and maneuver.

1.1.2 Formation Flying Technologies

The fundamental idea of formation flying is to lower the risk of single large spacecraft in the harsh space environment, to increase the stability of the mission operation, and to cut the huge development budget and time. At present, common large satellites’ life-scyece from initiation to launch normally lasts five to ten years, which is far behind the development step of innovative concept and technology. As conventional spacecrafts employ a combination of payload and other supporting subsystems, one minor mistake in the hardware or software may lead to the mission’s failure. In addition, since current satellite design and manufacturing involves no comprehensive and powerful self-repair function block, all the satellites are out of use once their lifetime is over. The solution is to develop distributed spacecrafts system based on fractionated spacecraft modules.

First of all, the principal theoretical problem of formation flying is the dynamics of multiple spacecrafts in the Earth orbit. Basically, this problem comes from the three-body problem that describes the motion of three point masses under their mutual gravitational interactions. The classical problem can be seen from a wide range of astronomical facts such as the Moon’s traveling around the Earth as perturbed by the Sun. From this point of view, the motion of stars together with their planets in a specific constellation or even the Universe can be regarded as an N-body problem. However, as proved by Poincare, the
three-body problem is non-integrable, that is, there is no general solution to it. Therefore, necessary simplifications are made according to the order of magnitude of certain terms in the original dynamic motion equations. For example, in a three-body system of the Earth, the Moon and one man-made satellite, it is obvious that the man-made satellite is much smaller in mass than the other two bodies. Such condition leads to the restricted problem directly. Now, let us imagine two identical satellites in the Earth orbit instead. In this system, the total mass of the two satellites is still much smaller than that of the Earth. With this consideration, Hill problem (1878) and the linearization model studying space rendezvous and docking with short separation distance between by Clohessy and Wiltshire (1960) become the basis of formation flying dynamics. The Hill-Clohessy-Wiltshire (HCW) equations have high fidelity because the relative motion and the time scale is relatively small and the perturbation does not affect so much.

Secondly, autonomous formation flying requires extremely precise state of each satellite in the group. In future missions, formation control must first determine the precise position of each satellite according to the Earth Inertial Coordinates in real time. Some innovative positioning technology was proposed such as low-cost Differential Carrier Phase Positioning System (DGPS). This kind of LEO satellite navigation system also employs GPS Wide Area Augmentation System (WAAS) as a differential GPS correction [19]. However, for missions in MEO, GEO with tight formation, GPS signals may be blocked by vehicles nearby, thus the GPS visibility may not be sufficient for high accuracy estimation. Recently, NASA proposed high-accuracy formation flying in the Terrestrial Planet Finder (TPF) mission with the magnitude of centimeter of pre-determined trajectories for synchronized observation. The main technical issue is to develop a suite of sensors to ensure formation position acquisition and communication. There are problems in the following aspects: first is the robustness of spacecrafts accommodation and interference of indirect signal reflected off from the configuration of spacecraft structure; second, auto-calibration is necessary, without requiring complex spacecraft attitude maneuver; and third, how to avoid crash in case of temporary communication failure. The solution to the challenges above is to develop a new generation of signal structure, in which range errors can be reduced and the integrated sensor with inter-spacecraft communication will also enhance formation stability [20]. Moreover, large expensive onboard transceivers are no longer suitable for small satellite formation. Low Power Transceiver (LPT) development is proposed by NASA for simultaneous communications and autonomous navigation.

Another essential part of formation flying technology is control. Control architecture, algorithms and advanced trajectory planning are demanded. The task also requires collision avoidance, fuel consumption minimization and minimum sensing and communication. In designing formation flying control, the size of a formation needs to be defined firstly. As sensing and communication load increase dramatically with large number of satellites. Next, the precision level of control should be considered as high-precision formation flying requires the performance accurate to the level of meters. They are two primary parameters for the formation flying architecture. Researchers from Jet Propulsion Laboratory (JPL) classify formation flying architecture into five categories: Multiple Input Multiple Output (MIMO), Leader/Follower, Virtual Structure, Cyclic and Behavioral. The Leader/Follower type is the most studied. In this architecture, formation flying control problem can be reduced to individual tracking problem [21]. In formation flying control algorithm, critical
problems are stability condition, robustness of the system and how to lower intercommunication requirements within satellites. In specific missions such as Earth observation, formation algorithm must account for not only the sensing range of individual spacecrafts, but also the overlapped or uncovered sensing ranges of each members in the group. What adds to the difficulties is the coupled spacecrafts attitude and relative positions, which leads directly to the complexity of the control algorithm in guidance and estimation.

Advanced trajectory planning needs advanced precise formation control methods. Furthermore, how to optimize it in the nonlinear model is becoming the focus of researchers [22]. For example, in remote sensing, formation rotation as a virtual rigid body and “U-V” coverage for synthetic aperture observation are more complex than in-track formation. Besides formation reconfiguration that consists of formation maneuver will be accomplished using distribute control method [23]. Highly distribute control system was proposed by researchers with non-linear control methods.

As to actuators, the propulsion system acts an important role in formation flying missions. Conventional satellite may not have a propulsion system due to mission requirements. Gyros and momentum wheels are sufficient for the attitude control. Although their mechanical structure is simple and easy for onboard integration, chemical thrusters will be replaced by electric thrusters. Electric thrusters have their superiority compared with conventional ones in mass, effectiveness and efficiency. In future, small satellites missions will not be able to employ cold gas thrusters because of the relevant heavy propellant. In 1998, NASA Solar Electric Propulsion Technology Application Readiness (NSTAR) 30 cm diameter thruster system was validated on the Deep Space 1 (DS-1) mission. This ion thruster was the main propulsion of the 1000 pound spacecraft, and it turned out to be 10 times more efficient than its alternative, a chemical one by reducing almost 90 percent of the mass. Another Hall effect thruster selected for primary propulsion was the PPS-1350 on Small Mission for Advanced Research in Technology 1 (SMART-1). It was launched by ESA in 2003. The shortcomings of these electric thruster mainly are their great power requirements to accelerate propellant. Ten years ago, researchers in NASA started to develop a new generation of Pulsed Plasma Thruster (PPT), in which solid propellant provides more flight stability than liquid. Recently, Micro Pulsed Plasma Thruster (µPPT) has become the hotpot of next generation space propulsion system especially for small satellites. In the EO-1 mission, the PPT thruster also demonstrated the ability of precise pitch attitude control. But a difficulty of the µPPT is how to minimize all the components to enable the application in small satellites formation flying.

Figure 1.7: DS-1 ion thruster [7]
1.1.3 Simulators Development

In formation flying, simulation is the key step to analyze mission, emulate space environment, validate control design. Among the simulators, there are mainly two fundamental types of orbit propagators for formation flying simulation. One is based on the modified HCW equations; the other is based on the classical orbital elements. The former propagator calculates the relative motion displacement directly in the LVLH frame and it is easier to implement in comparison with the latter. The fidelity of the modified HCW equations greatly affects the accuracy of this propagator. Currently, the modified HCW equations contain the effect of $J_2$ perturbation and nonlinear differential gravity [24]. Besides, the effects of aerodynamics drag could also be taken into consideration in the modified HCW equations. In this propagator, solar radiation is not included; the disturbance of the Sun and the Moon gravity can be also ignored if the formation is in LEO orbit. Generally, if the target problem is within the small separation distance and time scale, propagator based on the modified HCW equations are a good choice for its simplicity and good fidelity [25].

Propagators based on classical orbital elements (COE), use real time numerical computation with Keplerian orbital dynamics. To satisfying precise positioning and long-term requirements, these orbital propagators are developed by some institutions and commercial companies. Other open-source orbital dynamic libraries are also available in the internet helping users create their own applications to solve problems in astrodynamics, space mission design, navigation and ground tracking. The implementation of real time propagator is a huge project because a great deal aspects should be well considered and organized.

Firstly various systems of time and coordinates should be clearly defined, and the transformations between them should also be configured. In numerical computation time is an independent variable, and calculations of different parameters utilize different time systems and time formations. It is common that computation of spacecrafts’ motion is based on the Coordinated Universal Time (UTC), while the mutation and precession motion of the Earth is usually studied in the Barycentric Dynamical Time (TDB) system. Besides, transformation from Gregorian calendar date formation to Julian Date (JD) formation is also necessary in real time simulation in astronautics. If the simulation task is related to GPS, another specified time system for GPS is used. Additionally, the Earth Inertial Coordinates system used in simulation usually is Geocentric Equatorial Inertial Coordinate (GEI) system for epoch $J2000.0$ which is often referred as GEI $J2000$ [26]. The commonly used geo-potential model of the Earth is World Geodetic System (WGS)-84 [27]. While treating relative motion of formation flying spacecrafts, transformation between the LVLH frame and GEI is critical for analysis of the relative position and velocity.

Secondly, the choice of multifarious mathematic models of astrodynamics could be the most critical point in a propagator development. The two-body (Keplerian motion) model which only considers the force of gravity from the Earth is widely employed for simplicity. Furthermore, some perturbation models add $J_2/J_4$ perturbation and account for secular variations in the orbital elements due to the Earth oblate configuration [28]. Simplified General Perturbations Satellite Orbit Model 4 (SGP4) is another important algorithm of computation of LEO satellite developed by NASA/NORAD. What it takes into consideration are secular and periodic variations from the Earth oblate configuration, solar and lunar gravitational effects and aerodynamic drag. Open code sources written by C, Fortran
and MATLAB is available from the Center for Space Standards and Innovation (CSSI), a research arm of Analytical Graphics, Inc (AGI). Unfortunately, the comments in some functions of the library are not sufficiently complete with unknown reasons which make it really difficult for potential users [29]. Besides, High-Precision Orbit Propagator (HPOP) which can handle elliptical, parabolic and hyperbolic orbits at various altitudes from LEO to the orbit of the Moon was developed by world leading professional spaceflight simulation software provider AGI in Satellite Tool Kit (STK) to solve complex dynamics real-world problems [28].

Thirdly, it is necessary to find a proper numerical calculation algorithm for propagator implementation in coding. Runge-Kutta method is an often-used approximation of solutions of ordinary differential equations. The order of the Runge-Kutta solver greatly affects the performance of the propagator. Moreover, Gauss-Jackson is also an available switch in calculating and it could be combined with Runge-Kutta method in real application [30]. A problem is the running-time consumption of numerical integral algorithm in such a huge real-time simulation project no matter which algorithm is utilized.

Finally, 3-D visualization for orbits and trajectories also plays an important role in space flight simulation. One of the common simulation software - MATLAB is good at matrix computation, but not at 3-D visualization especially in real-time. In real-time visualization, it is possible to use JAVA and there is an open JAVA library source online - JAVA Astrodynamics Toolkit (JAT) [31]. The main problem is that if a simulation concentrates on control of formation flying, too much effort on 3-D visualization will definitely take much time of a development team or person. However, 3-D visualization could for sure promote the result presentation.

1.2 Motivation

As presented above, formation flying of multiple spacecrafts has been an essential of interest in space technology. This thesis will be the first to investigate the formation flying in the Department of Computer Science VII at the University of Würzburg. We concentrate on a special scenario: an accompany satellite (Follower) monitor and inspect a target satellite (Leader). The target may be non-cooperative. In such applications the accompany satellite needs to fly nearby the target satellite. Normally, the separation between these two satellites are within the range of 10 km level. This issue concerns with space orbit dynamics, control theory under the condition of existence of perturbation in the space environment.

1.3 Contribution and Structure of the Thesis

1.3.1 Contributions

The contribution of this project is: first, systematic investigation of different formation planning methods; second, development of a simulation platform based on MATLAB which can simulate the short-term LEO formation flying; third, implementation of different control strategies together with filtering and analysis of their performances; finally different formation scenarios simulation. One point should be noted that due to the time constrain and
other technique problems, the simulation work based on the COE propagator still needs exploration so that it is not presented in the report.

**Figure 1.8: Thesis systematic structure**

### 1.3.2 Structure of Thesis

Chapter 2 introduces formation flying dynamics with fundamental astrodynamics, orbit perturbation theory and Hill-Clohessy-Wiltshire (HCW) Equations as well as their modified versions.

In chapter 3, we analyze formation initialization, keeping and reconfiguration planning. Varieties of formation configuration are proposed for different usages. Then depending on formation maneuver complexity, straight-line transfer and automatic transfer are proposed.

Chapter 4 gives the formation control issues. First, state-space representation and feedback control laws are constructed; then, we introduce optimal control with a mature and representative method - Linear Quadratic Regulation (LQR); and it’s followed by a nonlinear control - Sliding Mode Control (SMC) which will show it power in control system uncertainty in communication failures with combined with Kalman filter for state estimation and noise reduction.

Chapter 5 presents the results of simulation. We select one typical formation style for each scenario, discuss the controller performance. In the end, one comprehensive formation mission is presented.

Finally, chapter 6 concludes all the work and gives the open points.
2 Formation Flying Dynamics

In this chapter, dynamics of formation are introduced: first, reference coordinates are defined; then we give the basis of astrodynamics; next is the orbit perturbations theorem; finally, the HCW equations are derived as the cornerstone of formation flying dynamics.

2.1 Reference Coordinates

2.1.1 Earth Centered Inertial (ECI) Coordinates

The Earth Centered Inertial frame system is a non-rotating reference without acceleration. Its origin is in the center of the Earth. The x-axis is defined aligned with the Earth’s mean equator and equinox. The z-axis is directing along the celestial north pole perpendicular to the equatorial plane. The y-axis completes a right hand orthogonal frame with the previous two axes within equatorial plane (Figure 2.1).

2.1.2 Earth Spherical (ES) Coordinates

The Earth Spherical coordinates frame is frequently used in computation of the gravity perturbation. In the ECI frame, the position of a satellite can be represented using spherical coordinates \((r, \theta, \phi)\) (Figure 2.2). Here \(r\) is the distance from the Earth center to the satellite, angle \(\theta\) is the satellite longitude and angle \(\phi\) is the satellite latitude.

Figure 2.1: ECI coordinates (OXYZ) and LVLH coordinates (oxyz)  
Figure 2.2: Earth Spherical Coordinates
2.1.3 Earth Centered Fixed (ECF) Coordinates

The Earth Centered Fixed coordinates frame is sometimes referred as a “conventional terrestrial” system. The origin of ECF coordinates denotes the mass center of the Earth. The \(z\)-axis is defined as being parallel to the Earth rotational axes, pointing toward north. The \(x\)-axis points the 0° latitude, 0° longitude at the time origin. Then the \(y\)-axis is perpendicular to \(x\)-axis in the equatorial plane. This means the ECF coordinates rotate with the Earth around its rotation axis.

2.1.4 Earth Centered Orbit (ECO) Coordinates

The introduction of the Earth Centered Orbit coordinates help us compute the position of a satellite in the ECI coordinates. The origin of the ECO coordinates locates in the center of the Earth. The \(x\)-axis points the position of satellite in orbital plane. The \(z\)-axis is normal to the orbit plane. The \(y\)-axis completes a right hand orthogonal frame.

2.1.5 Local Vertical Local Horizontal (LVLH) Coordinates

The Local Vertical Local Horizontal coordinates system has its origin at the center of mass of the reference satellite. The \(x\)-axis is along the position vector of the reference satellite in ECI frame. The \(z\)-axis is pointing along the orbital angular momentum vector of the reference satellite. The \(y\)-axis also completes a right hand orthogonal frame to be aligned along the velocity of the reference satellite. It should be noted that the LVLH coordinates are based on the trajectory of the center of the satellite relative to the ECI coordinates. The LVLH coordinates are distinguished from the LVLH (CBF) in which the reference coordinate is the Earth Centered Fixed coordinates (Figure 2.1).

2.2 Fundamental Astrodynamics

In this section, Kepler’s Laws are presented together with Newton’s Universal Gravitation Law; besides, three-body problem is briefly discussed; in the end, classical orbital elements are given.

2.2.1 Kepler’s Laws

In the early 16\textsuperscript{th} century, German mathematician and astronomer Johannes Kepler discovered laws of planetary motion. These mathematical laws describe the motion of planets in the solar system and they also apply to man-made satellites motion (Figure 2.3):

- First law: The orbit of every planet is an ellipse with the Sun at one focus.
- Second law: The line joining the planet and the Sun sweeps out equal areas during equal intervals of time.
- Third law: The square of the orbital period of a planet is directly proportional to the cube of the semi-major axis of its orbit.
2.2.2 Universal Gravitation and Three-Body Problem

Most analysis of celestial and spacecraft orbits is based on Newton’s law of universal gravitation, which is an empirical physical law describing gravitational attraction between bodies with mass. It states the following:

- Any two point masses attract each other by a force pointing along the line intersecting both. The magnitude of the force is directly proportional to the product of the two masses and inversely proportional to the square of the distance between the point masses.

The mathematic approach is:

\[ F = \frac{G m_1 m_2}{r^2} \]  

(2.1)

where \( F \) is the magnitude of the gravitational force, \( G \) is the gravitational constant, \( m_1 \) is the mass of the first point mass, \( m_2 \) is the mass of the second point mass, and \( r \) is the distance between the two point masses.

Now we consider the two-body problem. In this idealized situation, only two bodies exist and both of them can be treated as point masses (Figure 2.4). Using Newton’s second law and the law of universal gravitation, we obtain:

\[
\begin{align*}
\vec{F}_1 &= m_1 \ddot{\vec{r}_1} = G m_1 m_2 \frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} \\
\vec{F}_2 &= m_2 \ddot{\vec{r}_2} = G m_1 m_2 \frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} = -\vec{F}_1
\end{align*}
\]  

(2.2)

(2.3)

The Earth-satellite two-body system is based on the premise that the magnitude of the Earth mass is much greater than the mass of the satellite. Assuming that the Earth is spherically symmetric, the acceleration of the satellite motion is:

\[
\ddot{\vec{r}} = -\frac{G M_E}{r^3} \vec{r} = -\frac{\mu}{r^3} \vec{r}
\]

(2.4)

\[
\mu \equiv G M_E
\]

(2.5)

where \( \mu \) is the Earth’s gravitational constant.

The three-body problem, also known as Euler’s three-body problem in classical dynamics, is to find the solution of motion of three point masses under their gravitational interactions.
The application of Newton’s second law and law of universal gravitation also gives the following equations:

\[
\begin{align*}
\vec{F}_1 &= m_1\ddot{\vec{r}}_1 = Gm_1m_2\frac{\vec{r}_2 - \vec{r}_1}{|\vec{r}_2 - \vec{r}_1|^3} + Gm_1m_3\frac{\vec{r}_3 - \vec{r}_1}{|\vec{r}_3 - \vec{r}_1|^3} \\
\vec{F}_2 &= m_2\ddot{\vec{r}}_2 = Gm_1m_2\frac{\vec{r}_1 - \vec{r}_2}{|\vec{r}_1 - \vec{r}_2|^3} + Gm_2m_3\frac{\vec{r}_3 - \vec{r}_2}{|\vec{r}_3 - \vec{r}_2|^3} \\
\vec{F}_3 &= m_3\ddot{\vec{r}}_3 = Gm_1m_3\frac{\vec{r}_1 - \vec{r}_3}{|\vec{r}_1 - \vec{r}_3|^3} + Gm_2m_3\frac{\vec{r}_2 - \vec{r}_3}{|\vec{r}_2 - \vec{r}_3|^3}
\end{align*}
\] (2.6)

(2.7)

(2.8)

However, Poincare proved the three-body problem is non-integrable which indicates that it’s impossible to find a general solution of the three-body problem. Necessary simplifications are made and restricted three-body problem generated as presented above.

The ideal situation of two-body and three-body motions does not exist in practice. From this point of view, all the Universe is actually within the N-body problem. In Earth orbit, there still exist additional gravitation from the Sun, the Moon and other celestial bodies. Satellites are perturbed every second of the motion. On High Earth Orbit (HEO) the effect of perturbation from the Sun and the Moon must be considered.

2.2.3 Classical Orbital Elements

Classical orbital elements are parameters frequently used to identify a specific orbit. They consist five constants and one quantity varying with time. With these 6 parameters, the position and velocity of satellite in the orbit is uniquely determined. Note that all these parameters are defined in the ECI coordinate.

- Semi-major Axis \((a)\): it is the distance between the geometric center of the orbital ellipse and the perigee passing through the center mass focus.
- Eccentricity \((e)\): the parameter describes how flattened the ellipse is compared with a circle.
- Inclination \((i)\): vertical tilt of the angular momentum vector of satellite orbit and the z-axis of the ECI coordinate.
• Right Ascension of Ascending Node (Ω): the angle between the vernal equinox vector and the ascending node.

• Argument of Perigee (ω): the angle from the ascending node to the orbital eccentricity vector. The eccentricity vector points from the Earth center to perigee with a magnitude equal to the eccentricity of the orbit.

• True Anomaly (ν): the angle from the eccentricity vector to the satellite position vector. Besides, mean anomaly (M) and eccentric anomaly (E) are also used in the calculation of orbit parameters.

The first two elements define the shape and the size of the ellipse (Figure 2.3). The parameters of i and Ω define the orientation of the orbital plane with respect to the ECI coordinate. The last two elements define the position of the satellite on the orbital plane (Figure 2.5).

![Figure 2.5: Classical orbital elements](image)

### 2.3 Orbit Perturbations

Satellites orbit perturbations come from the non-spherical Earth with non-homogeneous distribution of mass, atmospheric drag force and the existence of other celestial bodies. With the effect of perturbations, a satellite orbit will not simply be the two-body motion and it changes perpetually. Generally, two main aspects should be taken into consideration: one is the effect time of the disturbance; the other is the magnitude of the disturbance force with respect to the Earth gravity.

The variation in one of the orbital elements vs time can be illustrated in Figure 2.6. Secular variation represents a linear change with extremely long effect time relative to the period. Long-period variation lasts longer than one orbit period, while short-period variation has a period shorter than one orbit period. Different variations are included in analysis depending on the demands of the mission.
The magnitude of disturbance varies with the altitude of the satellite. In LEO orbit, $J_2$ and atmospheric drag are dominant, but in HEO orbit atmospheric drag could be ignored because of the dramatic drop of the atmospheric density in outer space (Figure 2.7). Third-body perturbations from the Sun and the Moon increase when the distance between the satellite and the Earth increases.

Figure 2.7: Relative importance of orbit perturbations

### 2.3.1 Perturbation Function

When analyzing the force exerted by celestial bodies on a satellite, the gravitational potential function is used as follows:

\[
\vec{F} = \text{grad}U = \nabla U \tag{2.9}
\]

\[
U = \frac{GM_0}{r} \tag{2.10}
\]

where $M_0$ is the mass of the central celestial body, $r$ is the distance between where the satellite locates to the center mass point. After perturbations are included, the gravitational
potential function can be rewritten:

\[ U = U_0 + R \quad (2.11) \]

\[ \ddot{r} = -\frac{\mu}{r^3} \dot{r} + \nabla R \quad (2.12) \]

where \( R \) is the perturbation potential function, referred as the perturbation function.\[ \text{[32]} \]

In the Earth spherical coordinate (Figure 2.2), the velocity and acceleration of a satellite under the condition of perturbation can be expressed:

\[ \dot{r} = \frac{dr}{dt} \vec{u}_r + r \frac{d\vec{u}_r}{dt} = \dot{r} \vec{u}_r + \dot{\theta} \cos \phi \vec{u}_\theta + \dot{\phi} \vec{u}_\phi \quad (2.13) \]

\[ \ddot{r} = (\ddot{r} - r \dddot{u}^2 \cos^2 \phi - r \dddot{u}^2) \vec{u}_r + (r \ddot{\theta} \cos \phi + 2r \dot{\theta} \sin \phi) \vec{u}_\theta + \]
\[ + (r \dddot{\phi} + 2r \dot{\theta} \dot{\phi} + r \dot{\theta}^2 \sin \phi \cos \phi) \vec{u}_\phi \quad (2.14) \]

where force \( \vec{u}_r \) is along the direction from the Earth center to the satellite, force \( \vec{u}_\theta \) is along the tangent at the satellite position on the latitude circle (East positive) and force \( \vec{u}_\phi \) completes a right hand orthogonal frame with former two vectors. Perturbation functions based on classical elements is included in the appendix.

### 2.3.2 \( J_2 \) Gravity Perturbation

When developing satellite motion equation in the satellite-Earth two-body system, we assume that the Earth is a perfect sphere with spherically symmetric mass distribution. For these reasons, the gravitational force acting on a satellite is only related to its distance to the Earth mass center with fixed nadir direction. Actually, the Earth is not perfectly spherical. It bulges at the equator and is flattening at the poles like an oblate pear with radius in a range from 6357 km to 6378 km. The means radius of the equator is about 21.4 km longer than that of the poles. Additionally, the distribution of the mass over the Earth is non-uniform. All these generates extra disturbance force in the velocity and orbital normal direction on the satellite.

Early researchers have development the theorem of Earth gravitational potential shown as follows \[ \text{[33]} \]

\[ U = \frac{\mu}{r} \left\{ 1 - \sum_{n=2}^{\infty} \left( \frac{R_E}{r} \right)^n [J_n P_n \sin \phi - \sum_{m=1}^{n} J_{nm} P_{nm} \sin \phi \cos m(\theta - \theta_{nm})] \right\} \quad (2.15) \]

Here \( P_n \) and \( P_{nm} \) are Legendre polynomials.

\[ \begin{cases} 
P_n(x) = \frac{1}{2^n n!} \frac{d^n}{dx^n} (x^2 - 1)^n \\
P_{nm}(x) = (1 - x^2)^{\frac{n}{2}} \frac{d^m}{dx^m} P_n(x)
\end{cases} \quad (2.16) \]

The parameter \( J_n \) refers zonal harmonic coefficient, \( J_{nm} \) is sectorial harmonic coefficient and \( \theta_{nm} \) is a parameter related to longitude \( \theta \). In LEO orbit, the effect of the sectorial harmonic terms could be ignored. Only considering the \( J_2 \) zonal harmonic terms, Equation 2.15 can
be rewritten with the central gravity term omitted.

\[
U_{J_2} = -\frac{\mu J_2 R_E^2}{2r^3} (3\sin^2 \phi - 1)
\]  

(2.18)

Thus, \( J_2 \) gravity perturbation functions are derived in the LVLH coordinate.

\[
\begin{align*}
F_x &= -\frac{3\mu J_2 R_E^2}{2r^4} [1 - 3\sin^2 i \sin^2(\omega + \nu)] \\
F_y &= -\frac{3\mu J_2 R_E^2}{2r^4} \sin^2 i \sin 2(\omega + \nu) \\
F_z &= -\frac{3\mu J_2 R_E^2}{2r^4} \sin 2i \sin(\omega + \nu)
\end{align*}
\]  

(2.19 - 2.21)

The Earth gravitational potential causes periodic variations in all the classical orbital elements. However, secular variations in right ascension of ascending node (\( \Omega \)), argument of perigee (\( \omega \)) and true anomaly (\( \nu \)) are dominant. In other words, \( J_2 \) gravity perturbation affects the classical orbital elements except semi-major axis (\( a \)), eccentricity (\( e \)) and inclination (\( i \)). Mean variations due to \( J_2 \) gravity perturbation of elements goes as follows:

\[
\begin{align*}
\dot{a} &= 0 \\
\dot{e} &= 0 \\
\dot{i} &= 0 \\
\dot{\Omega} &= -\frac{3n J_2}{2(1-e^2)^2} \frac{R_E^2}{a} \cos i \\
\dot{\omega} &= -\frac{3n J_2}{2(1-e^2)^2} \frac{R_E^2}{a} \left( \frac{5}{2} \sin^2 i - 2 \right) \\
\dot{M} &= n - \frac{3n J_2}{2\sqrt{(1-e^2)^3}} \frac{R_E^2}{a} \left( \frac{3}{2} \sin^2 i - 1 \right)
\end{align*}
\]  

(2.22 - 2.27)

where \( n \) is the angular velocity of the satellite.

2.3.3 Atmospheric Drag

Although the atmosphere at hundreds kilometers altitude is extremely thin, during the impact between the high-speed satellite and the atmosphere particles, momentum transfer happens, accumulates and affects satellites’ motion greatly. This process generates atmospheric drag which is nonconservative.

The derivation of a mathematical model of atmospheric drag is based on the following assumptions [34]:

- The momentum of particles hitting at the surface is totally lost to the surface.
- The mean thermal motion of the atmosphere is much smaller than the speed of the spacecraft relative to the local atmosphere.
- For spinning vehicles, the relative motion between surface elements is much smaller than the speed of the mass center.
The equation of atmospheric drag can be stated as:

\[ \vec{F}_A = -\frac{1}{2} C_d \rho S \vec{v} \]  

(2.28)

where \( C_d \) is the drag coefficient, \( \rho \) is the local atmospheric density, \( S \) is the effective surface area and \( \vec{v} \) is the relative spacecraft velocity.

\[ S = A \cos \alpha \]  

(2.29)

where \( A \) is the local surface area, \( \alpha \) is the angle between surface normal direction and velocity direction.

Additionally, there are various atmospheric models in which the exponential atmospheric density model is frequently used [35].

\[ \rho = \rho_0 e^{(h_0 - h)/H} \]  

(2.30)

where \( \rho \) is the density at a specified altitude, \( h \) is the specified altitude, \( h_0 \) is the reference altitude and \( H \) is the scale height.

Because of the energy dissipation induced by atmospheric drag, the orbit altitude decreases during lifetime. A rough estimation of the orbit height attenuation in LEO is [32]:

\[ h(t) = h(t_0) \ln \left[ e^{h(t_0)/H} - \frac{C_d S \rho_0}{mH} \sqrt{\mu R_E (t - t_0)} \right] \]  

(2.31)

where \( h(t_0) \) is the initial of altitude and \( m \) is the mass of satellite.

### 2.3.4 Third-Body Perturbations

For geostationary satellites, the third-body perturbations must be taken into consideration because the disturbing forces from the Sun and the Moon have roughly the same magnitude with the gravitational perturbation in such a high altitude. The third-body perturbation is related to the N-body problem mentioned in previous section. For the complexity of the third-body perturbation, concise analysis is introduced.

Suppose point \( o \) (Figure 2.8) in the figure below is the origin without acceleration in the inertial space. Let \( \vec{r}_{ij} \) represent the vector pointing from point mass \( i \) to point mass \( j \), subscripts \( e, m, s \) and \( 1 \) denote the Earth, the Moon, the Sun and the satellite respectively. The accelerations of the Earth and the satellite are:

\[
\begin{align*}
\vec{a}_{1o} & = \frac{1}{m_1}(\vec{F}_{1e} + \vec{F}_{1m} + \vec{F}_{1s}) \\
\vec{a}_{eo} & = \frac{1}{m_e}(\vec{F}_{e1} + \vec{F}_{em} + \vec{F}_{es})
\end{align*}
\]  

(2.32)

(2.33)

Thus, the satellite acceleration relative to the Earth is obtained:

\[ \vec{a}_{e1} = \vec{a}_{1o} - \vec{a}_{eo} = -G(m_1 + m_e) \frac{\vec{r}_{1e}}{r_{1e}^3} + Gm_s (\frac{\vec{r}_{es}}{r_{1s}^3} - \frac{\vec{r}_{es}}{r_{es}^3}) + Gm_m (\frac{\vec{r}_{em}}{r_{1m}^3} - \frac{\vec{r}_{em}}{r_{em}^3}) \]  

(2.34)
Because $m_1 \ll m_e$, $r_{1s} \approx r_{es}$ and $r_{1m} \approx r_{em}$, the magnitude of perturbations due to the Sun and the Moon are:

$$
\begin{align*}
F_{1s} &= Gm_1m_s\left(\frac{r_{1s}}{r_{es}^3}\right) \quad (2.35) \\
F_{1m} &= Gm_m m_s\left(\frac{r_{1m}}{r_{em}^3}\right) \quad (2.36)
\end{align*}
$$

Using $\varepsilon$ for the ratio of disturbing force to the Earth gravity on the geostationary satellites, following values indicate the reason why third-body perturbation should be included in HEO.

$$
\begin{align*}
\varepsilon_s &= \frac{m_s}{m_e}\left(\frac{r_{es}}{r_{1s}}\right)^3 \approx 0.75 \times 10^{-5} \\
\varepsilon_m &= \frac{m_m}{m_e}\left(\frac{r_{em}}{r_{1m}}\right)^3 \approx 1.63 \times 10^{-5} \\
\varepsilon_{j2} &\approx 3.7 \times 10^{-5}
\end{align*}
$$

### 2.3.5 Solar Radiation

In the space environment, spacecrafts are also affected by the solar radiation. Some of the solar radiation is absorbed while the other is reflected. There are two different principles for describing the radiation reflect on a surface, namely specular reflection and diffuse reflection. In specular reflection, the incoming particles have an elastic impact with the surface regulated by the reflection law; in diffuse reflection, atmospheric particles penetrate the satellite surface material, interact with the surface molecules, and finally re-emitted at a number of random angles. This energy transfer process generates the force on the objects.

The radiation force acting on the surface $dA$ of a satellite is [32]:

$$
d\vec{F} = p\,dA|\cos \alpha| [(1 - c)\vec{u}_i - c'\vec{u}_f]
$$

where $p$ is the mean solar radiation pressure, $\alpha$ is the ray incoming angle, $u_i$ and $u_f$ are the unit vector of incoming and reflecting ray respectively, and $c$ is the absorb ratio while $c'$ is the reflecting ratio at the surface in Figure 2.9.
Due to the complexity of the solar ray acting on the satellite surface, we assume that the solar radiation pressure has the same direction with the incoming light beams. Thus, the total solar radiation pressure $\vec{F}_r$ the satellite goes:

$$\vec{F}_r = -KpA\vec{S}$$  \hspace{1cm} (2.41)

where $A$ is the effective sectional surface area, $K$ is solar radiation coefficient relative to specific surface material and satellite shape, $p$ is the mean solar radiation pressure, and $\vec{S}$ is the unit vector from the satellite center to the Sun center. In the geo-stationary orbit, the ratio of solar radiation pressure to the Earth gravity is:

$$\varepsilon_r = \frac{KpA}{GM_e m/r^2} \approx 0.2 \times 10^{-5}$$  \hspace{1cm} (2.42)

which indicates that the solar radiation pressure is as importance as $J_2$, third-body perturbations in HEO.

## 2.4 Hill-Clohessy-Wiltshire (HCW) Equations

Currently, two main methods are employed in dealing formation flying relative dynamics: first is to analyze relative motion among satellites directly in local coordinate; second, application of classical orbital elements (COE method) can find the absolute positions of satellites in the ECI coordinates, then we can transform them to local coordinate to compute the relative states. The former is the HCW equations, the development of which has its origin in Hill’s investigation (1878) about the relative motion of the Moon to the Earth. Later, a linearized form of these equations introduced by Clohessy and Wiltshire (1960) successfully solved the spacecrafts rendezvous and docking problem with application in the Project Gemini (1965 – 1966). Compared to the COE method, the HCW method is straightforward, easy for implementation, and requires less computation. However, approximation and linearization cost the loss of fidelity in the development of relative dynamics equations.
For long-term situation, the latter is superior while for short-term applications, the HCW equations are still an important mathematical tool in analyzing and handling formation flying issues. The HCW equations are based on the Leader/Follower architecture that is widely and frequently used today. It is also referred as Chief/Deputy, Master/Slave, or Target/Chaser. The Leader/Follower architecture is adopted in this thesis work.

2.4.1 Unperturbed HCW Equations

In the following derivation [10], subscript 1 and 2 stand for the leader and the follower satellite respectively. Assumptions are: first, both leader and follower are in unperturbed orbits; second, the follower is driven by external forces which come from the on-board thrusters. Therefore, motion equations are:

\[
\begin{align*}
\ddot{r}_1 &= -\mu \frac{\dot{r}_1}{r_1^3} \\
\ddot{r}_2 &= -\mu \frac{\dot{r}_2}{r_2^3} + \vec{f}
\end{align*}
\]  
(2.43)

\[
\ddot{a}_{21} = \ddot{r}_2 - \ddot{r}_1 = \mu \left[ \frac{\dot{r}_1}{r_1^3} - \frac{\dot{r}_2}{r_2^3} \right] + \vec{f}
\]
(2.44)

where \(\ddot{a}_{21}\) denotes the follower’s acceleration relative to the leader. Define \(\vec{\rho} = \vec{r}_2 - \vec{r}_1\) as relative position vector from the leader to the follower in the LVLH coordinate below (Figure 2.10).

![Formation flying in the LVLH coordinate](image)

\[
a_{\rho} = \ddot{\rho} = \frac{d^2\rho}{dt^2} + 2\vec{\omega} \times \frac{d\vec{\rho}}{dt} + \frac{d\vec{\omega}}{dt} \times \vec{\rho} + \vec{\omega} \times (\vec{\omega} \times \vec{\rho})
\]
(2.46)

In the right hand side of Equation 2.46, the second term is the Coriolis effect due to the rotation of the LVLH coordinate relative to the ECI coordinate. Assuming the leader is in a circular orbit without angular acceleration, we have:

\[
\begin{align*}
\vec{\rho} &= [x \ y \ z]^T \\
\vec{\omega} &= [0 \ 0 \ n]^T \quad \text{with} \quad n = \sqrt{\frac{\mu}{r_1^3}}
\end{align*}
\]  
(2.47)

(2.48)
Substituting above into Equation 2.46, we obtain:

\[
\vec{a}_\rho = \begin{bmatrix}
\ddot{x} - 2n\dot{y} - n^2x \\
\ddot{y} + 2n\dot{x} - n^2y \\
\ddot{z} + n^2z
\end{bmatrix} \quad (2.49)
\]

Rewrite Equation 2.45 as following:

\[
\vec{a}_21 = \vec{a}_\rho = \frac{\mu}{r_1^3} \left[ \vec{r}_1 - r_3 \frac{\vec{r}_2}{r_2^3} \right] + \vec{f} \quad (2.50)
\]

where we use Taylor expansion (1st order) to the term \( \vec{r}_2^3 \):

\[
\frac{\vec{r}_2^3}{r_2^3} = \frac{\vec{r}_1 + \vec{p}}{r_1} = \frac{\vec{r}_1 + \vec{p}}{\sqrt{2(\vec{r}_1 \cdot \vec{p} + \rho^2)^3/2}} = \frac{\vec{r}_1 + \vec{p}}{r_1^3} \left[ 1 - \frac{3}{2} \left( \frac{2\vec{r}_1 \cdot \vec{p}}{r_1^3} \right) \right] + O(\rho^2) \quad (2.51)
\]

Ignoring high-order terms and substituting it into Equation 2.50 yields:

\[
\vec{a}_\rho = \frac{\mu}{r_1^3} \left[ -\vec{p} + 3(\vec{r}_1 \cdot \vec{p}) \cdot \frac{\vec{r}_1}{r_1} \right] + \vec{f} \quad (2.52)
\]

with \( \vec{r}_1/r_1 = [1 \ 0 \ 0]^T \), the linear form of \( \vec{a}_\rho \) follows:

\[
\vec{a}_\rho = \frac{\mu}{r_1^3} \begin{bmatrix}
2x \\
-\dot{y} \\
-\ddot{z}
\end{bmatrix} + \vec{f} \quad (2.53)
\]

Combine Equation 2.49 and 2.53 together, the unperturbed HCW equations are:

\[
\begin{align*}
\ddot{x} - 2n\dot{y} - n^2x &= f_x \quad (2.54) \\
\ddot{y} + 2n\dot{x} &= f_y \quad (2.55) \\
\ddot{z} + n^2z &= f_z \quad (2.56)
\end{align*}
\]

where \( f_i \ (i = x, y, z) \) is the component in the LVLH coordinate.

### 2.4.2 Modified HCW Equations

In the development of the unperturbed HCW equations, several aspects should be emphasized: first, \( r_1 \) approximates to \( r_2 \) for the small relative distance between the leader and the follower, this effect is called nonlinear gravity \[24\]; second, the fidelity of the unperturbed model is cut by the omission of disturbance. In the modified HCW equations, \( J_2 \) and atmospheric drag disturbances are added into the equations.

Modified HCW equations with \( J_2 \) non-spherical Earth effect follows \[10\]:

\[
\begin{align*}
\ddot{x} - 2n\dot{y} - (5c_2^2 - 2)n^2x &= f_x \quad (2.57) \\
\ddot{y} + 2n\dot{x} &= f_y \quad (2.58) \\
\ddot{z} + k^2z &= f_z \quad (2.59)
\end{align*}
\]
with the parameters

\[ s = \frac{3J_2R_E^2}{8r_1^2}[1 + 3\cos(2i_1)] \]  
\[ (2.60) \]

\[ c = \sqrt{1 + s} \]  
\[ (2.61) \]

\[ k = n\sqrt{1 + s} + \frac{3J_2R_E^2}{2r_1^2}\cos^2 i_1 \]  
\[ (2.62) \]

In the development of atmospheric drag within the modified HCW equations [10], atmospheric drag exerting on unit mass of the follower is derived. Substitute the following to Equation 2.28

\[ v = \sqrt{v_x^2 + (v_y + v_1)^2 + v_z^2} \approx v_1 \]  
\[ (2.63) \]

\[ \vec{v} = \dot{\vec{r}} + \vec{\omega} \times \vec{r} = \begin{bmatrix} \dot{x} - ny \\ \dot{y} + nx \\ \dot{z} \end{bmatrix} \]  
\[ (2.64) \]

For simplicity, local atmospheric density is treated as a constant. Therefore the atmospheric drag in the LVLH coordinate can be expressed as:

\[
\begin{align*}
  f_x &= -\frac{\rho v_1}{2B}(\dot{x} - ny) \\
  f_y &= -\frac{\rho v_1}{B}(\dot{y} + nx) \\
  f_z &= -\frac{\rho v_1}{2B}\dot{z}
\end{align*}
\]  
\[ (2.65-2.67) \]

Finally, we combine above terms together and the modified HCW equations follows:

\[
\begin{align*}
  \ddot{x} - 2nc\dot{y} - (5c^2 - 2)n^2x &= -\frac{\rho v_1}{2B}(\dot{x} - ny) \\
  \ddot{y} + 2nc\dot{x} &= -\frac{\rho v_1}{B}(\dot{y} + nx) \\
  \ddot{z} + k^2z &= -\frac{\rho v_1}{2B}\dot{z}
\end{align*}
\]  
\[ (2.68-2.70) \]

where the ballistic coefficient \( B = m_2/(SC_D) \).

Above all, the introduction of reference coordinates, fundamental astrodynamics and the orbit perturbations is mainly for the derivation of the HCW equations and their modified versions. In the modified HCW equations, loss of the fidelity is the pay for linearization driven by intension for simplicity. For this reason, the modified HCW equations are strongly recommended for the short-term small-separation formation flying analysis. In the next chapter, we will discuss the formation motion planning which is greatly dependent on the unperturbed HCW equations.
3 Formation Motion Planning

Formation motion planning is a preliminary step in formation flying design. A well-designed relative motion trajectory will first satisfy mission requirements, improve its performances, and reduce fuel consumption. Formation motion includes formation keeping, formation initialization and formation reconfiguration.

3.1 Formation-Keeping Planning

The unperturbed HCW equations are utilized for formation keeping motion planning. Actually, from a mathematic point of view, formation motion planning is to find the free-force solution of these equations. There exist only several types of formation orbits which are realistic due to the nature of orbit mechanics. One of the key features of formation flying is that the period of formation approximates the period of the Leader’s orbit. Additionally, the initial condition of the formation keeping motion can significantly affect the performance. The special initial condition means the only one or two point on the relative orbit can satisfy the access to the keeping motion. This concept is similar to “launch window” that is to describe a time period for rocket launch.

3.1.1 Free-force Solution to the HCW Equations

The unperturbed HCW equations are inhomogeneous linear 2nd-order ordinary differential equations. Let the external driving force equal to zero,

\[
\begin{bmatrix}
  f_x \\
  f_y \\
  f_z
\end{bmatrix} = 0
\]  

(3.1)

so that the analytical solution can be derived by using Laplace transform.  

\[
\begin{align*}
  x(t) &= \frac{x_0}{n} \sin(nt) - (3x_0 + \frac{2y_0}{n}) \cos(nt) + (4x_0 + \frac{2y_0}{n}) \\
  y(t) &= \frac{2x_0}{n} \cos(nt) + (6x_0 + \frac{4y_0}{n}) \sin(nt) - (\frac{2x_0}{n} - y_0) - (6nx_0 + 3\dot{y}_0)t \\
  z(t) &= z_0 \cos(nt) + \frac{\dot{z}_0}{n} \sin(nt)
\end{align*}
\]  

(3.2)  

(3.3)  

(3.4)

where \([x_0 \ y_0 \ z_0 \ \dot{x}_0 \ \dot{y}_0 \ \dot{z}_0]^T\) is the initial condition of the follower’s state. Obviously, analytical solution in \(x\) and \(y\) axis are coupled, with three types of components - trigonometric term, constant term and secular term. To cancel the effect of useless terms becomes the trick in trajectory design.
3.1.2 In-plane Formation

Among formation trajectories, in-plane formation is the simplest one in which a group of satellites occupy the same orbit or orbit plane. One example is following flying, in which Leader and Follower share the same orbit with a certain separation in between (Figure 3.1). The relative velocity and acceleration of Follower to Leader are both zero. The separation is the initial condition \( y_0 \). In addition, it can be regarded as: Leader and Follower have the same classical orbital elements except mean anomaly.

\[
\nu_1 = \nu_2 + \frac{y_0}{a_1}
\]  

(3.5)

Another example (Figure 3.2) is that Follower’s orbit latitude is a little bit larger than Leader’s. They are close in the same orbital plane with a certain separation in \( x \)-axis of the LVLH coordinates.

\[
a_1 = a_2 - x_0 \quad x_0 = \Delta a
\]  

(3.6)

As presented by Shenyhou-7 and its companion satellite BX-1, \( x - y \) plane ellipse formation is more complex than the above ones. Setting the initial state of Follower as \([x_0 \ 0 \ 0 \ 0 \ -2nx_0 \ 0]^T\) \( x_0 = 2000 \) m, \( x - y \) plane ellipse formation shows in Figure 3.3. In this type formation, Follower, usually small satellite, can perform multi-direction monitoring activities about Leader within a certain range.

3.1.3 In-track Formation

In-track formation is the formation type in which both satellites share the same ground track. Due to the rotation of the Earth, satellites have to occupy slightly different orbital
planes separated by right ascension of ascending node. Imagine that Leader is at the ascending node at the point \( A \) over the Earth’s equator at time \( t \); and Follower passes the same point at \( (t + \Delta t) \). The difference in anomaly is given by \( \Delta \theta = \frac{y_0}{a_1} \) and the different in time to reach the same ground track point is \( \Delta t = \frac{\Delta \theta}{n} \). Furthermore, the solutions to the free-force HCW equations of the in-track formation are [10]:

\[
\begin{align*}
  x(t) &= 0 \\
  y(t) &= y_0 \\
  z(t) &= -y_0 \frac{\omega_E}{n} \sin i \cos(nt)
\end{align*}
\] (3.7), (3.8), (3.9)

where \( \omega_E \) is the rotation speed of the Earth.
3.1.4 Space Circular Formation

In this motion, the trajectory of Follower relative to Leader is designed to be a circle in space. For example, set the initial state of Follow as \( [x_0 \ 2\dot{x}_0/n \ \sqrt{3}x_0 \ \dot{x}_0 - 2nx_0 \ \sqrt{3}\dot{x}_0] \), \( x_0 = 2000m \), \( \dot{x}_0 = 0.2m/s \), to cancel the constant and secular terms in the solutions. Here, the value of \( \dot{x}_0 \) affects the radius of the space circle. The ideal situation is that the relative velocities can be set as zeros, so that it would minimize the secular terms in the analytical solutions of the unperturbed HCW equations. In fact, it’s really hard to reach such kind of idealized state. The solution is to find a “relay” orbit between previous formation orbit and the current formation keeping orbit.

![Figure 3.5: Space circular formation](image)

3.1.5 Projected Circular Formation

For some Earth observation missions, circle trajectory is useful. In this type of formation, the trajectory of Follower relative to Leader is ellipse in the space, but circle in \( y - z \) plane which means it is projected circle on the ground. Setting the initial state of Follower as: \( [x_0 \ 2\dot{x}_0/n \ 2x_0 \ \dot{x}_0 \ 0 \ 2\dot{x}_0] \), \( x_0 = 0m \), \( \dot{x}_0 = -1m/s \), the projected circular formation is shown in Figure 3.6.

3.2 Formation Initialization and Reconfiguration Planning

As presented in the state-of-the-art section, most satellite involved in formation flying are small satellites. Those satellites are launched by means of piggy-back which means Followers are carried on Leader originally. Before performing formation, initialization are demanded to perform the satellites group deployment. Then Follower will track the pre-designed
relative orbit around Leader. This process is referred as formation keeping or formation maintain which is most critical in formation motion, because the formation keeping takes the largest part of formation motion time.

3.2.1 Hohmann Transfer

The Hohmann transfer is to use two engine impulse to maneuver a spacecraft between two circular orbits. The path is designed to be half of an ellipse which touches both the original orbit and the final orbit. It’s been proven that Hohmann transfer is the most fuel-saving transfer method between two coplanar orbits [36]. The transfer is initiated by firing the engine at the point $A$ to accelerate it to follow the ellipse. When the spacecraft reaches the final orbit, the engine will give a instant thrust to make its velocity meet the new orbit’s requirement at the point $B$ (Figure 3.7). Hohmann transfer also bring the spacecraft from higher orbit to the lower orbit. The process as well as the calculation is the inverse of the previous one. The Hohmann transfer is theoretically based on two-body problem in which the potential energy and the kinetic energy of the Earth-satellite system is conservative. Moreover, another assumption is that high thrust could be provided to minimize the amount of extra fuel. Low thrust engines can only perform approximation of a Hohmann transfer orbit by creating a gradual enlargement of the initial orbit through carefully timed engine firings. This will require a $\Delta v$ that is up to 141% greater than the two-impulse transfer [37].

Therefore, the $\Delta v$ budget for the Hohmann transfer can be computed as follows:
where $r_A$ and $r_B$ are, respectively, the radii of the departure and arrival circular orbit. Whether maneuver into a higher or lower orbit, by Keplerian Third Law, the time taken in transfer is:

$$t_H = \frac{1}{2} \sqrt{\frac{4\pi^2 a_H^3}{\mu}} = \pi \sqrt{\frac{(r_A + r_B)^3}{8\mu}}$$

where $a_H$ is the length of Hohmann transfer orbit semi-major axis.

Hohmann transfer is suitable for the initialization motion planning of the parallel formation and the reconfiguration from following formation to parallel formation for the coplanar maneuver in these processes. However, one of the disadvantages of Hohmann transfer is that the transfer time is fixed, and equals to half of the period of the transfer ellipse period; the other is that, in practice, two-impulse can not transfer the spacecraft to the formation orbit precisely under the condition of perturbations.

### 3.2.2 Straight Line (in LVLH) Transfer

Currently, most researchers use the straight line (in the LVLH coordinates) transfer to maneuver between two different orbits for simple formation styles [38, 26]. When the transfer time equals to half of the reference orbit period, in the ECI coordinates, the absolute orbit is almost the same with the elliptic orbit. The absolute orbit under this condition is dependent on the transfer time although the relative trajectory is always straight line. Figure 3.8 shows the initialization of parallel formation, and the transfer time is half the reference orbit period in which the absolute maneuver trajectory of Follower is Hohmann transfer orbit; while Figure 3.9 illustrates the initialization of parallel formation with transfer time.
less than half of the reference orbit period \((\frac{1}{2}P_r)\).

![Figure 3.8: Straight line transfer \((t = \frac{1}{2}P_r)\)](image)

![Figure 3.9: Straight line transfer \((t < \frac{1}{2}P_r)\)](image)

### 3.2.3 Automatic Transfer

Automatic transfer in formation flying maneuver means Follower will search the path from the starting point to the target point automatically. This maneuver significantly depends on the design of the controller which is based on the trajectory tracking problem. Thus it’s really difficult to pre-analyze the performance of the transfer trajectory. Even with the same controller, different control accuracy and transfer time will affect the trajectory. Actually, in control theory, automatic transfer could be explained as: originally, there is a large state error for the trajectory tracking so that the path to the target point is dependent on how the controller eliminates the error in a given time or unlimited time.

Other aspects which should be noted are: first, due to the control accuracy and the consideration of control consumption, usually the automatic transfer will not send Follower to the target point precisely; second, the transfer trajectory sometimes is complex which may not be on one single plane; finally, until now, it’s extremely difficult to find a optimal automatic transfer trajectory for universal control method \([39, 40, 22]\).

Above all, this chapter discussed the formation motion planning which will be the control input (desired trajectory). In the next chapter, we will investigate how to design the controllers whose performance will greatly depends on the pre-designed trajectories.
4 Formation Flying Control

Formation flying control is based on the HCW equations. It was shown that the open-loop spacecraft relative dynamics are inherently unstable \[41\]. If there is no control input, a nonzero initial condition or an external disturbance will cause the two satellites to drift \[42\]. As a complex dynamics system, formation control have many inputs and outputs with a coupled manner. It is essential to use state-space approach to analyze the system and design the controller with control methods.

In this chapter, first state-space representation of the HCW equations and the feedback control law will be introduced; then, linear optimal control method (LQR) will be applied; it’s followed by a nonlinear method - SMC which will show its superior on the uncertainties control; finally, we will give the procedure of discrete-time Kalman filter which finds the optimal estimation of the system state.

4.1 State-Feedback Control Law

In Leader/Follower architecture, control problem is reduced to individual tracking problem with controllers in individual spacecrafts. The principle of formation flying control is the state-feedback control law, which is the basis to design controller for desired trajectory tracking.

4.1.1 State-Space Representation

In a Multi-Input-Multi-Output (MIMO) system, the continuous state-space representation is as follows:

\[
\dot{x} = Ax + Bu \tag{4.1}
\]

\[
y = Cx + Du \tag{4.2}
\]

where \(x\) is the state vector of the system, the dot is the first-order time derivative, \(u\) is the control vector or input vector and \(y\) is the output vector. Here, matrix \(A\) is state matrix, \(B\) is control matrix or input matrix, \(C\) is output matrix and \(D\) is feedforward matrix.

In the HCW equations, the position and velocity vectors \(\begin{bmatrix} \dot{\vec{r}} & \dot{\vec{r}} \end{bmatrix}^T\) can be chosen to be the state vector. Do not consider the observation. Then the continuous state-space representation is obtained as following (see the end of chapter 2) (the representation of modified HCW equations is very similar, not shown here):

\[
\dot{x} = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} x + \begin{bmatrix} B_1 \\ B_2 \end{bmatrix} u \tag{4.3}
\]
where \( x = [x \ y \ z \ \dot{x} \ \dot{y} \ \dot{z}]^T \), \( u = [f_x \ f_y \ f_z]^T \), \( A_{11} = 0_{3 \times 3} \), \( A_{12} = I_{3 \times 3} \).

\[
A_{21} = \begin{bmatrix} 3n^2 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -n^2 \end{bmatrix}, \quad A_{22} = \begin{bmatrix} 0 & 2n & 0 \\ -2n & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}, \quad B_1 = 0_{3 \times 3}, \quad B_2 = I_{3 \times 3}.
\]

\[
y = I_{6 \times 6}x \quad (4.4)
\]

Additionally, continuous state-space representation is transformed into discrete state-space form for the numerical computing. Suppose the sampling time is \( T \), \( k = 0, 1, 2, \ldots k \in \mathbb{N} \), the discrete state-space representation is provided below [13]:

\[
x(k + 1) = \Phi(T)x(k) + \Gamma(T)x(k) \quad (4.5)
\]

where

\[
\begin{cases}
\Phi(T) = e^{AT} \\
\Gamma(T) = \int_0^T e^{AT}dtB
\end{cases} \quad (4.6)
\]

\[
\begin{cases}
\Phi(T) = e^{AT} \\
\Gamma(T) = \int_0^T e^{AT}dtB
\end{cases} \quad (4.7)
\]

### 4.1.2 State Feedback

Normally, there are two main feedback control laws in state space: one is state feedback; the other is output feedback. Due to its behavior for monitoring and managing, the state-feedback control is frequently employed in formation flying [44]. Assume that all the state components can be used in feedback and set the control input to be a linear function of the state vector as below:

\[
u = v - Kx \quad (4.8)
\]

where \( v \) is the reference vector with the same dimension with the state vector \( x \), \( K \) is the real feedback-gain matrix. Note that in the previous chapter, formation motion trajectory design is discussed which means the desired relative position and velocity vectors (\( \vec{r}_d \) and \( \vec{\dot{r}}_d \)) can be given in advanced. Therefore, there exist the desired state \( x_d \) and the real state \( x_r \) provided by the orbit propagator. Introduce the desired state into the feedback to find a difference with the real state, and Equation 4.8 can be rewritten to be [15]:

\[
u = v - K(x_r - x_d) \quad (4.9)
\]

Letting the reference vector to be null, we obtain the formation flying state feedback law:

\[
u = K(x_d - x_r) = K\Delta x \quad (4.10)
\]

Figure 4.1 depicts the state feedback block diagram. Because \( v = 0 \), the desired state \( x_d \) can be seen as the control input, and \( y \) as the control output. In our formation dynamics, a perfect observer makes \( C = I \) and consequently, the real state \( x_r \) generated by a orbit propagator will be the control output. In the following sections, \( y_d \) denotes the desired output which, in fact, is the desired state \( x_d \) for the same reason. However, in some cases, \( y \) is still adopted for consisting with the reference.
4.2 Quadratic Optimal Control

Optimal control is the standard method for solving dynamic optimization problems by consideration of their economics. In aerospace, the economic problems such as the cost of the fuel and propellant are critical. It is necessary to find a control law for a given system such that a certain optimality criterion is achieved. In an optimal control problem, a set of differential equations describes the paths of the control variables that minimize the cost functional, which is a function of state variables. If the cost functional is a quadratic functional, then it refers to quadratic optimal control. In this theory, one of the main tools is the Linear Quadratic Regulation (LQR).

4.2.1 Linear Quadratic Regulation (LQR)

In the state-feedback law, the LQR algorithm is an effective way of finding the feedback gain. First, for a continuous-time linear system, the quadratic cost functional is defined as [46]:

\[ J = \frac{1}{2} \int_{t_0}^{t_f} \left[ x^T(t)Q(t)x + u^T(t)R(t)u \right] dt + \frac{1}{2} x^T(t_f)Q_{t_f}x(t_f) \] (4.11)

where \( Q(t) \) is the positive semi-definite state-weighting matrix, \( R(t) \) is the positive definite control-weighting matrix and \( Q_{t_f} \) is the positive definite terminal-weighting matrix. In practice, matrix \( Q \) and matrix \( R \) usually are set to be symmetric matrices. In the right-hand side of Equation (4.11) let \( L_x = x^TQ(t)x \). If \( x \) is the state error vector, \( L_x \) is the cost functional to measure the error. Likewise, term \( L_u = u^T(t)R(t)u \) is the cost functional to constrain the control input in the dynamic process. Term \( L_{x_{t_f}} = \frac{1}{2} x^T(t_f)Q_{t_f}x(t_f) \) is the terminal error functional which constrains the cost for terminal error. The objective of the optimal control is \( J \rightarrow min \) which, in its core, tends to use effective control input to achieve the given accuracy requirement. Then next step is to use Principle of the Minimum and to construct a Hamilton function. For the finite-horizon continuous-time LQR, feedback control gain \( K \) is given by [46]:

\[ K = R^{-1}B^TP(t) \] (4.12)

where matrix \( P \) is found by solving the continuous-time differential Riccati equation. The Riccati equation is given below:

\[ \dot{P}(t) + A^TP(t) + P(t)A - P(t)BR^{-1}B^TP(t) + Q = 0 \] (4.13)
The boundary condition is provided:

\[ P(t_f) = Q_t \]  \hspace{1cm} (4.14)

The performance of the LQR is greatly depending on the weighting matrixes. Points below should be noted: first, if the control cost was relatively extremely low, a small control-weighting matrix \( R \) could be given to enlarge the control signal for a better dynamical performance; second, in fact, the control cost for spaceflight is expensive due to various reasons. It is essential to find a small control input to achieve the accuracy requirements [47]. The choice of the control-weighting matrix and the error-weighting matrix becomes the key in the LQR controller design and implementation.

### 4.2.2 Trajectory Following LQR

The aim of trajectory following LQR is to make the system output \( y(t) \) follow the pre-designed trajectory with the consideration of fuel consumption or other cost. First we introduce the continuous-time trajectory following LQR design and then we will give the application in engineering.

Assume that the desired output is \( y_d(t) \), the state error vector can be given as:

\[ e(t) = y_d(t) - y(t) = y_d(t) - Cx(t) \]  \hspace{1cm} (4.15)

The quadratic cost functional is defined as

\[ J = \frac{1}{2} \int_{t_0}^{t_f} [e^T Q(t)e + u^T R(t)u] dt \]  \hspace{1cm} (4.16)

If \( t_f \) is of sufficient magnitude but not infinite, the linear time-invariable system trajectory following LQR is [16]:

\[ u_{eq} = -R^{-1}B^T P x(t) + R^{-1}B^T g \]  \hspace{1cm} (4.17)

where matrix \( P \) and matrix \( g \) are subjected to the equations below:

\[ \begin{align*}
A^T P + PA - PBR^{-1}B^T P + C^T Q C &= 0 \\
\dot{g} &\approx (PBR^{-1}B^T - A^T) - C^T Q y_d
\end{align*} \]  \hspace{1cm} (4.18)

Figure [4.2] shows the trajectory following LQR block diagram. Note that we can obtain \( g(t) \) by using \( \dot{g} \) and \( g(t_0) \).

### 4.2.3 Discrete-time LQR

Theoretically, discrete-time LQR controller should be applied in the discrete-time based computation of formation flying control. Consider the discrete-time state vectors and control vectors:

\[ \begin{align*}
\{x_1(k), x_2(k), \ldots, x_n(k)\}, (k = 1, 2, \ldots, N) \\
\{u_1(k), u_2(k), \ldots, u_n(k)\}, (k = 1, 2, \ldots, N)
\end{align*} \]  \hspace{1cm} (4.20) \hspace{1cm} (4.21)

With the dramatic increase of the number of the variables, the number of the boundary equations in discrete-time LQR are \( N \)-times larger than that of the continuous-time LQR.
This means the scale of the problem has been greatly enlarged which cause a huge load in computation. Due to the nature of our system, feedback gain $K$ can be found by the continuous-time method with state feedback law (the same with output feedback in our system). With this method, we only need to calculate the feedback control gain once and it has been proved in the previous research simulation [45].

4.3 Sliding Mode Control (SMC)

Sliding mode control, a kind of Variable-Structure Control (VSC), is a nonlinear control method that applies a high-frequency switching control effort. Because it has low sensitivity to disturbances and plant parameters variation, sliding mode control is an efficient tool to control complex high-order dynamics plant operating under uncertainties in real application or from the inaccurate representation of system [48].

4.3.1 SMC Control Law

The SMC could appear in a dynamic system which is governed by ordinary differential equations. In SMC control law, there exists a gain in the feedback path, which switches between two inverse values at a high frequency. Thus the system state switches frequently (theoretically infinite frequency). This motion is referred as sliding motion.

In order to explain the basis of SMC, the simplest first-order tracking-relay system will be taken as an example [48]:

$$
\dot{x} = f(x) + u
$$

(4.22)

where $f(x)$ is the bounded function with $|f(x)| \leq f_0 = \text{constant}$. The control as a relay function of the tracking error $e = r(t) - x$ with $r(t)$ the reference input is given:

$$
u = u_0 \text{sign}(e) \quad u_0 = \text{constant}$$

(4.23)

Obviously, the values of $e$ and $de/dt = \dot{e} = \dot{r} - f(x) - u_0 \text{sign}(e)$ have different signs if $u_0 > f_0 + |\dot{r}|$. It means that the magnitude of the tracking error decays at a finite rate and the error will be reduced to zero after a finite time $T$. The motion for $t > T$ is called sliding motion.
Now we start with the conventional control problem with a linear time-invariant multidimensional system:

\[
\dot{x} = Ax + Bu \tag{4.24}
\]

where \( x \) and \( u \) are \( n \)- and \( m \)-dimensional state and control vectors respectively \((n \geq m \geq 1)\). Matrices \( A \) and \( B \) are constant. The system is assumed to be controllable. Moreover, the sliding surface vector \( s \) is given as:

\[
s = C_e x \tag{4.25}
\]

where matrix \( C_e \) with dimension \( m \times n \) is the switching function matrix. Another important assumption is that matrix \( C_e B \) is nonsingular. This assumption ensures the successful application of sliding mode control and the determinant of \( C_e B \) \((\det(C_e B))\) measures how control is efficiently achieved \([49]\). Next task is to find the control input which can make the state trajectory tend towards the sliding surfaces under the condition function below:

\[
\begin{cases}
\dot{s}_i > 0, & s_i < 0 \quad (4.26) \\
\dot{s}_i < 0, & s_i > 0 \quad (4.27)
\end{cases}
\]

where \( s_i \) is the element of the sliding surfaces vector \( s \). Thus, the SMC control law is given:

\[
\begin{cases}
u_i(x) = u_i^+(x), & s_i > 0 \quad (4.28) \\
u_i(x) = u_i^-(x), & s_i < 0 \quad (4.29)
\end{cases}
\]

\[
u = [u_1 \ u_2 \ldots u_i]^T \quad i = 1, 2, \ldots, m \tag{4.30}
\]

Depending on different state feedback forms, system dynamics and sliding surfaces design, we have various forms of SMC control laws. Generally, there exist following forms: relay form, nonlinear relay form, unit-vector form and so on \([49]\).

### 4.3.2 Sliding Surfaces Design

Actually selection of the sliding surfaces is to find a suitable switching function matrix for the control task. However, in a MIMO system, sometimes it is extremely difficult to design the sliding surfaces due to the huge dimension of the state. Finding sliding surfaces or switching manifold \([48]\) becomes easier for systems represented in so-called regular form. Based on the regular form, some method such as eigenvalue placement can be applied in sliding surfaces design.

In a discrete linear time-invariance multidimensional system, the state-space representation is:

\[
x(k + 1) = \Phi x(k) + \Gamma u(k) \tag{4.31}
\]

with sliding surfaces

\[
s(k) = C_e x(k) \tag{4.32}
\]

Since \( \text{rank}(\Gamma) = m \), matrix \( \Gamma \) can be partitioned as:

\[
\Gamma = \begin{bmatrix} \Gamma_1 \\ \Gamma_2 \end{bmatrix} \tag{4.33}
\]
where \( \Gamma_1 \in \mathbb{R}^{(n-m) \times m}, \Gamma_2 \in \mathbb{R}^{m \times m} \) with \( \det(\Gamma_2) \neq 0 \). The nonsingular coordinate transformation follows:

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = Tx
\]  \hspace{1cm} (4.34)

\[
T = \begin{bmatrix}
    I_{n-m} & -\Gamma_1 \Gamma_2^{-1} \\
    0 & \Gamma_2^{-1}
\end{bmatrix}
\]  \hspace{1cm} (4.35)

This step is to reduce the system to the regular form in which we have:

\[
\begin{bmatrix}
    \Phi_{11} & \Phi_{12} \\
    \Phi_{21} & \Phi_{22}
\end{bmatrix}
\]

\[
\Gamma = \begin{bmatrix}
    0 \\
    \Gamma^*
\end{bmatrix}
\]  \hspace{1cm} (4.36)

(4.37)

where \( \Phi_{11} \in \mathbb{R}^{(n-m) \times (n-m)} \). The system can be expressed as follows:

\[
\begin{align*}
    x_1(k+1) &= \Phi_{11}x_1(k) + \Phi_{12}x_2(k) \\
    x_2(k+1) &= \Phi_{21}x_1(k) + \Phi_{22}x_2(k) + \Gamma^*u(k)
\end{align*}
\]  \hspace{1cm} (4.38) \hspace{1cm} (4.39)

\[
s(k) = C_{e1}x_1(k) + C_{e2}x_2(k)
\]  \hspace{1cm} (4.40)

Now we transform the state vector to a special form:

\[
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} \rightarrow \begin{bmatrix}
    x_1 \\
    s
\end{bmatrix}
\]  \hspace{1cm} (4.41)

equivalently,

\[
\begin{align*}
    x_1(k) &= x_1(k) \\
    x_2(k) &= C_{e2}^{-1}s(k) - C_{e2}^{-1}x_1(k)
\end{align*}
\]  \hspace{1cm} (4.42) \hspace{1cm} (4.43)

Consequently, the system is transformed as:

\[
\begin{align*}
    x_1(k+1) &= (\Phi_{11} - \Phi_{12}C_{e2}^{-1}C_{e1})x_1(k) + \Phi_{12}C_{e2}^{-1}s(k) \\
    s(k+1) &= [(C_{e1}\Phi_{11} + C_{e2}\Phi_{21}) - (C_{e1}\Phi_{12} + C_{e2}\Phi_{22})C_{e2}^{-1}C_{e1}]x_1(k) + (C_{e1}\Phi_{12} + C_{e2}\Phi_{22})C_{e2}^{-1}s(k) + C_{e2}\Gamma^*u(k)
\end{align*}
\]  \hspace{1cm} (4.44) \hspace{1cm} (4.45)

The sliding mode should be enforced in the manifold:

\[
\begin{align*}
    s(k) &= C_{e1}x_1(k) + C_{e2}x_2(k) = 0 \\
    s(k+1) &= C_{e1}x_1(k+1) + C_{e2}x_2(k+1) = 0
\end{align*}
\]  \hspace{1cm} (4.46) \hspace{1cm} (4.47)

Thus we obtain the sliding mode motion equation:

\[
x_1(k+1) = (\Phi_{11} - \Phi_{12}C_{e2}^{-1}C_{e1})x_1(k)
\]  \hspace{1cm} (4.48)

In this condition, the equivalent control input is:

\[
u_{eq}(k) = (C_{e2}\Gamma^*)^{-1}[(C_{e1}\Phi_{11} + C_{e2}\Phi_{21}) - (C_{e1}\Phi_{12} + C_{e2}\Phi_{22})C_{e2}^{-1}C_{e1}]x_1(k)
\]  \hspace{1cm} (4.49)

About the problem how to find a switch function matrix, eigenvalue placement is one of the methods. For example, we can treat \( x_2 \) as input:
\[
\begin{aligned}
\begin{cases}
x_2 = Kx_1 \\
K = -C_{e_2}^{-1}C_{e_1}
\end{cases}
\end{aligned}
\] (4.50)

After finding $K$ according to the requirement of sliding motion dynamics, the switching function matrix is

\[
C_e = [C_{e_1}, C_{e_2}] = C_{e_2}[-K, I]
\] (4.52)

where $I$ is the unit matrix.

### 4.3.3 Reaching Law

The precondition of a SMC system to get close to the sliding surface is Equation 4.28 and Equation 4.29. However, these preconditions can not show how the state motion get close to the sliding surfaces. In SMC design, to ensure the system reach and enter the sliding motion at a finite time is the most critical problem. One method is to introduce the reaching law. Generally, the reaching law is to describe the reaching rate of motion towards the sliding surface. With this method, analysis about the performance of the reaching motion is becoming easier.

The equations below are the frequently used reaching law [49]:

- **Constant speed**
  \[
  \dot{s} = -\varepsilon s \text{sgn} s \quad (\varepsilon > 0)
  \] (4.53)

- **Exponential**
  \[
  \dot{s} = -\varepsilon s \text{sgn} s - ks \quad (\varepsilon > 0, k > 0)
  \] (4.54)

- **Power-law**
  \[
  \dot{s} = -k |s|^\alpha s \text{sgn} s \quad (k > 0, 1 > \alpha > 0)
  \] (4.55)

Based on the exponential reaching law, in the discrete MIMO system, we have:

\[
s(k+1) - s(k) = -\varepsilon Ts\text{sgn}(s(k)) - qTs(k)
\] (4.56)

where $\varepsilon$ and $q$ are diagonal matrices with positive components, and $T$ is the sampling time. Thus the discrete sliding surfaces are:

\[
s(k+1) = C_e x(k+1) = C_e \Phi x(k) + C_e \Gamma u(k)
\] (4.57)

Substitute it into Equation 4.54 and find the control input:

\[
u(k) = (C_e \Gamma)^{-1}[(I - qT)s(k) - \varepsilon Ts\text{sgn}(s(k)) - C_e \Phi x(k)]
\] (4.58)

The parameter $q$ mainly affect the speed of the motion towards the sliding surfaces. The larger the components of $q$, the faster the motion will be. However, quick motion may lead to the relatively huge input which causes the chattering problem in the SMC. The function sign matrix $\varepsilon$ is to reduce the effect of the external disturbance and noise. Meanwhile the design of these parameters are depending on the system distribution and real application experience.
4.3.4 Trajectory Following SMC

In formation flying keeping control, the Leader/Follower architecture simplify the complex control issue to the trajectory following problem. Trajectory following SMC is a nonlinear approach in trajectory following.

Assuming the given desired state trajectory is $x_d(k)$ in a discrete system, the sliding surface can be written as:

$$s(k) = C_e e(k) = C_e(x_d(k) - x(k)) \quad (4.59)$$

In the next time step $(k+1)$, the sliding surface is derived:

$$s(k+1) = C_e(x_d(k+1) - x(k+1))$$
$$= C_e(x_d(k+1) - \Phi x(k) - \Gamma u(k)) \quad (4.60)$$

Thus, the trajectory following control law can be expressed as:

$$u(k) = (C_e \Gamma)^{-1}(C_e x_d(k+1) - C_e \Phi x(k) - s(k+1)) \quad (4.61)$$

4.4 Kalman Filter

Since its publication in 1960 by R. E. Kalman, the recursive optimal filter has been applied in a wide range of engineering. Given a linear dynamic system driven by stochastic processes, the Kalman Filter provides an efficient optimal way of extracting a signal buried in noise by estimating the system state. It has the significant property that it can continue to run and provide recursive state estimation in discrete time which is straightforward for digital computation [50]. In our application, suppose that Follower’s position/velocity information relative to Leader is provided by sensors on Leader. Normally, the failures of communication may cause the “death” of the control. In order to solve this problem, the Kalman Filter is used in the feedback loop to find the estimation of state information under the condition of communication failures. Even with frequent relative position measurements, the Kalman Filter will also provide the best estimation of the relative information between Follower and Leader.

4.4.1 Discrete-time Kalman Filter

First, we assume that our dynamic system is a Kalman System which means the system is controllable and observable. For the linear discrete-time system driven by stochastic processes, the state-space representation is below [51]:

$$x(k+1) = \Phi x(k) + \Gamma u(k) + \gamma \omega(k) \quad (4.62)$$

where $\omega$ is the system random noise vector, $\gamma$ is the system random noise matrix (identity matrix in our system). The linear observer can be written as:

$$y(k+1) = Hx(k) + \nu(k) \quad (4.63)$$

where $\nu$ is the observation noise. For stochastic processes assumption, the noise is zero-mean Gaussian random noise, additionally, it’s white noise. That is the noise at time $t_k$ is
independent on the noise at time \( t_{k+1} \).

\[
\begin{align*}
\left\{ \begin{array}{l}
f(\omega) = N(0, \sigma_\omega) \\
f(\nu) = N(0, \sigma_\nu)
\end{array} \right. 
\tag{4.64}
\end{align*}
\]

Define the covariance matrix of the estimation error as \( P \), similarly the covariance matrix of the process noise and the observation noise as below:

\[
\begin{align*}
P(k) &= E[\tilde{x}(k)\tilde{x}^T(k)] \\
Q(k) &= E[\omega(k)\omega^T(k)] \\
R(k) &= E[\nu(k)\nu^T(k)]
\end{align*}
\tag{4.66-4.68}
\]

where \( \tilde{x} = \hat{x} - x \), \( \hat{x} \) is the state estimate vector, \( x \) is the true state vector.

Thus the linear discrete-time Kalman filter procedure follows [51]:

- Start with an initial estimate \( \hat{x}^+_0 \) and it’s accuracy \( P^+_0 \).
- Propagate that estimate and it’s accuracy to time \( t_1 \):

\[
\begin{align*}
\hat{x}_1^- &= \Phi \hat{x}_0^+ + \Gamma u_0 \\
P_1^- &= \Phi P_0^+ \Phi^T + \gamma Q \gamma^T
\end{align*}
\tag{4.69-4.70}
\]

- Begin Kalman Filter loop \( k = 1 \).
- Update Kalman gain

\[
K_k = P_k^- H_k^T [H_k P_k^- H_k^T + R_k]^{-1}
\tag{4.71}
\]

- Update covariance estimate

\[
P_k^+ = [I - K_k H_k] P_k^-
\tag{4.72}
\]

- Take the measurement \( y_k \), find the optimal state estimate

\[
\hat{x}_k^+ = \hat{x}_k^- + K_k (y_k - H_k \hat{x}_k^-)
\tag{4.73}
\]

- Propagate the state and covariance to the next time step

\[
\begin{align*}
\hat{x}_{k+1}^- &= \Phi \hat{x}_k^+ + \Gamma u_k \\
P_{k+1}^- &= \Phi P_k^+ \Phi^T + \gamma Q \gamma^T
\end{align*}
\tag{4.74-4.75}
\]

- Continue loop

### 4.4.2 Linear Quadratic Gaussian (LQG) Control

The combination of Kalman filter and LQR method is referred as linear-quadratic-Gaussian (LQG) control. The LQG method can provide the optimal control for a linear dynamics system which is perturbed by Gaussian noise. Here, Kalman filter for linear system is also referred as linear-quadratic estimator (LQE). A word of caution, LQG optimality does not automatically ensure good robustness properties [52]. This point will be discussed in the next chapter.
4.5 Controllers Implementation

Above all, we have introduced control strategies and the Kalman filter, then the implementation of control is presented. Actually, we have two parallel controllers - LQG and SMC with Kalman filter in Figure 4.3 and Figure 4.4. In the SMC implementation, \( \hat{y} \) is actually the estimation state \( \hat{x} \). Concept of the SMC implementation is shown but not the details.

Above all, the trajectory control methods and state optimal estimation filtering are introduced. Together with the dynamics plant (orbit propagator), the control simulation loop is complete. In the next simulation step, simulator and results will be presented.
5 Simulation

In this chapter, simulation results will be presented. Firstly, the HCW simulator development is introduced; then, scenarios simulation together with the results are shown. In the scenarios simulation, we choose a typical formation style for each formation phase. Among those phases, most attention is payed on the formation keeping scenario in which LQR and SMC are applied, and communication failures are simulated as well. In the end, we combine all the phases to a complete and comprehensive mission with automatic path planning.

The expected results are: on one hand that is the LQR will give the best performance for the linear dynamics compared with the nonlinear method and on the another hand in communication failures, the SMC controller is supposed to weaken the effect of the system uncertainties. The LQR will also perform the optimal automatic path finding.

5.1 HCW Simulator

In this thesis, the HCW simulator is mainly based on the modified HCW equations in which atmospheric drag and \( J_2 \) perturbation are added as disturbances. The linear HCW simulator takes the most simulation work load for its good fidelity as explained above. The overview of the HCW simulator is as follows: The desired trajectory generated by unperturbed HCW equations will be the control input. The difference of the desired state vector and the optimal estimation of state vector given by Kalman filter will be forwarded to the controller (LQR or SMC), thus control force is produced which will go to the propagator for the next time-step state generation. The observed state is the control output.

![HCW simulator function block diagram](image)

5.2 Scenarios Simulation

In all cases of the simulation, the initial condition of Leader/Follower is shown in Table 5.2. We set the satellites formation in the LEO orbit with 600 km altitude, 60° inclination. Due to the close separation between each satellite, for each satellite the local atmospheric density is assumed to be the same, and independent on the longitude variance. Simulation
Table 5.1: Leader’s initial condition

<table>
<thead>
<tr>
<th></th>
<th>$a$ (km)</th>
<th>$e$ ($\circ$)</th>
<th>$i$ ($\circ$)</th>
<th>$\Omega$ ($\circ$)</th>
<th>$\omega$ ($\circ$)</th>
<th>$\nu$ ($\circ$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Leader/Follower</td>
<td>600</td>
<td>60</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.2: Follower mass and other parameters

<table>
<thead>
<tr>
<th></th>
<th>Mass (kg)</th>
<th>$\rho$ (kg/m$^3$)</th>
<th>$S$ ($m^2$)</th>
<th>$C_d$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Follower</td>
<td>50</td>
<td>$1.04 \times 10^{-13}$</td>
<td>0.16</td>
<td>1</td>
</tr>
</tbody>
</table>

5.2.1 Following Formation Initialization

We start with the simplest following formation: at time $t_0$, Leader and Follower are in the initial position of the LVLH coordinates; at time $t_f$, Follower will arrive at the [0, 2000, 0]. In this process, we don’t consider the process noise and observer noise. The measurement is assumed to be perfect. First, the transfer time is set to be $0.5p$ ($p$ is the period of Leader orbit), which in fact is the Hohmann transfer time. The LQR controller is justified to make the position error not greater than 30 m (Figure 5.2). The $y$-axis thrust pulse at the beginning is to accelerate the spacecraft from relative stationary motion, and then the control effort is to cancel the disturbance during the process. Precise position control is easier to achieve in such simple formation style because the controller just focus on one single axis position control if there is no velocity requirement. The parameters of LQR are chosen as: $Q = \text{diag}([1 \ 1 \ 1 \ 1 \ 1 \ 1])$, $R = \text{diag}([1/n^2 \ 1/n^2 \ 1/n^2]) \times a$ ($a = 0.5$) ($n$ is the reference satellite orbit period) based on the engineering experiences [45, 47]. Coefficient $a$ is adjusted to control the accuracy. The advantage of LQR is that it’s really convenient to adjust the parameters for the control accuracy, while in nonlinear control things are becoming quite difficult.

Then, with the transfer time $t_f = 0.1p$ and the same LQR control parameters, more control effort is needed to realize the initialization process. Moreover, the control accuracy decreases if control parameters keeps the same with the previous one (Figure 5.3). The relative trajectory in the LVLH coordinates is shown in Figure 5.4. It indicates that with the LQR controller, Follower can track the pre-designed motion under the perturbations. Obviously the transfer time affects the absolute trajectory and also greatly affects the control impulse output and control accuracy. Thus, the optimal initialization time is between $0.2p$ and $0.8p$ (Figure 5.5), because the control cost for shortening the transfer time is too high when $t_f \leq 0.1 \approx 9\text{min}$ and meanwhile it cost the control accuracy. It is not recommended to initialize the formation in such short time, otherwise the initial state error will accumulate and affect the next formation phase. All these aspects are the trade-off in initialization design. One aspect should be noted that: in following formation initialization, the control parameters can ensure the accuracy we need so that comparisons of different parameters will be included in this section. For formation keeping control in next section, control consumption will be compared with different time and different control parameters (control accuracies).
Figure 5.2: Control accuracy and control consumption ($t_f = 0.5p$)

Figure 5.3: Control accuracy and consumption ($t_f = 0.1p$)
Figure 5.4: Following initialization relative trajectory \( t_f = 0.1p \)

Figure 5.5: Control accuracy/consumption vs. time
5.2.2 Space Circle Formation Keeping with LQR

In this simulation, we will consider the noise and communication failure during formation keeping in which Kalman filter will perform the estimation to help Follower’s tight tracking with Leader. Stochastic noise are added into the formation dynamics to simulate the realistic condition. Because the disturbance has been already in the dynamics, the system noise which represents the periodic disturbance originally now turns out to lose its meaning. The measurement noise is the noise in the observing process.

Initial condition: \( \mathbf{x} = [2000 \ 369.3 \ 3464.1 \ 0.2 \ -4.3 \ 0.3]^T \). LQR controller parameters: \( Q = \text{diag}([1 \ 1 \ 1 \ 1 \ 1 \ 1]) \) \( R = \text{diag}([1/n^2 \ 1/n^2 \ 1/n^2] \ast a) \) (\( a = 12000 \)) to make the position error \(< 300\text{m}\). The setting of the parameters is due to engineering experience [45, 47]. From Figure 5.6 Kalman filter performs good estimation in 3-axis position; otherwise, precise position control would not complete under the existence of noise. What should be emphasized is that we assume the system dynamics are precisely known together with the information of noise. Figure 5.7 shows that control effort in \( z \)-axis is dominant which coincides the relative motion with the direction perpendicular to the reference orbit plane. Extra thrust is needed to perform \( z \)-axis motion and cancel the disturbance. Then we modify control-weighting matrix \( R \), let the control accuracy reach 175 m (Figure 5.9) for the comparison with the previous trajectory (Figure 5.8).

Then we simulate with different formation keeping time \( t = 1p, 2p, 5p, 10p \) while we change the controller parameters to meet different control accuracy. The comparison is illustrated in Figure 5.10. In short-time formation keeping, the relationship between control consumption and accuracy can been roughly seen as a linear function. However, as the

![Figure 5.6: Position Kalman estimation (partial enlargement)](image-url)
time goes by, state error accumulation might cause extra control effort for the long-term formation keeping especially when the control accuracy is limited in 100 m.

Figure 5.7: Control accuracy and consumption

Figure 5.8: Space circle formation relative trajectory (control accuracy 300m)

Figure 5.9: Space circle formation relative trajectory (control accuracy 175m)

Now, communication accidence is supposed to happen in the formation keeping process. During the communication failure, we assume Follower cannot get the control signal from Leader which in our case means the measurement from the observer is totally lost. Control calculation is purely dependent on the state estimation by Kalman filter. In order
to compare with the situation without communication failure, we choose the LQR control parameter which can provide the accuracy of 300 m.

Figure 5.11 depicts that in the beginning of the formation maintain, a communication failure happens with a time-span of about 3000 s. Follower keeps tracking with the desired trajectory with the state estimation provided by Kalman filter. The position displacement in three axes keeps increasing, although extra control effort has been made (Figure 5.12). After the recovery of observer signal, Kalman filter can work for the optimal estimation quickly and effectively and maneuver or relative large thrust is not required for adjustment theoretically. In all, this is the best method during the communication signal lost with a precondition - the system dynamics must be precisely known as well as the information of system noise. In fact, spacecrafts usually carry backups for sensors or communication equipments which can provide reliability sufficient for the mission. Because currently no methods could precisely estimate the real dynamics, long-time communication failure may be a catastrophe for formation flying with short-distance separation.

Figure 5.13 compares the control accuracy and consumption with/without communication failure. With the early communication failure, maximum position error shoots up and its average value is over 60% larger than the value without communication failure. Mean position error different is significant in the early period; while in the later phase of the entire keeping time, both mean position error are almost same. For control consumption, it’s obvious that with communication failure extra control effort is required to keep the control accuracy roughly same as the one without communication failure.
Figure 5.11: Position Kalman estimation under early communication failure (partial enlargement)

Figure 5.12: Control accuracy and consumption under early communication failure
5.2.3 Space Circle Formation Keeping with SMC

Now, under the condition of noise and Kalman filter, SMC is applied with parameters: $C_e = [0.001 \ast \text{diag}([10 \ 50 \ 800]) \ 500 \ast \text{diag}([1 \ 1 \ 1])]$, $\varepsilon = 0.005$, $q = 0.0055$ and $\Delta = 50$. The form of the parameters is from sliding surfaces design. The value is from engineering experience [47]. The setting of $\Delta$ is the boundary control which lets the sliding motion is approached in a certain range without switching too much. Figure 5.13 depicts that the nonlinear controller can also make Follower's tracking with Leader to the desired accuracy. And the huge control pulse is needed when large maneuver takes place. Relative trajectory is shown in Figure 5.15 with 150 m accuracy.

Figure 5.16 depicts that around mean control accuracy 70 m, nonlinear controller does not have the advantage over LQR, and even with the same control effort, SMC is not able to provide better control accuracy considering the SMC parameter might not be the optimal. This is because the dynamics, we are not able to optimize the sliding surface with method provided in [49]. The discussion is: due to our linear dynamic plant and the nature of LQR method analyzed in previous chapter, LQR can perform optimal control under the given control consumption or the given accuracy. Additionally, the LQR method is much easier for design than SMC, especially in the control of accuracy.

However, with communication failure, nonlinear SMC shows its power in governing system uncertainty. Figure 5.17 depicts the position estimation by Kalman filter and Figure 5.18 shows position error is effectively limited under the large uncertainty caused by early communication failure. Figure 5.19 indicates that SMC is more powerful than LQR for the system nonlinear condition. With relative extra 10% consumption, SMC can provide more control accuracy (5% of system scale, 100% of the accuracy of LQR).
Figure 5.14: Control accuracy and consumption

Figure 5.15: Space circle formation keeping trajectory (SMC, accuracy 150 m)

Figure 5.16: Comparison between LQR and SMC
Figure 5.17: Position Kalman estimation under early communication failure (SMC, partial enlargement)

Figure 5.18: Control accuracy and consumption (SMC)
5.2.4 Formation Reconfiguration

Generally, formation reconfiguration is one kind of spacecraft maneuver which is similar to the one in formation initialization. However, usually the initial condition in reconfiguration is more complex with relative velocity and acceleration. Automatic path planning is adapted to find an optimal trajectory between the two distinguished states with LQR controller.

In this simulation, we will find how to reconfigure following formation to $y - z$ formation. First, the following flying state is set as $[0, 5000, 0, 0, 0, 0]^T$ and the $y - z$ formation entry point is set to be $[0, -3324, 0, 1.8, 0, 3.6]$. Noise is also included with Kalman filter. Figure 5.20 and Figure 5.21 show that Follower finds the maneuver path automatically switching from following flying to $y - z$ circle formation. The maneuver time is about $t = 0.4p$. Note that choosing the right “maneuver window” is extremely critical for formation reconfiguration. The ”maneuver window” means that the entry point of the latter formation should be as close as possible to the former formation maneuver point (Figure 5.22). From control theory, it can be explained as finding an optimal initial state error. Moreover, Figure 5.23 and Figure 5.24 compare the initial velocity error’s effect on the formation reconfiguration. Definitely, relatively stationary state is best for maneuver so that the following flying orbit might be the optimal intermediate transfer orbit between two complex formation styles.
Figure 5.20: Reconfiguration control accuracy and consumption

Figure 5.21: Relative trajectory (small initial position error)

Figure 5.22: Relative trajectory (large initial position error)
5.2.5 Formation Flying Mission

Finally, we combine all the above scenarios control methods into a comprehensive formation mission with LQG methods. The mission is divided as four phases: the first is the initialization to $x-y$ ellipse formation; the second is the $x-y$ formation keeping; then, the reconfiguration to $y-z$ circle formation; the last is the $y-z$ formation keeping.

In the simulation, at the very first initial time, Follower and Leader are in the same original place; the initial time and $x-y$ ellipse keeping time are set to be $5.05p$; the recon-
figuration time and $y - z$ circle keeping time are 5$\mu s$ too. The figures below show the control accuracy and consumption (Figure 5.25), entire trajectory (Figure 5.26) and trajectory in each mission phase (Figure 5.27).

Figure 5.26: Mission trajectory

Figure 5.27: Mission phases trajectory
Above all, we present the formation initialization, keeping and reconfiguration and a comprehensive formation mission with the linear and nonlinear control strategies, simulate the communication failure with the solution of Kalman filter.

The summary of the results is:

- In formation initialization, for simple formation the straight line method is recommended for its simplicity and convenience; for complex formation style as in the mission above, automatic path finding is recommended. The best initialization time is within the range of $0.2p \sim 0.8p$.

- For short-time formation, a linear controller is more suitable than a nonlinear controller for the linearized plant.

- When meeting system uncertainties, a nonlinear control is necessarily demanded, meanwhile filtering strategy such as Kalman filter should also be employed to the state estimation.

- In the mission, the connection between two different phases is really important in order to provide less control consumption and better control accuracy. “Maneuver window” suggests that we should try to make the starting point of each phases as close as possible.
6 Conclusions and Future Work

6.1 Conclusions

The paper focuses on formation control issues based on the Leader/Follower architecture. However, the selection of the orbit propagator is also very important as explained in the chapter of introduction. For these reasons, the works of this master project is summarized as follows:

- The propagator based on the modified HCW Equations was more suitable for theoretical analysis of formation flying. It was simpler for implementation, easier for debugging and the most important is that it’s had good fidelity in short-term (less than one day) simulation which could fully satisfy the demands of trials of different control strategies.

- In the LVLH, straight line path planning was proper for simple formation initialization such as following formation, parallel formation. When its transfer time equals $0.5p$, it turned out to be Hohmann transfer. For complex formation path planning, automatic transfer is suggested, which with LQR can find the fuel-optimal path in given transfer time. Moreover, a transition orbit lying in the following orbit with relative stationary state was a good choice for maneuver.

- In short-term formation flying, if there existed a motion component in the direction perpendicular to the nominal orbit plane, then control effort from a linear controller dominated in this direction; while the situation of nonlinear controller was different. Taking SMC for example, sliding motion enforced the motion from the current state to the desired state by nonlinear method, and it could distribute the control effort in three axes directions equally.

- Nonlinear control strategy didn’t show its advantage over linear dynamics system, especially when LQR has provided the optimal control. When the two aspects-accuracy and control consumption needed to be balanced, it was really hard to adjust the nonlinear controller parameters for an optimal solution. Things became straightforward when LQR was applied.

- When large uncertainties took place in the dynamics system, SMC shows its great superior in reducing the effect of uncertainties. This was proved in the communication failure situation, especially in the early stage of formation when Kalman filter did not acquire the entire observation information.

- For a comprehensive formation mission, it’s been proven that well-designed reconfiguration/maneuver path would affect the entire performance of control. Consequently, trade-off between two formation style should be made in the mission analysis.
6.2 Future Work

Some open points emerged in the research of this project.

- Finding a propagator for long-term deep-space simulation. Unfortunately, at present only numerical solution can provide long-term high-fidelity simulation fit for deep-space formation mission.

- The COE propagator is sensitive to the initial condition provided by the transform from the LVLH coordinates to the ECI coordinates. Currently, how to provide the accurate ECI position and velocity still needs exploration.

- Research on the formation flying based on the non-circular reference orbit is becoming the focus. In such condition, dynamic system is no longer linear, in which finding a fuel-optimal path is not easy.

- Formation with multi-followers and the adaptive collision-avoidance control are also the future work.
Appendix A

Perturbation Functions Based on Classical Orbital Elements

\[
\frac{da}{dt} = \frac{2}{n\sqrt{1 - e^2}}[f_x e \sin \nu + f_y (1 + e \cos \nu)] \quad (A.1)
\]

\[
\frac{de}{dt} = \frac{\sqrt{1 - e^2}}{na}[f_x \sin \nu + f_y (\cos E + \cos \nu)] \quad (A.2)
\]

\[
\frac{di}{dt} = \frac{r \cos (\omega + \nu)}{na^2 \sqrt{1 - e^2} f_z} \quad (A.3)
\]

\[
\frac{d\Omega}{dt} = \frac{r \sin (\omega + \nu)}{na^2 \sqrt{1 - e^2} \sin i} f_z \quad (A.4)
\]

\[
\frac{d\omega}{dt} = \frac{\sqrt{1 - e^2}}{nae}[-f_x \cos \nu + f_y \cos \nu \frac{2 + e \cos \nu}{1 + e \cos \nu} \sin \nu] - \cos i \frac{d\Omega}{dt} \quad (A.5)
\]

\[
\frac{dM}{dt} = n - \frac{1 - e^2}{nae}[f_x (\frac{2er}{p} - \cos \nu) + f_y (1 + \frac{r}{p}) \sin \nu] \quad (A.6)
\]

where \(a\), \(e\), \(i\), \(\Omega\), \(\omega\), \(\nu/M/E\) are classical orbital elements; \((f_x, f_y, f_z)\) is the perturbation force vector in the LVLH coordinates; \(r\) is magnitude of the satellite’s position vector in the Keplerian orbit, \(p\) is the semi-latus rectum of the Keplerian ellipse. Note that under the two-body assumption \((f_x, f_y, f_z) = 0\), \(dM/dt = n\).
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